I. CONVENTIONS, DEFINITIONS, AND OPERATORS.

PHASORS

A time harmonic quantity $a\cos(\omega t+\varphi) = a\cos(2\pi ft+\varphi)$ may be written as

$$a\cos(\omega t + \varphi) = \mathrm{Re}[\ A\ \exp(j\omega t)] \qquad (1.1)$$

or

$$a\cos(\omega t + \varphi) = \mathrm{Re}[\ A\exp(-j\omega t)] \qquad (1.2)$$

The complex quantity A is called a *phasor* . The definition according to (1.1) follows the Electrical Engineering convention where $A = a\exp(j\varphi)$. Eq. 1.2 represents the Physics convention where $A = \exp(-j\varphi)$. The quantity $A\exp(j\omega t)$ or $A\exp(-j\omega t)$ is called the *analytic signal* . In this book we will follow the Physics convention when dealing with wave propagation.

COMPLEX PROFILE

We will often represent the cross section of an optical field propagating in the Z-direction, by the (physics convention) phasor E(x,y,z) such that

$$e(x,y,z,t) = \mathrm{Re}[E(x,y,z)\exp(-j\omega t)] \qquad (1.3)$$

where

$$E(x,y,z) = E_e(x,y,z)\exp(jkz) \qquad (1.4)$$

and E_e is called the *complex profile* of the field The constant k is called the propagation constant:

$$k = \omega/c = 2\pi/\lambda \quad (1.5)$$

with c the velocity of light and λ the wavelength.

In (1.4) e and E denote *scalar* electric fields. The word "scalar" means that the electric field has no direction: it is not *polarized* . From Maxwell's equations we know that to be incorrect; a real electric field is a *vector* . Nevertheless, in most the optics we are dealing with, the vector character of the field can be ignored. Strictly speaking, phasors represent *monochromatic* signals only. A signal with a narrow spectrum around ω is called a *quasi-monochromatic* signal. It is often represented by a *time-varying phasor* :

$$a(t)\cos[\omega t + \varphi(t)] = \mathrm{Re}\,[A(t)\exp(j\omega t)] \text{ or } \mathrm{Re}[A(t)\exp(-j\omega t)] \quad (1.6)$$

It is assumed that the time variation in A(t) is slow relative to the *carrier frequency* ω. An example of a time varying phasor is given by $A\exp(-j\Delta\omega t)$ where Δω is an offset frequency.

In communication systems engineering, quasi-monochromatic signals are called bandpasssignals, the time varying phasor is called the *complex envelope* ,and the analytic signal the *pre-envelope* [Ref 1.1]

OPERATORS

For convenience of notation we will use the following *italic* shorthand notations for operators:

F **two- dimensional Fourier transform:**

$F\{E(x,y)\} = A(k_x,k_y)$

F^{-1} two- dimensional inverse Fourier transform:

$F^{-1}\{A(k_x,k_y) = E(x,y)$

$F_X, F_Y, F_X^{-1}, F_Y^{-1}$ one-dimensional Fourier transforms.

If the context is clear, brackets will often be left out:

FF^{-1} E = E FF E(x,y) = E(-x,-y)

C OR * conjugation

$C\,E = E^*$

R or * convolution:

$R\,\{E_1, E_2\} = E_1 * E_2$

V or ⊗ correlation:

$V\,\{E_1,E_2\} = E_1 \otimes E_2$

S ( ) symbol exchange operator:

$S(cf_x)E(x) = E(cf_x)$

ANGLES

The direction of a vector like **k** in Fig. 1.1 will be denoted by azimuth and elevation angles φ and φ' (positive as shown here) or by direction cosine angles φ_x, φ_y and φ_z where

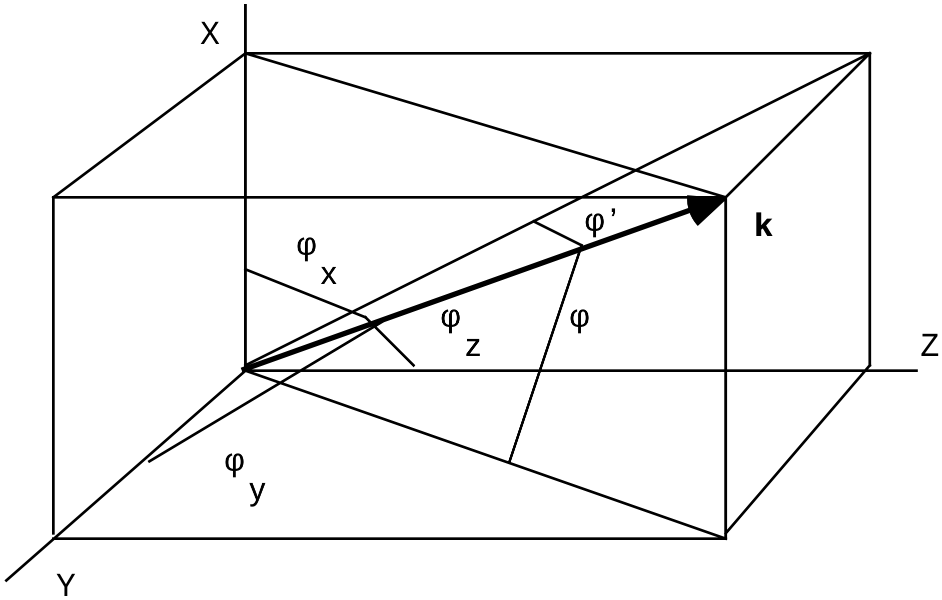

Fig.1.1. Showing direction cosine angles φ_x, φ_y, φ_z , azimuth angle φ and elevation angle φ'

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$$\cos^2\varphi_x + \cos^2\varphi_y + \cos^2\varphi_z = 1 \qquad (1.7)$$

$$\varphi = \pi/2 - \varphi_x, \qquad \varphi' = \pi/2 - \varphi_y \qquad (1.8)$$

INTENSITY

For simplicity we shall assume that in vacuum the intensity (W/m^2) of a propagating field $E(x,y,z)$ is given by $|E(x,y,z)|^2$, i.e we leave out proportionality constants.

NORMALIZATION

Normalized variables will be denoted by an overstrike. Variables are often (but not always) normalized with respect to the wavelength. In such a case $\bar{x} = x/\lambda$, etc. In other cases, variables may be defined with respect to some length L characteristic of the configuration (e.g interaction length or width of a wave guide). Then $\bar{x} = x/L$, and so on.

REFERENCES

1.1 S. Haykin, *Communication Systems* , 3rd ed., Wiley, New York, 1994