Human Performance Measures: Mathematics

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Abstract

This paper presents a number of measures aimed at mathematically quantifying certain characteristics associated with human performance. Such measures have typically relied on comparisons with empirical data collected from thousands of repetitive experiments and many rules of thumb. This paper defines such measures using a rigorous mathematical approach that is easily implemented into computer code. As a result, an ergonomist, a designer, or a clinician will be able to use such measures to evaluate a task where a human is to be involved. Furthermore, the long term goal of this research is to use such functions in an iterative algorithm to optimize ergonomic design (this will be presented in a follow-up report). The measures include reachability, dexterity, joint functionality, ranges of motion, effort, energy, force, and work, and power. Examples are illustrated.

Keywords: Human performance measures, reachability, functionality, dexterity, orientability, force, strength, energy, effort, stress, work

Introduction

Currently, there are over 500 distinct measures of human performance used to evaluate the functionality of the brain, limbs, and other functions. These measures evaluate different performance capacities or structural parameters associated with a specific domain. Some of these measures include the following: Central Processing (e.g., response speed, memory, etc.), Upper Extremity Motor Control (e.g., finger tapping speed, coordination, etc.), Lower Extremity Motor Control (simple reaction time), Isometric Strength (e.g., Neck Functional Units), Postural Stability (Balance), Steadiness (Tremor), Extremes of Motion (Range of Motion, Flexibility), Body Segment Lengths (Anthropometry), Tactile Sensation (Vibratory, Thermal, Electric Current), Speech and Hearing, and Hand Performance (Pinch Strength, Twisting), and many others.

It is evident that human performance measures have been extensively used in the clinical evaluation of patients, however, have only modestly been used in the evaluation of job related tasks. How well is a person qualified to do a certain job that requires significant dexterity? Will this person be subjected to more stress than that person if they were to perform a task that required a significant amount of arm strength? Such questions are currently addressed using empirical data, rules of thumb, and experience.

This paper is an attempt at formalizing the measurement of human performance using well-known rigorous mathematical analysis. The approach taken herein is adapted from the field of robotics where the author has developed a number of measures in the past decade to formalize the selection of robotic equipment for a given task. The work is precursor to a large scale effort to develop an ergonomic design optimization process where the measures developed in this paper will be utilized as cost functions in an iterative algorithm.

The aim is to define an equation that evaluates to a numerical value (a measure of performance). While numerical ratings of some human performance measures have been attempted (Johnson, et al. 1991), there are only a few rigorous measures that are based on sound mathematical formulations. Attempts at quantifying and providing predictive models most of which are statistical in nature have also appeared (Meister 1999 and Boff and Lincoln 1988).

There has been many recent reports addressing models (mostly empirical) for measuring and predicting human performance. For example, a simple calculator model of energy expenditure was developed in 1979 by Danielsson (1979).

Examples of human performance measures are: numerical rating of Dexterity (Johnson, et al. 1991 and Bishu, et al. 1993); human strengths documented in the form of a database (Mital and Kumar 1998), Seating comfort, visibility, and safety for human factors in automotive design (Zhang, et al. 1997), operator performance models for flight simulation were also developed (Jordan, et al. 1996), heart and respiratory rate measures (Nakajima, et al. 1996), human strengths of the trunk (flexion and extension) (Morras, et al. 1990), postural discomfort Corlett and Bishop (1976) and Bhatnager, et al. (1985), and pain (Satow and Taniguchi 1989).

The notion of having a sound rigorous mathematical formulation for human performance measure was first suggested by Kantowitz (1990) in response to an article by Meister, et al. (1990) and Meister (1990). Also on the issue of measurements of human performance measures, Enderwick (1990) discussed some pragmatic issues related to human performance measures.

In addition to the introduction of such qualitative measures for the evaluation of work related tasks, the concept of a measure in ergonomic design has become more pressing in recent years because of the ability to design using digital human simulation.

Modeling

A common modeling scheme typically used in the field of kinematics is adapted. We model human limbs and joints as a series of linkages joined by kinematic pairs. Joints can be composed of one or more degrees of freedom (DOF). For example, the elbow joint shown in Fig. 1 is only one degree of freedom and is modeled as a revolute joint. The wrist joint is a spherical joint modeled as three revolute intersecting joints and has 3DOF.

