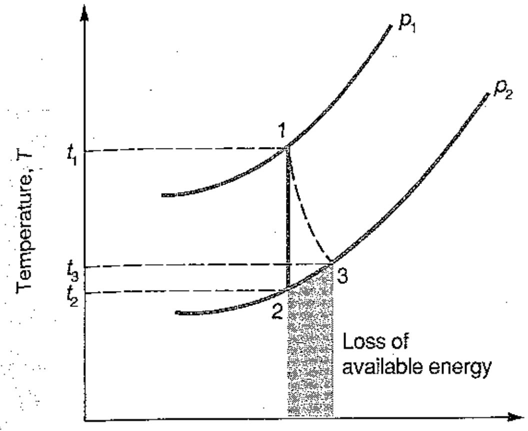


Fig. 11-12

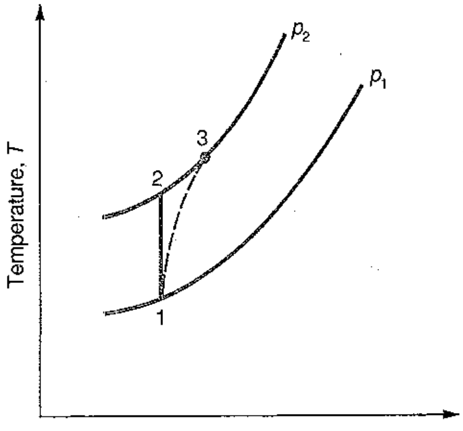


Fig. 11-13

CYCLES

The EIT afternoon problems will usually include a Rankine cycle problem and a vapor compression problem. Typically, there will be others involving a knowledge of the Brayton, Otto or Diesel cycles.

Rankine Cycle (Steam)

An ideal Rankine cycle with superheated steam into the turbine is shown in Fig. 11-14. The four open-system components are analyzed as follows:

a. Boiler

$$Q_{in} = h_2 - h_1 \quad \left(\frac{\text{kJ}}{\text{kg}} \text{ or } \frac{\text{BTU}}{\text{lb}}\right)$$

$$\dot{Q}_{in} = \dot{m}_{stm}(h_2 - h_1) \quad \left(\text{kW or } \frac{\text{BTU}}{\text{hr}}\right)$$

b. Turbine

$$W_T = h_3 - h_2 \text{ kJ/kg or BTU/lb}$$

$$\dot{W}_T = \dot{m}_{stm}(h_3 - h_2)\text{kW or BTU/hr}$$

c. Condenser

$$Q_{out} = h_3 - h_4 = C_p\Delta T \text{ kJ/kg or BTU/lb}$$

$$\dot{Q}_{out} = \dot{m}_{stm}(h_3 - h_4) = \dot{m}_{cw}C_p\Delta T_{cw} \text{ kW or BTU/hr}$$

d. Pump

$$W_p = h_1 - h_4 = v_4\,(p_1 - p_4) \text{ kJ/kg or BTU/lb}$$

$$\dot{W}_p = \dot{m}_{stm}(h_1 - h_4) \text{ kW or BTU/hr}$$

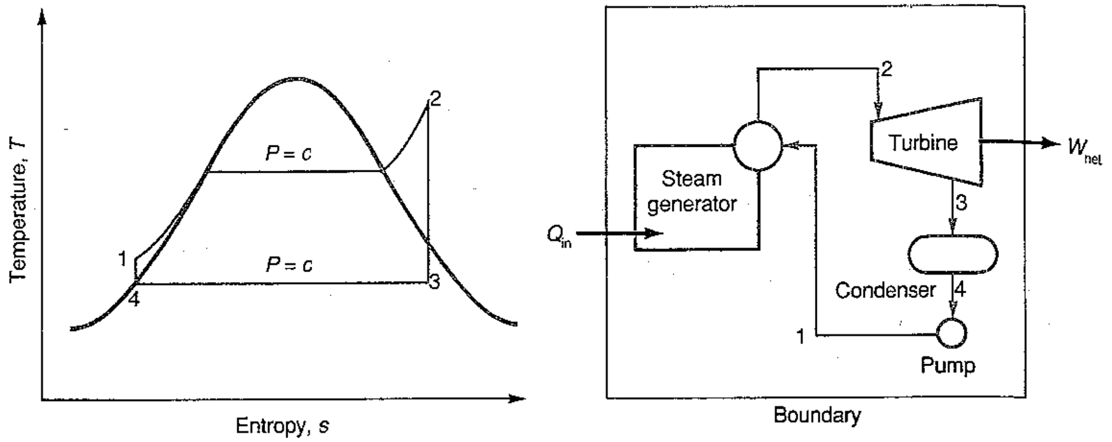


Fig 11-14

State points are found in property (steam) tables and/or a Mollier diagram. State 2 is usually given by pressure and temperature and either can be saturated or (normally) superheated. They can be found in the steam tables or a Mollier diagram. State 3 is found by using an isentropic process, so that entropy (State 2) and pressure (condensing) are known. This turbine expansion is best done on a Mollier diagram. State 4 is saturated liquid found in the tables at the condensing pressure. Also State 1—using an isentropic process—can be found by the tables but is more easily found by

$$h_1 = h_4 + v_4(p_1 - p_4)$$

The thermal efficiency is

$$\eta = \frac{W_T - W_P}{Q_{in}} = \frac{h_2 - h_3 - (h_1 - h_4)}{h_2 - h_1}$$

$$\eta_{approx} = \frac{h_2 - h_3}{h_2 - h_4} \text{ ignoring pump work}$$

Example 9

A Rankine cycle using steam has turbine inlet conditions of $P = 5$ MPa, $T = 500$ °C and a condenser pressure of 50 kPa. The turbine efficiency is 90% and the pump efficiency is 80%.

For both the ideal cycle and the cycle considering the component efficiencies, find a) the thermal efficiency (η), b) the turbine discharge quality (x), and c) the steam flow rate ($\dot{m}$) for 1 MW of net power.

