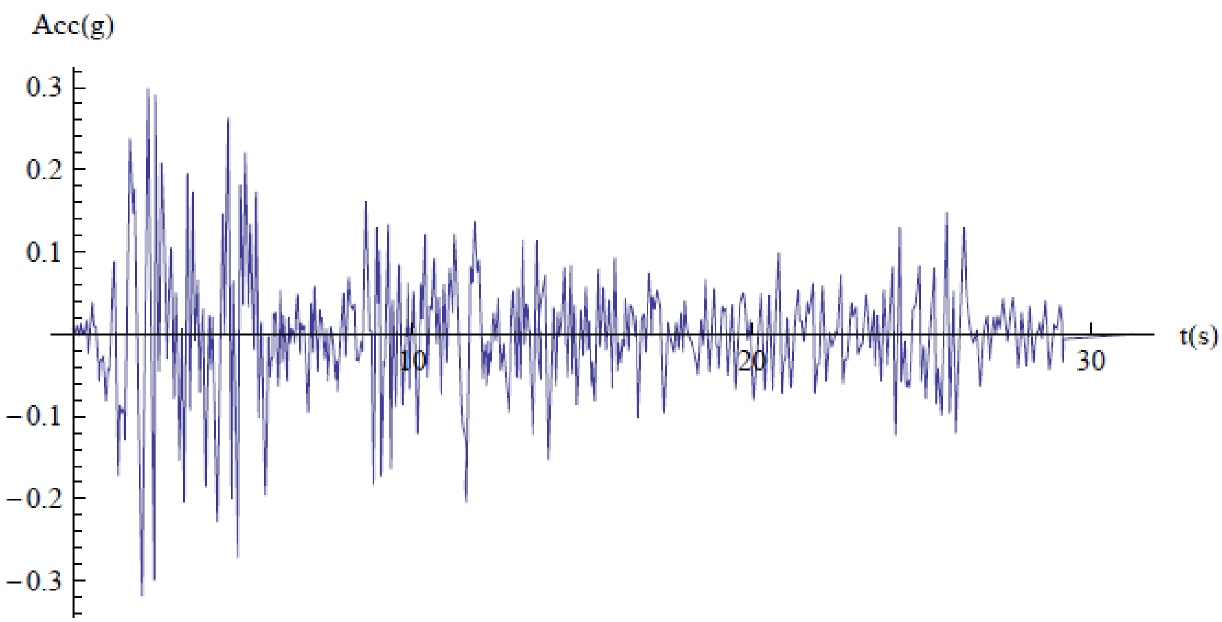
Lecture 18: Earthquake-Response Spectra

Reading materials: Sections 6.4, and 6.5

1. Introduction

The most direct description of an earthquake motion in time domain is provided by accelerograms that are recorded by instruments called *Strong Motion Accelerographs*.

The accelerograph records three orthogonal components of ground acceleration at a certain location.

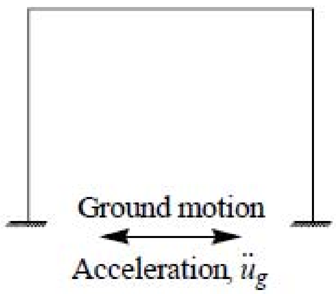


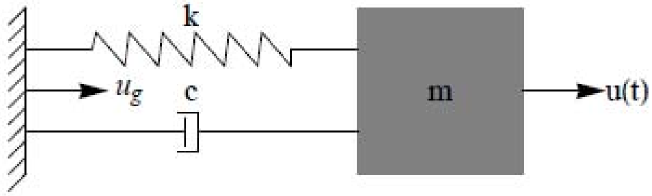
The peak ground acceleration, duration, and frequency content of earthquake can be obtained from an accelerograms. An accelerogram can be integrated to obtain the time variations of the ground velocity and ground displacement.

A response spectrum is used to provide the most descriptive representation of the influence of a given earthquake on a structure or machine.

2. Structures subject to earthquake

It is similar to a vehicle moving on the ground. In both cases there is relative movement between the vibrating system (structures or machines) and the ground.