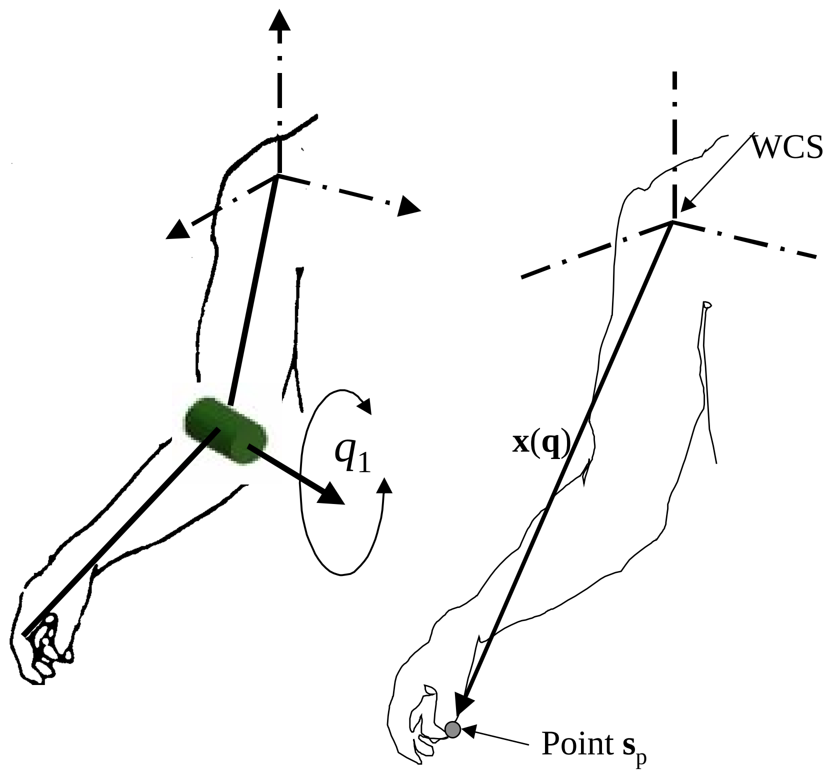

Fig. 1 Modeling of the elbow joint as a 1DOF revolute joint

In order to relate one body part to another in the same serial chain, we shall utilize the well-established Denavit-Hartenberg (DH) representation method, which offers a systematic method for characterizing the motion of a specified point on the hand with respect to a World Coordinate System (WCS) as treated in several books on the kinematics of robotics, e.g., Paul (1981), Asada and Soltine (1986), and Fu, et al. (1987). The method to establish the coordinate system of a link in a serial chain of links was developed by Denavit and Hartenberg in (1955). The method is a systematic approach to describing the rotational and translational relationships among adjacent links in the chain.

The DH representation results in a (4×4) homogeneous transformation matrix representing each link's coordinate system at the joint with respect to the previous link's coordinate system. Through several transformations, the last link can be related to the WCS. The DH notation of a rigid body depends on four geometric parameters associated with each link. These four parameters completely describe any revloute, prismatic, or spherical (three revolute) joint. The four parametrs are defined as follows: (the required notation is illustrated in [Fig. 2].

1. θ_i Joint angle, measured from the $\mathbf{X}_{i-1}$ to the $\mathbf{X}_i$axis about the $\mathbf{Z}_{i-1}$ (right hand rule applies). θ_i is a constant for a prismatic joint.
2. d_i Distance from the origin of the (i-1)th coordinate frame to the intersection of the $\mathbf{Z}_{i-1}$ axis with the $\mathbf{X}_i$ axis along $\mathbf{Z}_{i-1}$ axis, d_i is a constant for a revolute joint.
3. a_i Offset distance from the intersection of the $\mathbf{Z}_{i-1}$ axis with the $\mathbf{X}_i$ axis to the origin of the ith frame along $\mathbf{X}_i$ axis. (Shortest distance between the $\mathbf{Z}_{i-1}$ and Z_i axis).
4. α_i Offset angle from $\mathbf{Z}_{i-1}$axis to Z_i axis about the $\mathbf{X}_i$ axis (right hand rule)