Properties of Water (SI units): Superheated-vapor Table

v, cm³/g; u, kJ/kg; h, kJ/kg; s, kJ/(kg)(°K)

Temp., °C	v	u	h	s	v	u	h	s
	40 bars (4.0 MPa)(T_{sat} = 150.40 °C)				60 bars (6.0 MPa)(T_{sat} = 275.64 °C)			
500	86.43	3099.5	3445.3	7.0901	56.65	3082.2	3422.2	6.8803

Properties of Saturated Water (SI units): Pressure Table

v, ft³/lb; u and h, BTU/lb; s, BTU/(lb)(°R)

		Specific volume		Internal energy		Enthalpy			Entropy	
Press. bars P	Temp. °C T	Sat. liquid v_f	Sat. vapor v_g	Sat. liquid u_f	Sat. vapor u_g	Sat. liquid h_f	Evap. h_{fg}	Sat. vapor h_g	Sat. liquid s_f	Sat. vapor s_g
0.50	81.33	1.0300	3240	340.44	2483.9	340.49	2305.4	2645.9	1.0910	7.5939

Solution

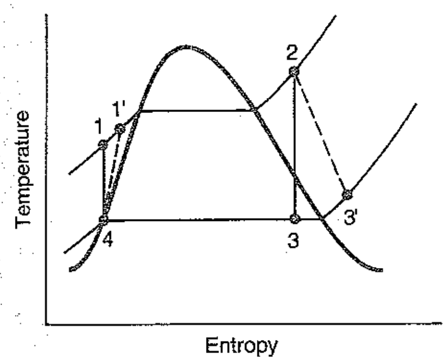
Starting with State 2 (turbine inlet), the properties (h and s) can be found either with steam tables or Mollier diagram

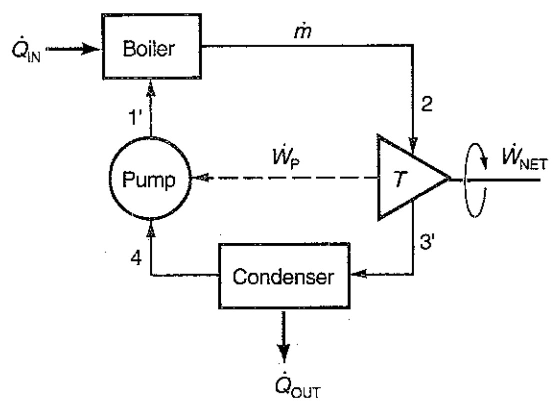
$$h_2 = 3434\frac{\text{kJ}}{\text{kg}}, \quad s_2 = 6.976\frac{\text{kJ}}{\text{kg} \bullet {}^\circ\text{K}}$$

The ideal turbine discharge (State 3) is found at $P = 50$ kPa and $s = 6.976$:

$$x = \frac{s - s_f}{s_{fg}} = \frac{6.976 - 1.091}{6.503} = 0.905$$

$$h_3 = h_f + xh_{fg} = 340.5 + (0.905)(2305) = 2426 \text{ kJ/kg}$$





Thermodynamics

Or these values can be read with less accuracy from the Mollier diagram.

The saturated liquid state leaving the condenser (State 4) is read from the tables:

$$h_4 = 340.5 \text{ kJ/kg}$$

$$s_4 = 1.091 \frac{\text{kJ}}{\text{kg} \bullet {}^\circ\text{K}}$$

$$v_4 = 0.00103 \text{ m}^3\text{/kg}$$

The compressed (subcooled) liquid leaving the pump (State 1) can be found in the tables if values are available for the condition. The usual approximation is to calculate

$$\begin{aligned} h_1 &= h_4 + v_4(p_4 - p_1) \\ &= 340.5 + 0.00103(5000 - 50) = 340.5 + 5.1 \\ &= 345.6 \text{ kJ/kg} \end{aligned}$$

For the ideal cycle

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{W_T - W_P}{Q_{in}} = \frac{(3434 - 2426) - 5.1}{3434 - 345.6}$$

$$= \frac{1008 - 5.1}{3088.4} = \frac{1002.9}{3088.4} = 0.325$$

$$x = 0.905$$

$$\dot{m} = \frac{1000 \text{ kW}}{W_{net}} = \frac{1000}{1002.9} = 1.0 \frac{\text{kg}}{\text{s}}$$

For the cycle considering the component efficiencies,

$$\eta_{turb} = \frac{h_2 - h_{3'}}{h_2 - h_3} \quad h_{3'} = 3434 - 0.9(3434 - 2426) = 2527 \text{ kJ/kg}; \quad h_{3'} = 2527 \text{ kJ/kg}, \quad P = 50 \text{ kPa}$$

$$x = \frac{h_{3'} - h_f}{h_{fg}} = \frac{2527 - 340.5}{2305.4} = 0.95$$

$$\eta = \frac{W_T - W_P}{Q_{in}} = \frac{h_2 - h_{3'} - h_{1'} - h_4}{h_2 - h_{1'}}$$

$$= \frac{(3434 - 2527) - \frac{5.1}{0.8}}{3434 - \left(340.5 + \frac{5.1}{0.8}\right)} = \frac{900.6}{3087} = 0.292$$

$$\dot{m} = \frac{1000}{900.6} = 1.11 \frac{\text{kg}}{\text{s}}$$

The pump work makes little numerical difference and usually can be ignored in the calculation of thermal efficiency and steam flow rate.

Thermodynamics

Vapor Compression Cycle (Refrigeration)

An ideal vapor compression cycle is shown in Fig. 11-15. The four open-system components are analyzed below:

a. Compressor

$$W_{in} = h_2 - h_1$$
$$P_{in} = \dot{m}_R(h_2 - h_1)$$

b. Condenser

$$Q_{out} = h_2 - h_3$$
$$\dot{Q}_{out} = \dot{m}_R(h_2 - h_3)$$

c. Expansion Valve

$$h_3 = h_4$$

d Evaporator

$$Q_{in} = h_1 - h_4$$

$$\dot{Q}_{in} = \dot{m}_R(h_1 - h_4)$$

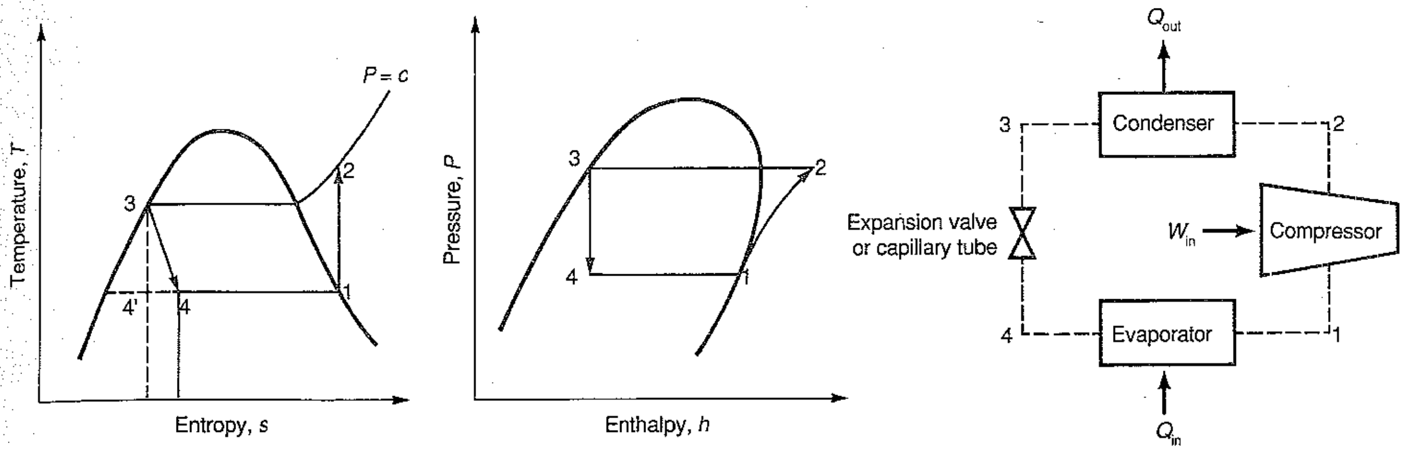


Fig. 11-15

The coefficient of performance (COP), if the cycle, used as a refrigerator, is

$$\mathrm{COP}_{\mathrm{REFR}} = \frac{Q_{in}}{W_{in}} = \frac{h_1 - h_4}{h_2 - h_1}$$

If the cycle is used as a heat pump

$$\mathrm{COP}_{\mathrm{HEAT\,PUMP}} = \frac{Q_{out}}{W_{in}} = \frac{h_2 - h_3}{h_2 - h_1} = \mathrm{COP}_{\mathrm{REFR}} + 1.0$$

The state points are found in property tables and/or property diagrams (P-h). State 1 is usually given as saturated vapor at a given pressure or temperature. State 2 is found by assuming an isentropic process so that the entropy and pressure (or corresponding condensing temperature) are known. This is best done on a P-h diagram. State 3 is saturated liquid at the given condensing pressure. State 4 is found at the same enthalpy as State 3 and at the evaporating pressure.

Thermodynamics

Example 10

An ideal vapor compression refrigeration cycle using R-12 operates between 100 kPa and 1000 kPa. Find the COP and the mass flow rate required for 100 kW of cooling.

Properties of Saturated Refrigerant 12, CCl_2F_2 (SI units): Temperature Table

v, cm^3/g; u, kJ/kg; h, kJ/kg; s, kJ/(kg)(°K)

		Specific volume		Internal energy		Enthalpy			Entropy	
Temp, °C T	Press., bars P	Sat. liquid v_f	Sat. vapor v_g	Sat. liquid u_f	Sat. vapor u_g	Sat. liquid h_f	Evap h_{fg}	Sat. vapor h_g	Sat. liquid s_f	Sat. vapor s_g
−30	1.0041	0.6720	159.38	8.79	158.20	8.86	165.33	174.20	0.0371	0.7170

Properties of Saturated Refrigerant 12, CCl_2F_2 (SI units): Pressure Table

v, cm³/g; u, kJ/kg; h, kJ/kg; s, kJ/(kg)(°K)

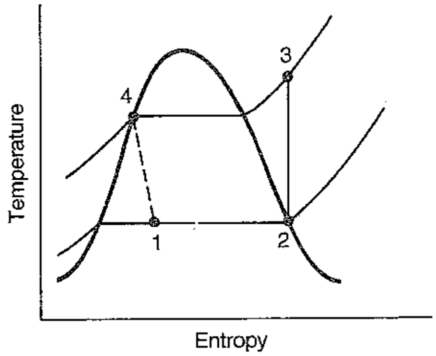
		Specific volume		Internal energy		Enthalpy			Entropy	
Press., bars P	Temp., °C T	Sat. liquid v_f	Sat. vapor v_g	Sat. liquid u_f	Sat. vapor u_g	Sat. liquid h_f	Evap. h_{fg}	Sat. vapor h_g	Sat. liquid s_f	Sat. vapor s_g
10.0	41.64	0.8023	17.44	75.46	186.32	76.26	127.50	203.76	0.2770	0.6820

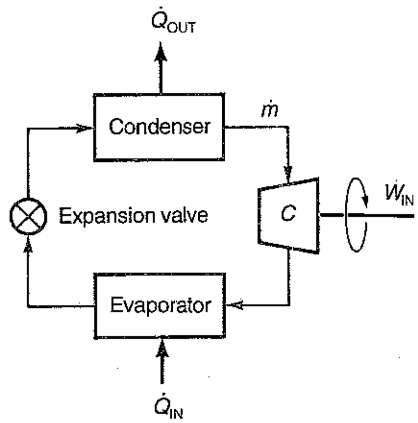
Properties of Superheated Refrigerant 12, CCl_2F_2 (SI units)

v, cm³/g; u and h, kJ/kg; s kJ/(kg)(°K)

Temp., °C	v	u	h	s	v	u	h	s
	10.0 bars (1.0 MPa)(T_{sat} = 41.64 °C)				12.0 bars (1.2 MPa)(T_{sat} = 49.31 °C)			
50	18.37	191.95	210.32	0.7026	14.41	188.95	206.24	0.6799
60	19.41	198.56	217.97	0.7259	14.48	189.43	206.81	0.6816

Solution





Thermodynamics

Refer to the accompanying T-s diagram and a schematic of the components. Properties from the R-12 table for saturated liquid at $T = -40$ °C, $h = 0$ kJ/kg.

State 2 (saturated vapor)

$h_2 = 174.20$ kJ/kg

$s_2 = 0.7170$ kJ/kg • °K

$T_2 = -30$ °C

State 3 ($P_2 = 1000$ kPa)

$s_2 = s_3 = 0.7165$ kJ/kg • °K

$h_3 = 214.9$ kJ/kg

$T_3 = 56$ °C

State 4 (saturated liquid)

$h_4 = 76.15$ kJ/kg

$T_2 = 41.6$ °C

State 1 (liquid + vapor)

$h_1 = h_4 = 76.15$ kJ/kg

$$\mathrm{COP_R}=\frac{\dot{q}_{evap}}{\dot{W}_C}=\frac{h_2-h_1}{h_3-h_2}=\frac{174.08-76.15}{214.9-174.08}=\frac{97.93}{40.82}=2.4$$

$$\dot{q}_{evap}=\dot{m}(h_2-h_1),\quad \dot{m}=\frac{100}{97.93}=1.02\frac{\mathrm{kg}}{\mathrm{s}}$$

Brayton Cycle (Gas Turbine)