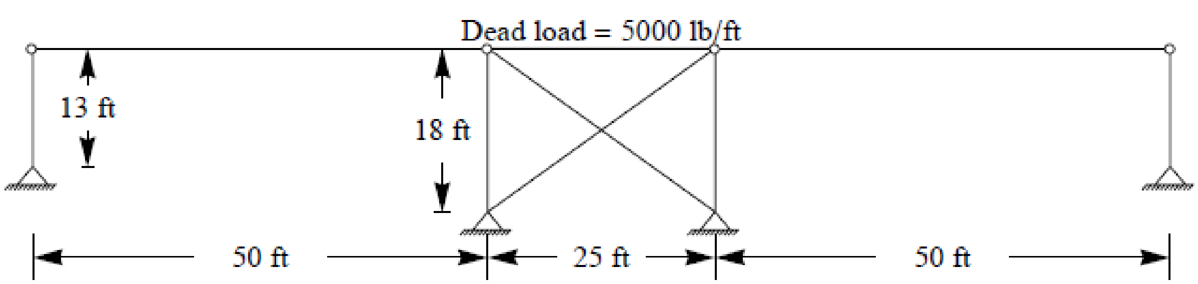


$$m\left(\ddot{u}+\ddot{u}_g\right)+c\,\dot{u}+k\,u=0 \quad \Longrightarrow \quad m\,\ddot{u}+c\,\dot{u}+k\,u=-m\,\ddot{u}_g$$

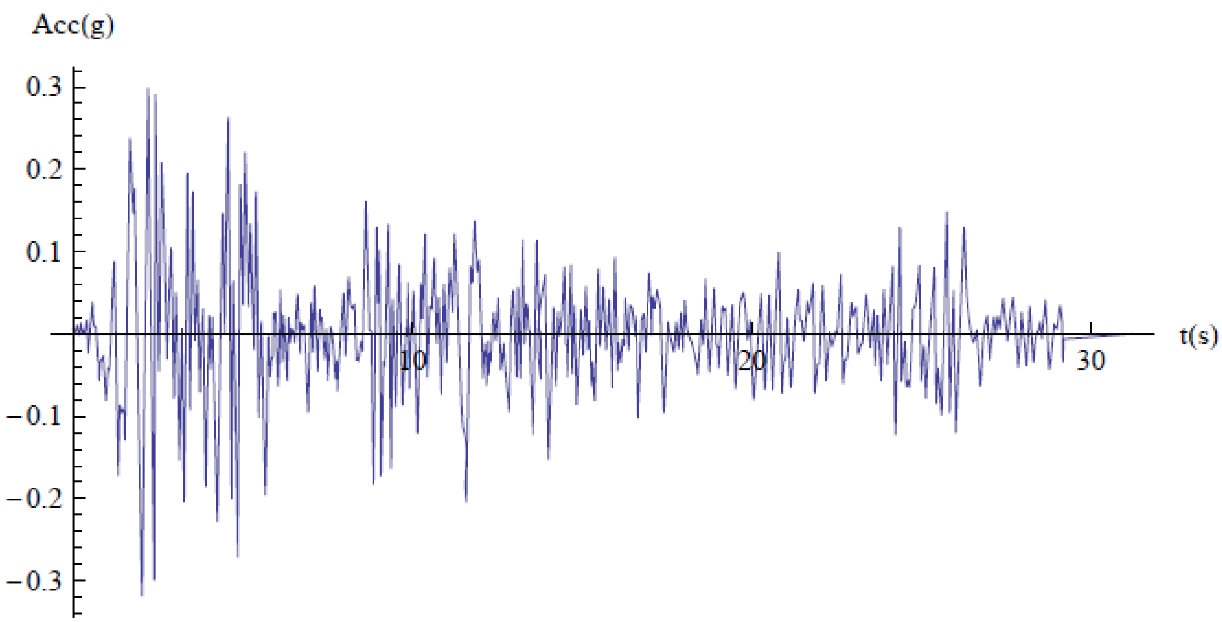
u_g(t) is the ground motion, while u(t) is the motion of the mass relative to ground.

If the ground acceleration from an earthquake is known, the response of the structure can be computed via using the Newmark's method.

Example: determine the following structure's response to the 1940 El Centro earthquake. 2% damping.