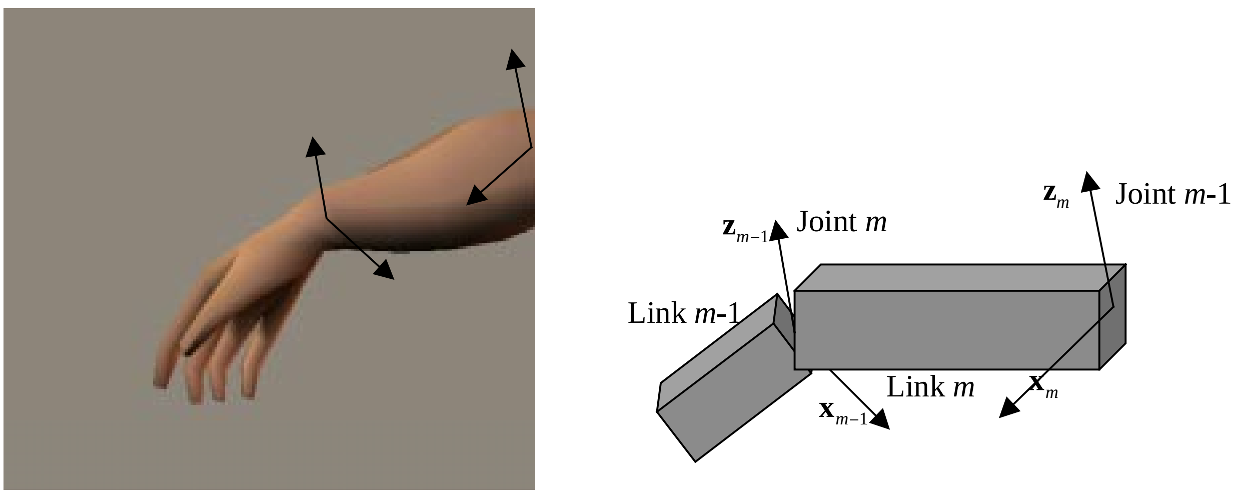

Figure 2: Denavit-Hartenberg Notation

Once the DH parameters have been established for each link in the chain, a homogeneous transformation matrix can be developed relating the ith frame with the (i-1)th frame.

The following successive transformations are performed:

1. Rotate about the $\mathbf{Z}_{i-1}$ axisan angle θ_i to align the $\mathbf{X}_{i-1}$ with the $\mathbf{X}_i$ axis.
2. Translate along the $\mathbf{Z}_{i-1}$ axis a distance d_i to bring the $\mathbf{X}_{i-1}$ and $\mathbf{X}_i$ into coincidence.

3. Translate along the $\mathbf{X}_i$ axis a distance of a_i to bring the two origins as well as the x axis into coincidence.
4. Rotate about the $\mathbf{X}_i$ axis an angle of α_i to bring the two coordinate systems into coincidence.

Each step above is expressed by a basic homogeneous rotation or translation matrix.

The product of these four matrices results in a composite homogeneous transformation matrix

$$^{i-1}\mathbf{A}_i = \mathbf{T}_{z,d}\mathbf{T}_{z,\theta}\mathbf{T}_{x,a}\mathbf{T}_{x,\alpha} \tag{1}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{i-1}\mathbf{A}_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

For a revolute joint (e.g., the elbow joint), the transformation matrix will be only a function of the joint cariable q such as $^{2}\mathbf{A}_3(q_3)$ where $^{2}\mathbf{A}_3$ is the matrix relating link number 2 to link number 3 and q_3 is the angle of joint 3. Following the popular notation adopted by most publications, we represent the homogeneous transformation matrix relating two adjacent links (*m-1*) and *m* as $^{m-1}\mathbf{A}_m$. For any sequence of consecutive **A** matrices, we have

$$^{m-1}\mathbf{A}_m \, ^{m}\mathbf{A}_{m+1} \cdots ^{m+n-1}\mathbf{A}_{m+n} = ^{m-1}\mathbf{T}_{m+n} \tag{3}$$

In particular, for a six axis manipulator, m=1, n=5, the position and orientation of the end-effector with respect to the World Coordinate System (WCS) (The base of the manipulator), is represented by:

$$^{0}\mathbf{A}_1(q_1)\,^{1}\mathbf{A}_2(q_2)\,^{2}\mathbf{A}_3(q_3)\,^{3}\mathbf{A}_4(q_4)\,^{4}\mathbf{A}_5(q_5)\,^{5}\mathbf{A}_6(q_6) = ^{0}\mathbf{T}_6(\mathbf{q}) \tag{4}$$

If $\mathbf{X}^{m-1}$ and $\mathbf{X}^{m}$ are the extended position vector of a point, referred to coordinate frames embedded in link (m-1) and (m) respectively, the relationship between the two vectors is given by

$$\mathbf{x}^{m-1}(\mathbf{q}) = {}^{m-1}\mathbf{T}_m(\mathbf{q})\mathbf{x}^m(\mathbf{q}) \tag{5}$$

Therefore, the vector function describing every point that can be reached within the reach envelope with respect to a World Coordinate System (WCS) is described by $\mathbf{x}(\mathbf{q}) = {}^{0}\mathbf{T}_m{}^{m}\mathbf{x}$, where $\mathbf{q}$ is the vector of joint variables.

Each of the following measures will be explained in detail. The development of a measure is aimed at obtaining a numerical value for a given situation. Each measure will be given a symbol and will evaluate to a numerical value.