An ideal Brayton cycle is shown in Fig 11-16. The four open-system components are analyzed below:

a Compressor

$$W_C=h_2-h_1=C_p(T_2-T_1)\quad \text{(Ideal Gas)}$$
$$\dot{W}_C=\dot{m}(h_2-h_1)$$

b Heater

$$Q_{in}=h_3-h_2=C_p(T_3-T_2)\quad \text{(Ideal Gas)}$$
$$\dot{Q}_{in}=\dot{m}(h_3-h_2)$$

c Turbine

$$W_T=h_3-h_4=C_p(T_3-T_4)\quad \text{(Ideal Gas)}$$
$$\dot{W}_T=\dot{m}(h_3-h_4)$$

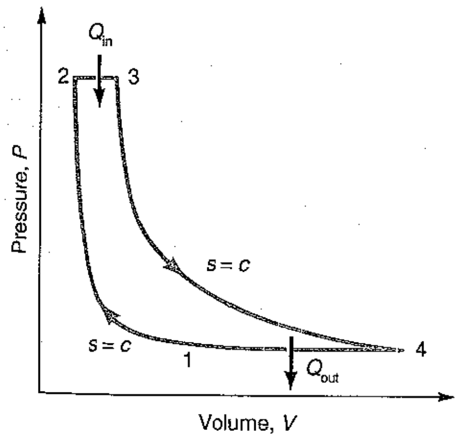
d Cooler

$$Q_{out}=h_4-h_1=C_p(T_4-T_1)\quad \text{(Ideal Gas)}$$
$$\dot{Q}_{out}=\dot{m}(h_4-h_1)$$

The thermal efficiency is

$$\eta=\frac{W_{net}}{Q_{in}}=\frac{W_T-W_C}{Q_{in}}=\frac{h_3-h_4-(h_2-h_1)}{h_3-h_2}$$

Thermodynamics



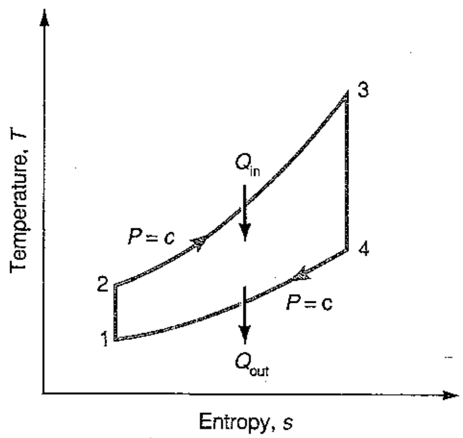


Fig 11-16

For an ideal gas

$$\eta = \frac{T_3 - T_4 - (T_2 - T_1)}{T_3 - T_2} = 1 - \frac{1}{\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}}$$

The state points are found by the ideal gas laws or air (gas) tables. State 3 is usually given by P and T. State 4 is found by an isentropic process; for an ideal gas

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}}$$

If the air tables are used,

$$P_{r_4} = P_{r_3}\left(\frac{P_4}{P_3}\right)$$

State 1 is usually given by T and P. State 2 is found by an isentropic process; for an ideal gas

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

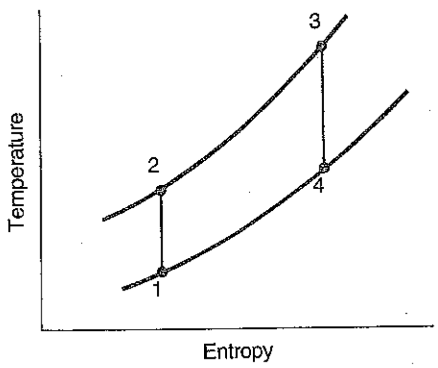
If air tables are used

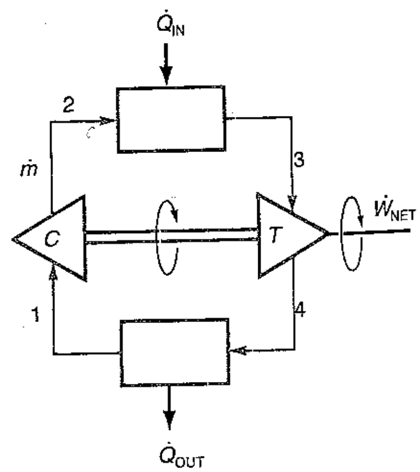
$$P_{r_2} = P_{r_1}\left(\frac{P_2}{P_1}\right)$$

Example 11

An air standard Brayton cycle has air entering the compressor at 27 °C and 100 kPa. The pressure ratio is 10 and the maximum temperature is 1350 °K. Find all state-point properties and the thermal efficiency. The value of k for ideal air is 1.4.

Solution





Refer to the accompanying sketch of the T-s diagram and schematic of components.

State 1

$P_1 = 100$ kPa

$T_1 = 300$ °K

$$v_1 = \frac{RT_1}{P_1} = 0.287 \times \frac{300}{100} = 0.861 \frac{\text{m}^3}{\text{kg}}$$

State 2

$P_2 = 10 \times P_1 = 1000$ kPa

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 300 \times 10^{0.286} = 580\ °\text{K}$$

$$v_2 = \frac{RT_2}{P_2} = \frac{0.287 \times 580}{1000} = 0.166 \frac{\text{m}^3}{\text{kg}}$$

State 3

$P_3 = 1000$ kPa

$T_3 = 1350$ °K

$$v_3 = \frac{RT_3}{P_3} = \frac{0.287 \times 1350}{1000} = 0.38 \frac{\text{m}^3}{\text{kg}}$$

State 4

$P_4 = 100$ kPa

$$T_4 = \frac{T_3}{\left(\frac{P_3}{P_4} \right)^{\frac{k-1}{k}}} = \frac{1350}{10^{0.286}} = 699\ °\text{K}$$

$$v_4 = \frac{RT_4}{P_4} = \frac{0.287 \times 699}{100} = \frac{2.01\ \text{m}^3}{\text{kg}}$$

$$\eta_{TH} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{|\dot{W}_T| - |\dot{W}_C|}{\dot{Q}_{in}} = \frac{\dot{m}(h_3 - h_4) - \dot{m}(h_2 - h_1)}{\dot{m}(h_3 - h_2)}$$

$$= \frac{C_p(T_3 - T_4) - C_p(T_2 - T_1)}{C_p(T_3 - T_2)} = \frac{(1350 - 699) - (580 - 300)}{1350 - 580}$$

$$= \frac{651 - 280}{770} = \frac{371}{770} = 0.482$$

Check: ($r_p \equiv$ pressure ratio: $p_1/p_2 = p_3/p_4$)

$$\eta_{TH} = 1 - \frac{1}{r_p^{\frac{k-1}{k}}} = 1 - \frac{1}{10^{0.286}} = 0.482$$

Thermodynamics

Otto Cycle (Gasoline Engine)

An ideal Otto Cycle is shown in Fig. 11-17. It consists of the following four processes:

1. An isentropic compression for 1–2.
2. A constant volume heat addition from 2–3.
3. An isentropic expansion from 3–4.
4. A constant volume heat rejection from 4–1.