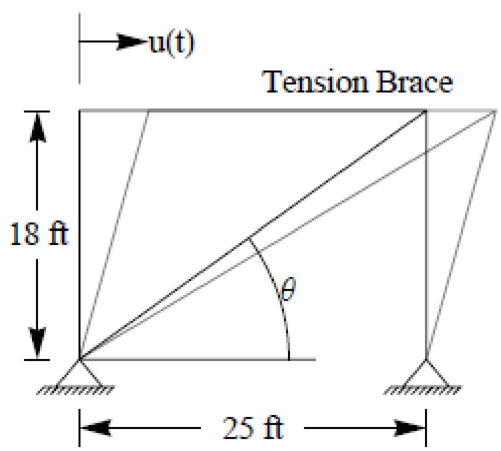
1940 EL Centro, CA earthquake



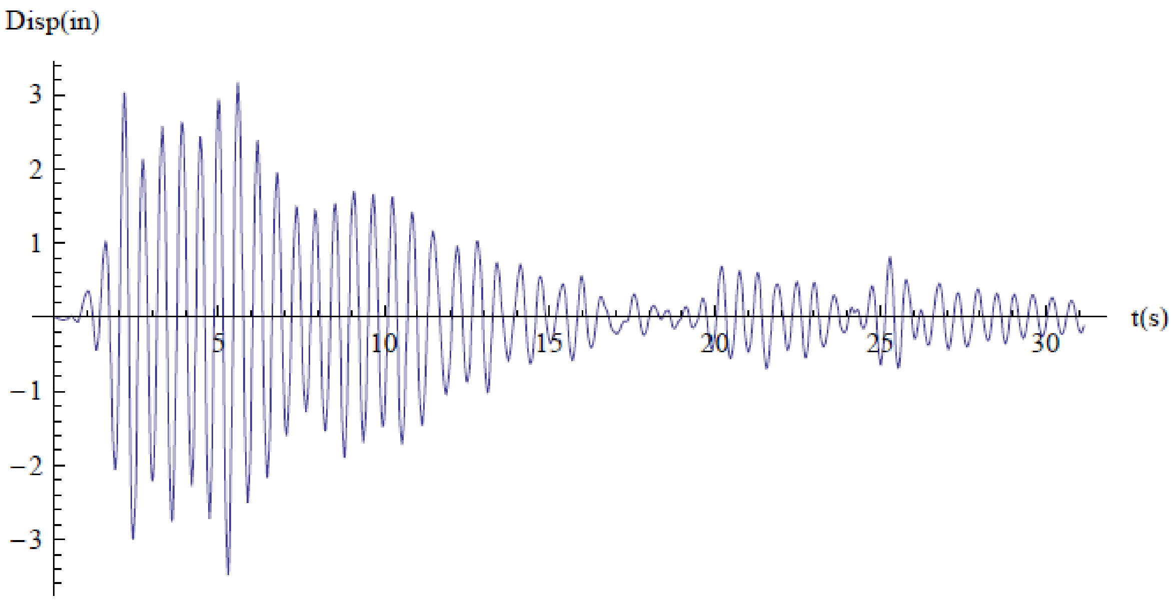
Maximum ground motion values:

Acceleration = 0.319 (g)   Velocity = 14.2 (in/s)   Displacement = 8.35 (in)

Only the bracing members resist the lateral load. Considering only the tension brace.



Neglecting the self weight of the members, the mass of the equivalent spring-mass system is equal to the total dead load.



3. Earthquake design spectra

Given a earthquake ground acceleration, there is no difficulty to compute response using Newmark's method as the previous example. Therefore, we can generate a response spectrum for that earthquake.

We are interested in the maximum relative displacement, velocity, and total accelerations. These quantities are generally referred to as