Reacheability

For a person at a given location (e.g., sitting), it is required to quantify the level of reachability possible by this person's arm. The reach envelope of a given limb depends on both its dimensions and its ranges of motion (joint limits). If we are able to analytically identify the reach envelope then the volume enclosed by this envelope is indeed a qualitative measure of the reach achievable by this person. In order to develop this measure, we shall use recent results to develop a rigorous mathematical method for delineating the boundary to the workspace generated by all possible motions of a human arm.

In order to illustrate the reasoning behind the analysis, consider the reach envelope of a simple model of an arm (restricted to move on the surface of a desk) as shown in Fig. 3. Note that the reach envelope in this case is only curves because of the planar surface restriction.

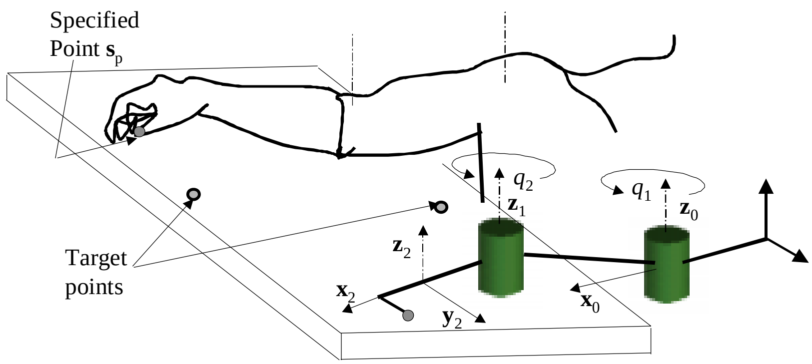

Figure 3 A simple 2DOF model of the arm and its corresponding envelope

In order to quantify the reach, we calculate the area enclosed by the boundary curves which provides a measure for every point that can be reached by this arm. However, to generalize this to 3-dimensions, where the reach envelope is a volume, it is necessary to compute the volume bound by the surfaces.

Example: A 9DOF Model of the Arm

Consider a person with the following dimensions and ranges of motion:

$-90^o \le q_1 \le 90^o$, $-110^o \le q_2 \le 120^o$, $-90^o \le q_3 \le 90^o$, $0^o \le q_4 \le 150^o$, $-60^o \le q_5 \le 60^o$, $-20^o \le q_6 \le 20^o$, $-180^o \le q_7 \le 0^o$, $-L \le q_8 \le L$, and $-L \le q_9 \le L$. The vector function $\mathbf{x}(\mathbf{q}) = \left[x(\mathbf{q}) \quad y(\mathbf{q}) \quad z(\mathbf{q}) \right]^T$ describing every point in the reach envelope is determined as:

$$\begin{aligned} x(\mathbf{q}) = & -20c_1c_3s_2 + 20s_1s_3 + 10c_4(-c_1c_3s_2 + s_1s_3) - 10c_1c_2s_4 + 5(c_6(c_5(c_4(-c_1c_3s_2 \\ & + s_1s_3) - c_1c_2s_4) + (c_3s_1 + c_1s_2s_3)s_5) + (-c_1c_2c_4 - (c_1c_3s_2 + s_1s_3)s_4)s_6 \end{aligned}$$

$$y(\mathbf{q}) = -20c_3s_1s_2 - 20c_1s_3 + 10c_4(-c_3s_1s_2 - c_1s_3) - 10c_2s_1s_4 + 5(c_6(c_5(c_4(-c_3s_1s_2 - c_1s_3) - c_2s_1s_4) + (-c_1c_3 + s_1s_2s_3)s_5) + (-c_2c_4s_1 - (-c_3s_1s_2 - c_1s_3)s_4)s_6$$

$$z(\mathbf{q}) = 20c_2c_3 + 10c_2c_3c_4 - 10s_2s_4 + 5(c_6(c_5(c_2c_3c_4 - s_2s_4) - c_2s_3s_5) + (-c_4s_2 - c_2c_3s_4)s_6)$$

For this 9DOF system (as shown in Fig. 4a), the reach envelope is generates and shown in Fig. 4b.