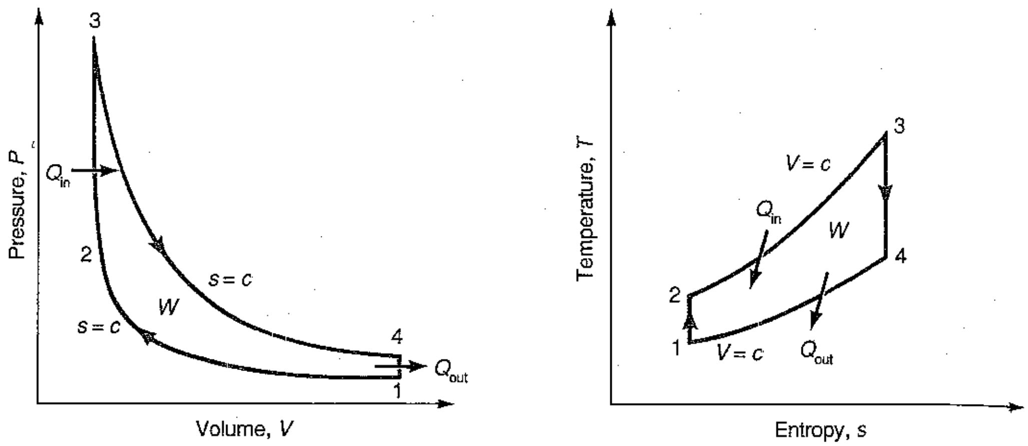


Fig 11-17

The Otto Cycle is an Air Standard Cycle, that is, one which uses ideal air as the working media and has ideal processes. An equipment sketch would consist only of a piston and cylinder, since it is a closed system cycle using a fixed quantity of mass. The four closed-system processes reduce to:

1. Isentropic compression

$$u_1 + q = u_2 - W$$
$$q = 0$$
$$W_{comp} = u_2 - u_1 = C_v(T_2 - T_1)$$

2. Heat addition

$$u_2 + Q_{in} = u_3 + W$$
$$W = 0$$
$$Q_{in} = u_3 - u_2 = C_v(T_3 - T_2)$$

3. Isentropic expansion

$$u_3 + q = u_4 + W$$
$$q = 0$$
$$W_{exp} = u_3 - u_4 = C_v(T_3 - T_4)$$

4. Heat rejection

$$u_4 + Q_{\text{out}} = u_1 + W$$
$$W = 0$$
$$Q_{\text{out}} = u_1 - u_4$$

The thermal efficiency is

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{W_{exp} - W_{comp}}{Q_{in}} = \frac{u_3 - u_4 - (u_2 - u_1)}{u_3 - u_2} = \frac{T_3 - T_4 - (T_2 - T_1)}{T_3 - T_2} = 1 - \frac{1}{r_c^{\,k-1}}$$

Note that r_c is the compression ratio, a ratio of the *volume* at the bottom of the piston stroke (bottom dead center) to the *volume* of the top of the stroke (top dead center). This is also equal to v_1/v_2

The state points are found by ideal gas laws or air tables: State 1 is usually given by T and P. State 2 is found by using an isentropic process; for an ideal gas

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = r_c^{\,k-1}$$

If air tables are used,

$$\frac{v_{r_2}}{v_{r_1}} = \frac{v_2}{v_1} = \frac{1}{r_c}$$

State 3 is usually found by knowing the heat addition:

$$Q_{in} = u_3 - u_2 = C_v(T_3 - T_2)$$

$$T_3 = \frac{q}{C_v} + T_2$$

State 4 is found by an isentropic process; for an ideal gas

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{k-1} = r_c^{\,k-1}$$

$$T_4 = \frac{T_3}{r_c^{\,k-1}}$$

With air tables one can use

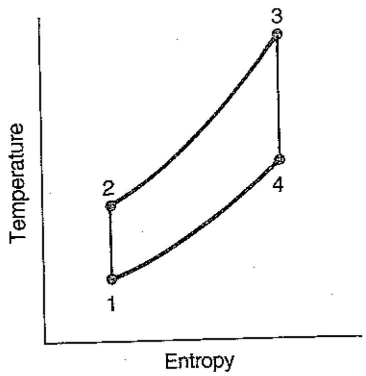
$$\frac{v_{r_3}}{v_{r_4}} = \frac{v_3}{v_4} = \frac{1}{r_c}$$

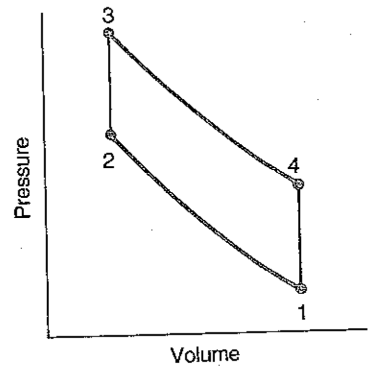
Thermodynamics

Example 12

An engine operates on an Air Standard Otto cycle with a temperature and pressure at the beginning of compression of 27 °C and 100 kPa, respectively. The compression ratio is 8.0, and the heat added is 1840 kJ/kg. Find the state point properties and the thermal efficiency. The properties for ideal air are $C_p = 1.0047$ kJ/kg-°K, $C_v = 0.717$ kJ/kg-°K, $R = 0.287$ kJ/kg-°K, and $k = 1.4$.

Solution





Refer to the sketch of the T-s diagram and P-V diagram

State 1

$P_1 = 100$ kPa

$T_1 = 300$ °K

$$v_1 = \frac{RT_1}{P_1} = 0.287 \times \frac{300}{100} = 0.861 \text{ m}^3/\text{kg}$$

State 2

$$v_2 = \frac{v_1}{10} = 0.0861 \text{ m}^3\text{kg}$$

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = 300 \times (8.0)^{0.4} = 689 \text{ °K}$$

$$P_2 = P_1\left(\frac{v_1}{v_2}\right)^{k} = 100 \times (8.0)^{1.4} = 1838 \text{ kPa}$$

State 3

$v_3 = v_2 = 0.0861$ m³/kg

$u_3 = u_2 + Q_{in}$

$Q_{in} = u_3 - u_2 = C_v(T_3 - T_2)$

$$T_3 = \frac{1840}{0.717} + 689 = 3255 \text{ °K}$$

$$P_3 = P_2\left(\frac{T_3}{T_2}\right) = 1838 \times \frac{3255}{689} = 8683 \text{ kPa}$$