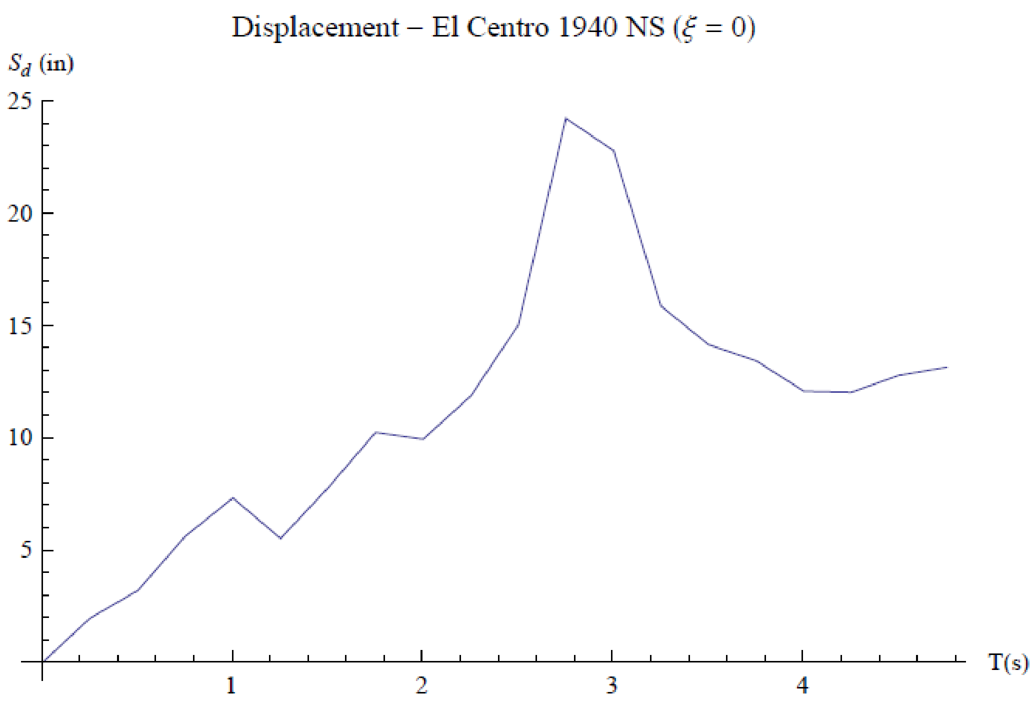
spectral displacement, $S_d = \max_t | u(t) |$

spectral velocity, $S_v = \max_t | v(t) |$

spectral acceleration, $S_a = \max_t | a(t) + a_g(t) |$

Earthquake response spectra are plots of these quantities as a function of undamped natural period vibration

Response spectra for the El Centro 1940 earthquake. (no damping situation)



- Time-histories of ground accelerations from different earthquakes are quite different, the resulting spectra will also be very different.

- We generate earthquake design spectra by averaging spectra from past earthquakes to design structures to resist earthquakes.

- Earthquake Design Spectra

<u>Pseudospectral quantities</u>

spectral displacement, $S_d = \max_t |\, u(t) \,|$

pseudospectral velocity, $S_{pv} = \omega\, S_d$

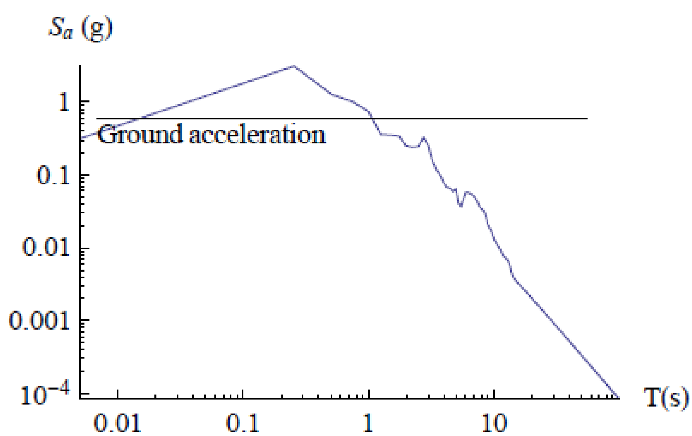
pseudospectral acceleration, $S_{pa} = \omega^2\, S_d$

First obtain pseudospectral velocity S_{pv}

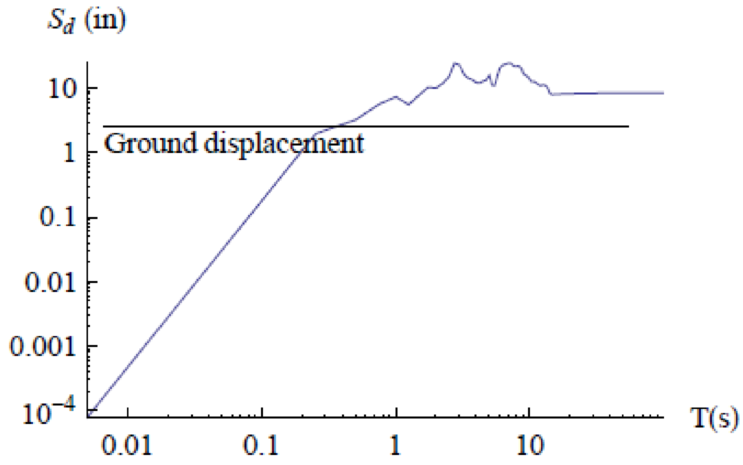
spectral displacement, $S_d = S_{pv} \,/\, \omega$