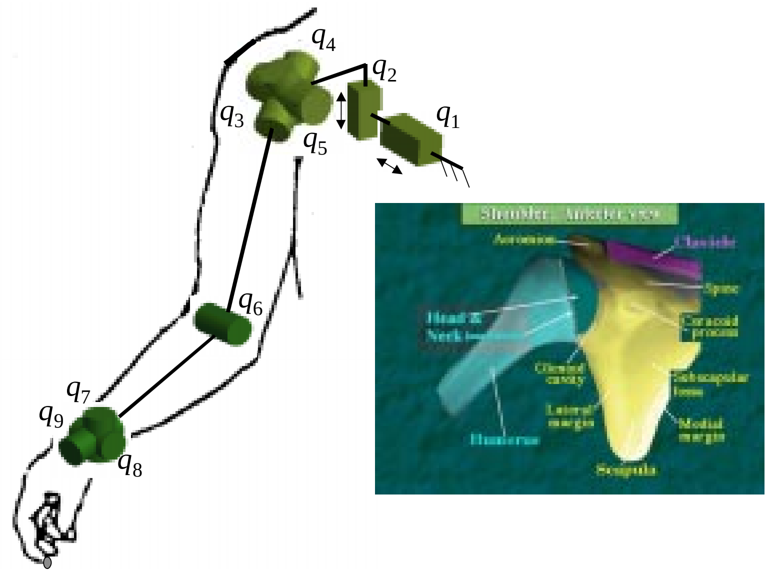

Fig. 4a The 9DOF arm model

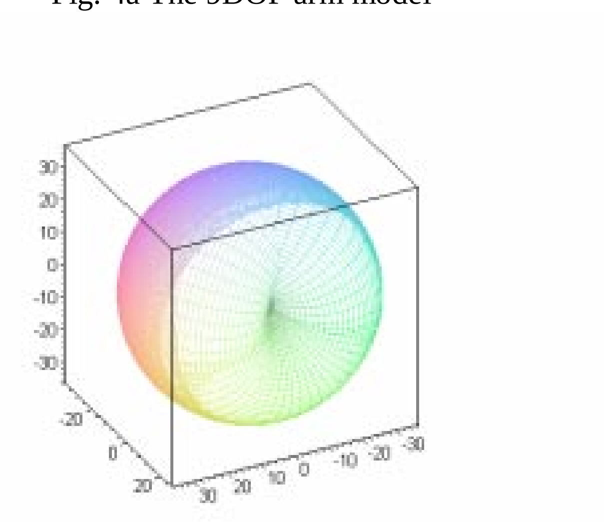

Fig. 4b The reach envelope of the 9DOF arm

In order to obtain a measure for the reach, we calculate the volume of the reach envelope. The total volume is a summation of all enclosures within as

$$\mathbf{V} = \mathbf{V1} \cup \mathbf{V2} \cup \mathbf{V3} \cup \mathbf{V4} \tag{6}$$

where each independent volume segment is calculated as follows:

$$\mathbf{V1} = 4/3\pi R^3 - \iint_{\sigma} R(x,y)\, dA \tag{7}$$

where the function *R(x,y)* is known from the analysis above.

$\mathbf{V2} = 2(2L\pi r^2 - r^2\theta)$; $\mathbf{V3} = r(1+\cos(\theta))(2L)^2$; and $\mathbf{V4} = 2(r^2 \sin 2\theta)L$

Joint functionality

Quantifying the workspace subtended by a given joint is a difficult problem but is of great significance towards the evaluation of joint functionality. To initiate the discussion, consider a simple 1-DOF joint such as the elbow. The motion due to the elbow is a simple arc. If the length of this arc is measured, it provides a measure of the functionality of the elbow joint (as shown in Fig. 5). However, the situation becomes more difficult when the joint's structure increases in sophistication.

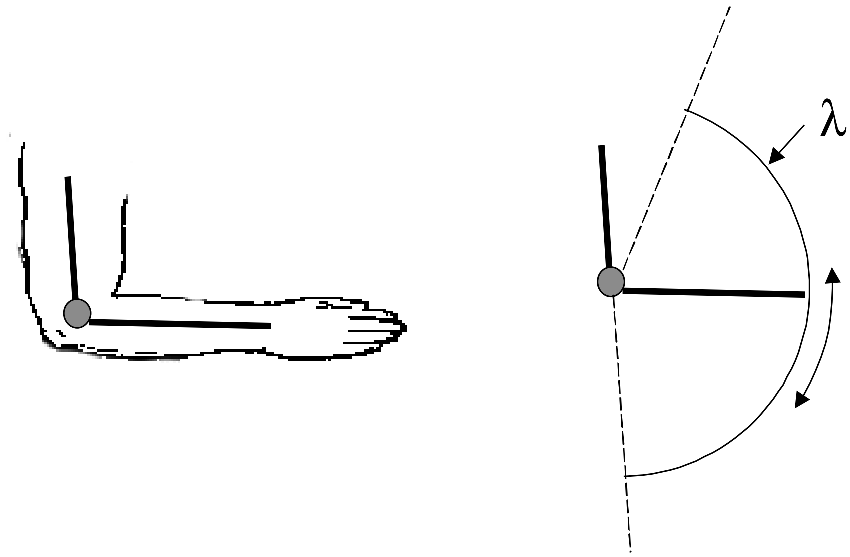

Fig. 5 The range of motion of an elbow

As a direct application of this formulation, it is now possible to visualize the progress of a certain joint via workspace analysis. Note that measurement techniques and devices are well established. However, the range of motion is typically given in terms of a set of numerical joint angle values. The progress is difficult to monitor. We will show in this example that it is now possible to visualize the progress through a series of plots that depict the mobility of the joint (its workspace). Consider for example the wrist joint and hand shown in Fig. 6, where the wrist has been modeled as a 3-DOF system.