State 4

$v_4 = v_1 = 0.861$ m³/kg

$$T_4 = \frac{T_3}{r_c^{\,k-1}} = \frac{3255}{(8.0)^{0.4}} = 1417 \text{ °K}$$

$$P_4 = \frac{P_3}{r_c^{\,k}} = \frac{8676}{(8.0)^{1.4}} = 472 \text{ kPa}$$

$$\eta_{TH} = \frac{W_{net}}{Q_{in}} = \frac{W_{3-4} - W_{1-2}}{Q_{in}} = \frac{C_v(T_3 - T_4) - C_v(T_2 - T_1)}{1840} = \frac{0.717(3255 - 1416) - 0.717(689 - 300)}{1840}$$

$$= \frac{1318 - 279}{1840} = \frac{1039}{1840} = 0.565$$

Check:

$$\eta_{TH} = \frac{Q_{in} - Q_{out}}{Q_{in}} = \frac{1840 - C_v(T_4 - T_1)}{1840} = \frac{1840 - 800.9}{1840} = 0.565$$

$$\eta_{TH} = 1 - \frac{1}{r_c^{\,k-1}} = 1 - \frac{1}{(8.0)^{0.4}} = 0.565$$

Diesel Cycle (Diesel Engine)

An ideal diesel cycle is shown in Fig. 11-18. It consists of the following processes:

1. An isentropic compression from 1–2
2. A constant pressure heat addition from 2–3
3. An isentropic expansion from 3–4
4. A constant volume heat rejection from 4–1

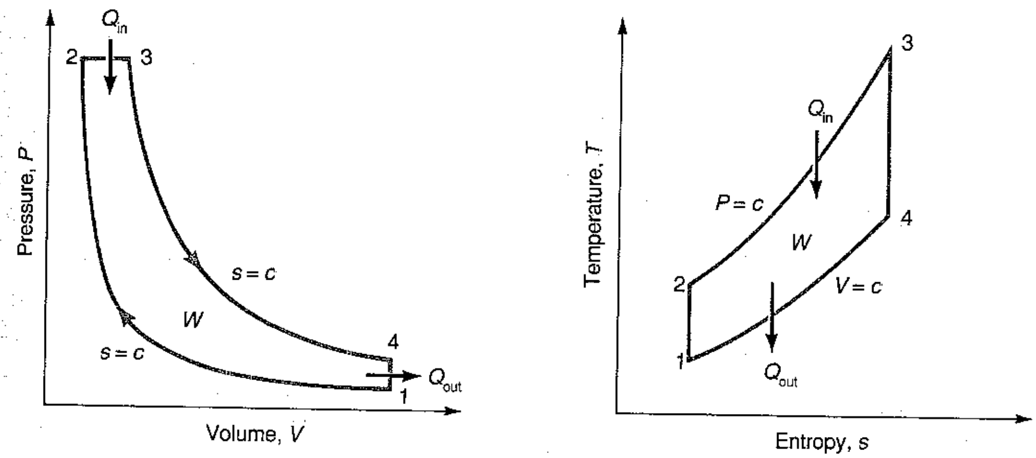


Fig. 11-18

Thermodynamics

The diesel cycle is an "Air Standard" cycle: it uses ideal air as the working medium and has ideal processes. An equipment sketch would consist only of a piston and cylinder since it is a closed system using a fixed quantity of mass. The four closed-system processes reduce to:

1. Isentropic compression

$$u_1 + q = u_2 + W_c$$

$$q = 0$$

$$W_c = u_1 - u_2 = C_v(T_1 - T_2)$$

2. Heat addition

$$u_2 + q = u_3 + W$$

$$W = P_3 v_3 - P_2 v_2 = R(T_3 - T_2)$$

$$Q_{in} = h_3 - h_2 = C_p(T_3 - T_2)$$

3. Isentropic expansion

$$u_3 + q = u_4 + W_E$$

$$q = 0$$

$$W_E = u_3 - u_4 = C_v(T_3 - T_4)$$

4. Heat rejection

$$u_4 + q = u_1 + W$$

$$W = 0$$

$$Q_{out} = u_1 - u_4 = C_v(T_1 - T_4)$$

The thermal efficiency is

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{W_{2-3} + W_{3-4} - W_{2-1}}{Q_{in}} = \frac{R(T_3 - T_2) + C_v(T_3 - T_4) - C_v(T_2 - T_1)}{C_p(T_3 - T_2)}$$

The thermal efficiency is sometimes stated in terms of the compression ratio (r_c) and the "cut-off ratio" (r_{co}). The term $r_{co} = v_3/v_2$ leading to

$$\eta = 1 - \frac{1}{r_c^{k-1}}\left[\frac{r_{co}^{\,k} - 1}{k(r_{co} - 1)}\right]$$

The state points are found by ideal gas laws or air tables State 1 is usually given by T and P State 2 is found by using an isentropic process; for an ideal gas

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = r_c^{\,k-1}$$

If air tables are used

$$\frac{v_{r_2}}{v_{r_1}} = \frac{v_2}{v_1} = \frac{1}{r_c}$$

State 3 is usually found by knowing the heat addition:

$$Q_{in} = h_3 - h_2 = C_p(T_3 - T_2)$$

$$T_3 = \frac{Q_{in}}{C_p} + T_2$$

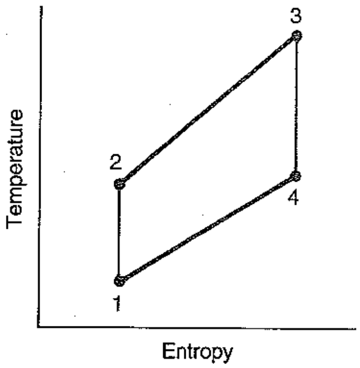
State 4 is found by an isentropic process

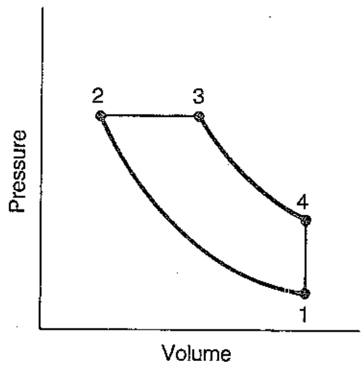
$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{k-1}$$

Example 13

An engine operates on an air standard Diesel cycle with temperature and pressure at the beginning of compression of 300 °K and 100 kPa, respectively. The compression ratio is 16 and the heat added is 1840 kJ/kg. The properties for ideal air are $C_p = 1.0047$ kJ/kg-°K, $C_v = 0.717$ kJ/kg-°K, $R = 0.287$ kJ/kg-°K, and $k = 1.4$.

Solution





Refer to the T-s and P-V diagrams.