pseudospectral acceleration, $S_{pa} = \omega\, S_{pv}$

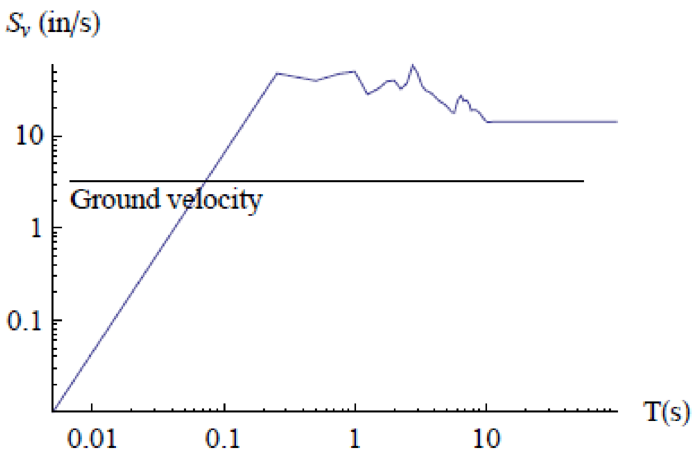
- Spectra for 1940 El Centro



For very low periods, the spectral acceleration plot shows essentially a constant range above the ground acceleration line.



For high periods, the spectral displacement plot shows essentially a constant range above the ground displacement line.



In the intermediate range, the spectral velocity plot shows essentially a constant range above the ground velocity line.

Design Pseudospectral velocity spectrum

Point	Period(s)	S_{pv}
1	.01	$\frac{(0.01)\,\ddot{u}_g}{2\pi}$
2	$1/33$	$\frac{(1/33)\,\ddot{u}_g}{2\pi}$
3	$1/8$	$\frac{(1/8)\,\alpha_A\,\ddot{u}_g}{2\pi}$
4	T_{va}	$\alpha_V\,\dot{u}_g$
5	T_{vd}	$\alpha_V\,\dot{u}_g$
6	10	$\frac{2\pi\,\alpha_D\,u_g}{10}$
7	33	$\frac{2\pi\,u_g}{33}$
8	100	$\frac{2\pi\,u_g}{100}$

$$T_{va} = \frac{2\pi\,\alpha_V\,\dot{u}_g}{\alpha_A\,\ddot{u}_g}$$

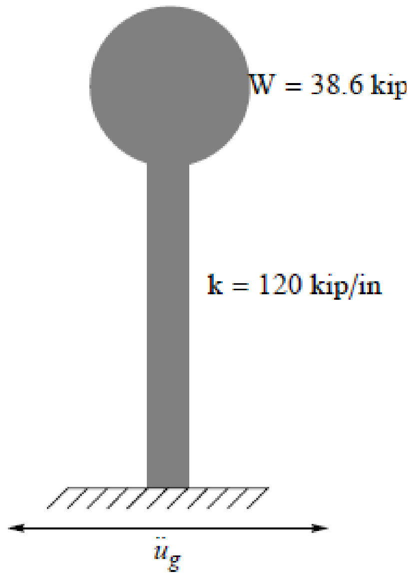
$$T_{vd} = \frac{2\pi\,\alpha_D\,u_g}{\alpha_V\,\dot{u}_g}$$

Amplification factors

- 50% probability of non-exceedance.
  - $\alpha_A = 3.21 - 0.68\ln(100\,\xi)$; $\alpha_V = 2.31 - .41\ln(100\,\xi)$; $\alpha_D = 1.82 - 0.27\ln(100\,\xi)$

Example An 80 ft tall water tower as modeled below is to be designed for a site with median probability of an earthquake similar to the Northridge, CA earthquake of 1994.

Determine design bending moment at the base. (2% damping)


W = 38.6 kip
k = 120 kip/in
$\ddot{u}_g$