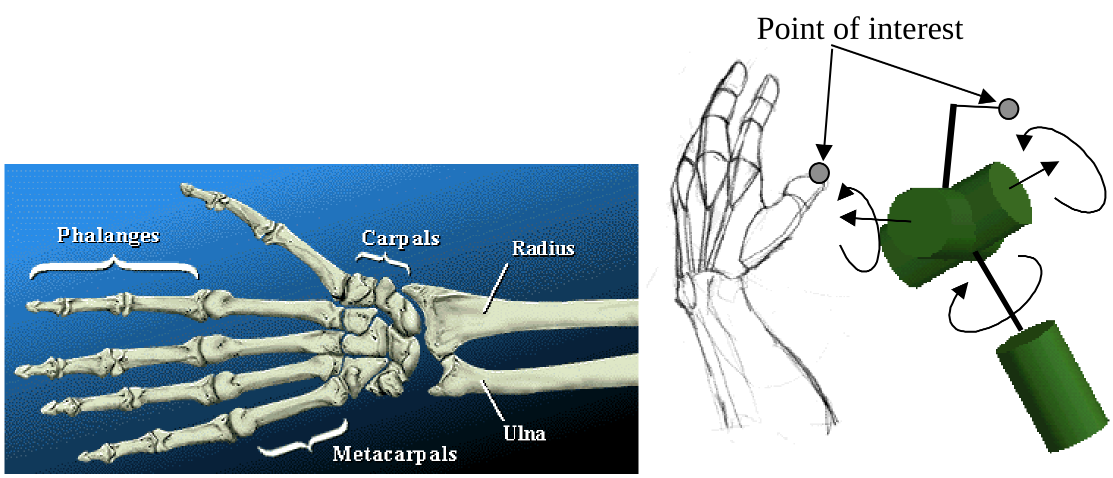

Fig. 6 (a) Wrist and hand (b) Modeling of the wrist joint

For an individual that has had a surgical procedure, the wrist joint motion may take weeks or months to return to normal or may be left with residual restrictions. Progress made, whether due to time alone or physical therapy, is measured using ranges of motion. Using the above formulation, not only visualization of the progress can effectively be made, but an accurate overall number can be used. Indeed, the surface area (or volume if the workspace is a volume), can be used to provide a good estimate. A normal joint range of motion for an adult is $-180^o \le q_1 \le 45^o$, $-70^o \le q_2 \le 80^o$, and $-20^o \le q_3 \le 40^o$, where the initial configuration of the hand is given as horizontal, thumb up, arm extended and away from the body. Using the above formulation, the resulting workspace is indeed a surface (a region of a spherical surface) as shown in Fig. 7.

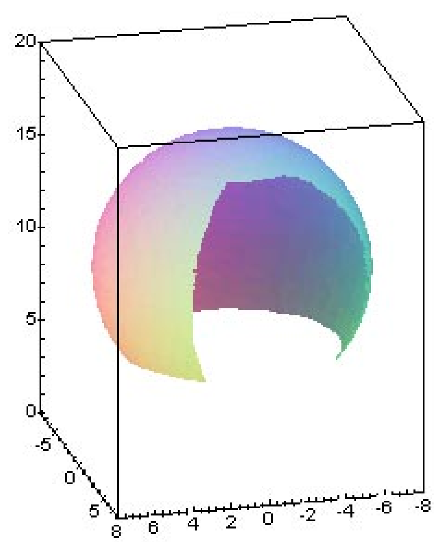

Fig. 7 The workspace of a point on the tip of the thumb with respect to the wrist

For a person who is undergoing physical therapy after a surgical operation, the functionality of the wrist may first be very limited. For example, immediately after the operation, the wrist joints may be limited to $-90^o \le q_1 \le 10^o$, $-30^o \le q_2 \le 30^o$, and $-10^o \le q_3 \le 20^o$, for which the workspace is shown in Fig. 8a. As the joint gains better mobility, the range of motion is increased and the progress is monitored by the visual workspace as shown in Figs. (8b-d), if an accurate measure is needed, the *surface area* obtained by an integration over the surface is performed. Note that this is only possible because of the ability to obtain equations of the boundary.