State 1

$$p_1 = 100 \text{ kPa}$$

$$T_1 = 300\ °\text{K}$$

$$v_1 = \frac{RT_1}{P_1} = 0.287 \times \frac{300}{100} = 0.861 \frac{\text{m}^3}{\text{kg}}$$

State 2

$$v_2 = \frac{v_1}{r_c} = 0.0538$$

$$T_2 = T_1 \times r_c^{k-1} = 300 \times 160.4 = 909\ °\text{K}$$

$$P_2 = P_1 \times r_c^{k} = 100 \times 161.4 = 4850 \text{ kPa}$$

State 3

$$P_3 = P_2 = 4850 \text{ kPa}$$

$$h_3 = h_2 + Q_{in}$$

$$T_3 = T_2 + \frac{Q_{in}}{C_p} = 909 + \frac{1840}{1.0047} = 2740\ °\text{K}$$

$$v_3 = v_2 \times \frac{T_3}{T_2} = 0.0538 \times \frac{2740}{909} = 0.162 \frac{\text{m}^3}{\text{kg}}$$

State 4

$$v_4 = v_1 = 0.861 \frac{\text{m}^3}{\text{kg}}$$

$$T_4 = \frac{T_3}{\left(\frac{v4}{v3}\right)^{k-1}} = \frac{2740}{\left(\frac{0.861}{0.162}\right)^{0.4}} = 1405\ °\text{K}$$

$$P_4 = \frac{P_3}{\left(\frac{v4}{v3}\right)^{k}} = \frac{4850}{\left(\frac{0.861}{0.162}\right)^{1.4}} = 468 \text{ kPa}$$

$$\eta_{TH} = \frac{W_{net}}{Q_{in}} = \frac{W_{2-3} + W_{3-4} - W_{1-2}}{Q_{in}} = \frac{R(T_3 - T_2) + C_v(T_3 - T_4) - C_v(T_2 - T_1)}{C_p(T_3 - T_2)}$$

$$= \frac{0.287(2740 - 909) + 0.717(2740 - 1405) - 0.717(909 - 300)}{1840}$$

$$= \frac{525.5 + 957.2 - 436.7}{1840} = \frac{1046.0}{1840} = 0.569$$

Check:

$$\eta_{TH} = \frac{Q_{in} - Q_{out}}{Q_{in}} = \frac{1840 - C_v(T_4 - T_1)}{1840} = \frac{1840 - 0.717(1405 - 300)}{1840} = \frac{1048}{1840} = 0.569$$

MISCELLANEOUS

Mixture of Gases

The composition of a closed mixture of gases may be expressed in terms of volume (mol) fractions or mass fractions. These are related through the component molecular weight (m.w.) and are best shown in tabular form. If volume fractions are given, convert to mass fraction:

Gas	Volume fraction	m.w.	Volume fraction × m.w.	Mass fraction
O_2	0.2	32	6.4	$\frac{6.4}{38.4} = 0.167$
N_2	0.2	28	5.6	$\frac{5.6}{38.4} = 0.146$
CO_2	0.6	44	26.4	$\frac{26.4}{38.4} = 0.687$
			38.4	1.000

If mass fraction is given, convert to volume fractions:

Gas	Mass fraction	m.w.	$\frac{\text{Mass fraction}}{\text{m.w.}}$	Volume fraction
O_2	0.1	32	0.00313	$\frac{0.00313}{0.03135} = 0.100$
N_2	0.6	28	0.0214	$\frac{0.0214}{0.03135} = 0.683$
CO_2	0.3	44	0.00682	$\frac{0.00682}{0.03135} = 0.217$
			0.03135	1.000

Thermodynamics

The mass fraction is sometimes called the gravimetic fraction. Component pressure and molecular weight are volume fraction functions; u, h, C_p, C_v, and R are mass fraction functions.

Heat Transfer

The three modes of heat transfer are conduction, convection, and radiation. The heat transfer "laws" are based on both empirical observations and theory but are consistent with the first and second laws of thermodynamics. That is, energy is conserved and heat flows from hot to cold.

Conduction

Conduction occurs in all phases of materials (Fig. 11-19). The equation for one dimensional, planar, steady-state conduction heat transfer is

$$q = k\,A\frac{(T_H - T_C)}{x} \quad \left(\text{watts}, \frac{\text{BTU}}{\text{hr}}\right)$$

The conductivity, k, is a property of the material and is evaluated at the average temperature of the material. The heat flow rate, q, is sometimes expressed as a heat *flux* q/A.

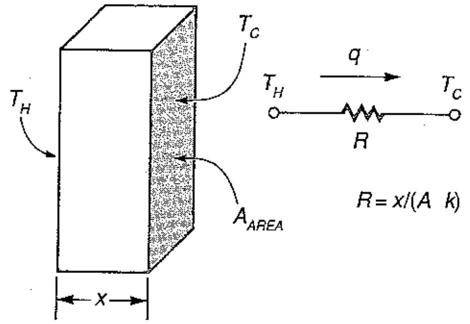


Fig 11-19

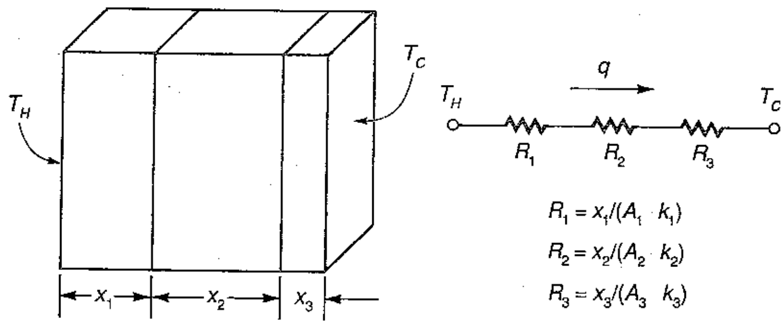


Fig 11-20

For multiple layers of different materials (Fig 11-20), as in composite structures, it is usually best to use an electrical analogy:

$$\dot{q} = q_1 = q_2 = \dot{q}_3$$

$$\dot{q}_1 = \frac{T_H - T_{x_1}}{R_1} \quad R_1 = \frac{x_1}{A_1 k_1}$$

$$\dot{q}_2 = \frac{T_x - T_{x_2}}{R_2} \quad R_2 = \frac{x_2}{A_2 k_2}$$

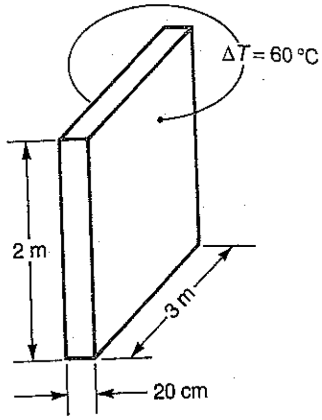
$$\dot{q}_3 = \frac{T_{x_2} - T_c}{R_3} \quad R_3 = \frac{x_3}{A_3 k_3}$$

$$\dot{q}_T = \frac{T_H - T_c}{R_1 + R_2 + R_3}$$

Example 14

A plane wall is 2 m high by 3 m wide and is 20 cm thick. It is made of a material which has a thermal conductivity of 0.5 $W/(m\ °K)$. A temperature difference of 60 °C is imposed on the two large faces. Find the heat flow, the heat flux and the conductive resistance

Solution



Refer to the sketch of the problem.

$$\dot{q} = \frac{kA(T_H - T_C)}{x} = \frac{0.5\times3\times2\times60}{0.20} = 900\ W$$

$$\frac{\dot{q}}{A} = \frac{900}{3\times2} = 150\frac{W}{m^2}$$

$$R = \frac{x}{kA} = \frac{0.2}{0.5\times3\times2} = 0.0667\frac{°K}{W}$$

Convection

Convection occurs at the boundary of a solid and a fluid (liquid or gas) when there is a temperature difference. The mechanism is complex and can be evaluated analytically only for a few simple cases; most situations are evaluated empirically. The equation for convective heat transfer is

$$q = hA\ (T_{surface} - T_{fluid})$$

The evaluation of h, the heat transfer coefficient, normally involves use of data correlated in the form of dimensionless parameters; for example, Nussult Number, Reynolds Number, Prandtl Number.

The conduction and convection mechanisms can be combined as shown in Fig. 11-21. So the temperature of the surface, T_s, is dependent on the relative magnitude of the two resistances.

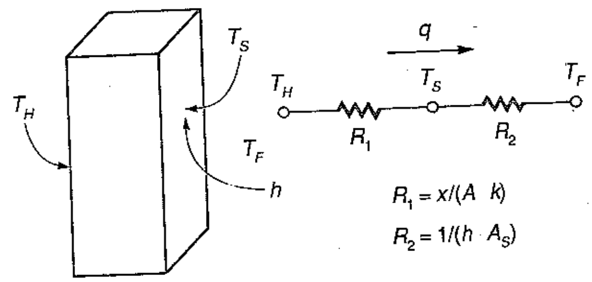
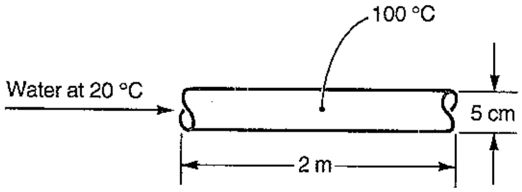


Fig. 11-21

Example 15

Water at an average temperature of 20 °C flows through a 5-cm diameter pipe which is 2 m long. The pipe wall is heated by steam and is held at 100 °C. The convective heat transfer coefficient is $2.2 \times 10^4\ W/(m^2\,°K)$. Find the heat flow, the heat flux and the convective resistance.

Solution



Refer to the sketch of the problem.

$\dot{q} = hA(T_H - T_C) = 2.2 \times 10^4 \times (\pi \times 0.05 \times 2) \times (100 - 20) = 5.53 \times 10^5\ W$

$\dfrac{\dot{q}}{A} = \dfrac{5.53 \times 10^5}{\pi \times 0.05 \times 2} = 1.76 \dfrac{MW}{m^2}$

$R = \dfrac{1}{hA} = \dfrac{1}{2.2 \times 10^4 \times \pi \times 0.05 \times 2} = 1.45 \times 10^{-4} \dfrac{°K}{W}$

Radiation

Radiation heat transfer occurs between two surfaces via electromagnetic waves and *does not* require an intervening medium to permit the energy flow. In fact, it travels best through a vacuum as radiant energy does from the sun. The equation for radiation energy exchange between two surfaces is:

$$q = \sigma A_1 F_e F_s \left(T_1^4 - T_2^4\right)$$

where the Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8} \dfrac{W}{m^2 °K^4}$ or $0.173 \times 10^{-8} \dfrac{BTU}{hr \bullet ft^2 \bullet °R^4}$, F_e is a factor which is a function of the emissivity of the two surfaces with a value from 0 to 1.0, and F_s is a modulus which is a function of the relative geometries of the two surfaces with a value from 0 to 1.0.

Note that the heat flow is not proportional to the linear temperature difference but is a function of the temperature of the surfaces to the fourth power.

The simplest and, by far, the most common case of radiation energy exchange occurs in the case of a small surface radiating to large surroundings. In this case, the equation simplifies to

$$q = \sigma A_1 \in_1 \left(T_1^4 - T_2^4\right)$$

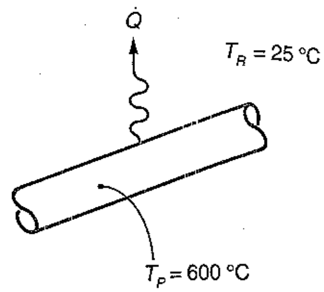
where $\in_1$ is the emissivity of the radiating surface.

Example 16

A steam pipe with a surface area of 5 m^2 and a surface temperature of 600 °C radiates into a large room (which acts as a black body), the surfaces of which are at 25 °C. The pipe gray-body surface emissivity is 0.6. Find the heat flow and heat flux from the surface to the room.

Solution

Refer to the sketch of the problem.



$$\dot{q}_{1-2} = \sigma A\, F_e\, F_s\left(T_1^4 - T_2^4\right)$$

For a gray body radiating to a block body enclosure

$$F_e F_s = \in_1$$

$$\dot{q}_{1-2} = 5.67\times10^{-8}\times5\times0.6\times\left(873^4 - 298^4\right) = 9.75\times10^4\, W$$

$$\frac{\dot{q}}{A} = \frac{9.75\times10^4}{5} = 1.95\times10^4\ \frac{W}{\text{m}^2}$$

Selected Symbols and Abbreviations

Symbol or Abbreviation	Description
C	specific heat
h	enthalpy, heat transfer coefficient
k	thermal conductivity
m	mass
P, p	total pressure, partial pressure
P_r	relative pressure
Q	heat taken in or given off
q	heat (energy transfer), emitted radiation
r_c	compression ratio
R	gas constant
$\overline{\text{R}}$	universal gas constant
R	thermal resistance
s	entropy
T, t	temperature
T_H	High or hot temperature
T_L, T_C	Low temperature, cold temperature
u	internal energy
V_r	relative volume
V, v	Volume
v	specific volume
W, w	Work
Z	compressibility factor
η	thermal efficiency of a heat engine
$\in$	emissivity