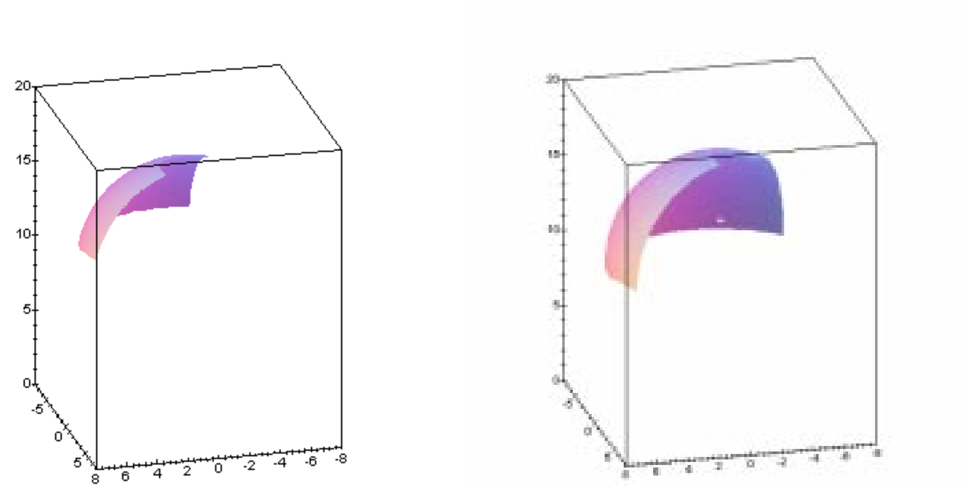

Fig. 18 (a) $-90^o \le q_1 \le 10^o$, $-30^o \le q_2 \le 30^o$, and $-10^o \le q_3 \le 20^o$,
(b) $-120^o \le q_1 \le 20^o$, $-40^o \le q_2 \le 45^o$, and $-13^o \le q_3 \le 25^o$

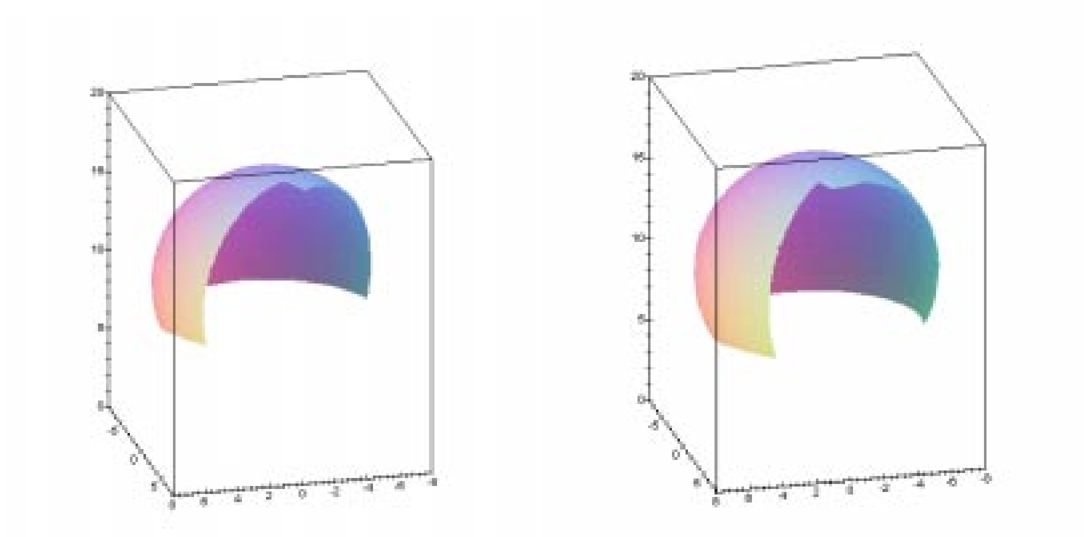

Fig. 8 (c) $-140^o \le q_1 \le 30^o$, $-50^o \le q_2 \le 60^o$, and $-16^o \le q_3 \le 32^o$
(d) $-160^o \le q_1 \le 40^o$, $-60^o \le q_2 \le 70^o$, and $-18^o \le q_3 \le 37^o$

Moreover, the surface area of the 9DOF can also be calculated and provides a strong measure of the functionality of a limb. The total surface area for the 9DOF example is

$$\mathbf{S} = \mathbf{S1} \cup \mathbf{S2} \cup \mathbf{S3} \cup \mathbf{S4} \qquad \textbf{(8)}$$

where

$$\mathbf{S1} = 4\pi r^2 - \iint_{\sigma} \sqrt{R_x^2(x,y) + R_y^2(x,y) + 1}\, dA$$

$$\mathbf{S2} = 2(2L)^2\textbf{;} \quad \mathbf{S3} = \pi(r\sin(\theta))^2\textbf{;} \text{ and } \mathbf{S4} = 8Lr\sin(\theta)$$

where $r = L1 + L2 + L3$, and $R(x,y)$ is the approximated envelope of 9 DOF human arm which lies over a closed region σ. $\theta = \theta(\mathbf{q})$, and $\mathbf{q} = [q_1 \quad \ldots \quad q_7]^{\mathbf{T}}$. For example, for r = 35, L=5, $\theta = \theta(\mathbf{q}) = Arc\cos(5/7)$, the total volume is **V** = 212165 $inch^3$ and the total surface area is **S** = 16259.4 $inch^2$

Dexterity (also called orientability or manipulability)

We define orientability or dexterity as a measure that quantifies the degree of possible orientations of an arm at a given target. Figure 9 illustrates a simple dexterity measure (although not simple enough to quantify this measure), which is a spherical surface centered at a given point in space. A person will be able to orient his/her hand at the target with a variety of orientations, all of which will intersect the spherical surface. All combinations of such orientations will penetrate the surface through regions. The sum of all these regions divided by the total surface area of the sphere will give a very methodical measure for dexterity. However, this seemingly simple algorithm is difficult to implement.

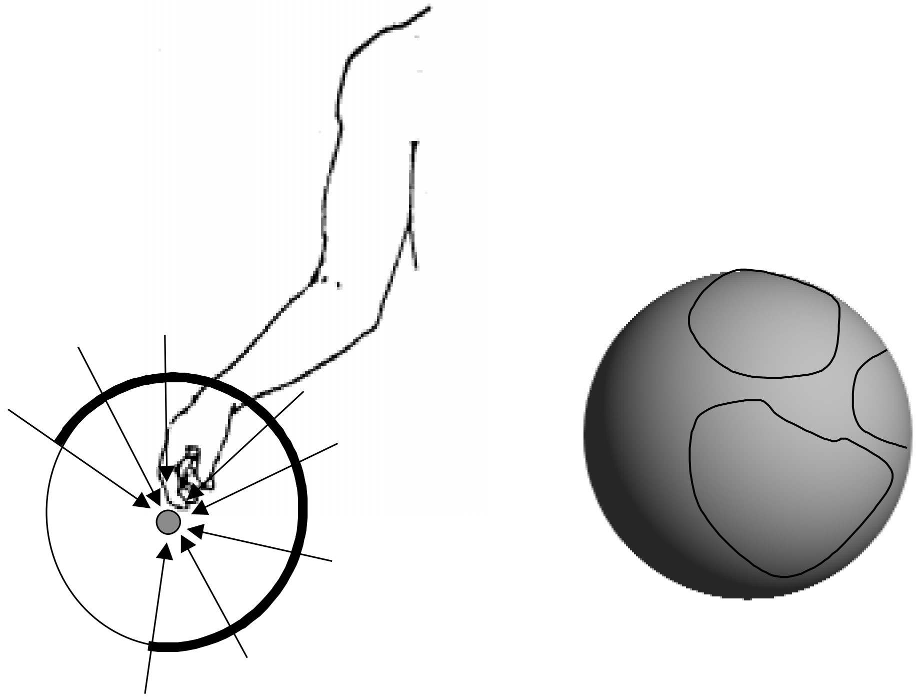

Fig. 9 A measure of dexterity at a given target

Human joints are constrained in motion and can typically be characterized by a set of inequality constraints in the form of $q_i^L \le q_i \le q_i^U$. Therefore, any formulation aimed at quantifying the dexterity of an arm near a point must include these measures.

In order to include joint limits in the formulation, we have used a parametrization (see Appendix A) to convert inequalities on q_i to equalities $q_i = \Psi(\lambda_i)$, where the new variables are defined by $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_n]^T \in \mathbf{R}^n$. For a point in space $\mathbf{x}_o$, the following $(n+3)$ augmented constraint equations must be satisfied

$$\mathbf{G}(\mathbf{q}^*) = \begin{bmatrix} \mathbf{x}(\mathbf{q}) - \mathbf{x}_o \\ \Psi(\boldsymbol{\lambda}) - \mathbf{q} \end{bmatrix}_{(n+3)\times 1} = \mathbf{0} \tag{9}$$

where the augmented vector of generalized coordinates is $\mathbf{q}^* = [\mathbf{x}^T \quad \mathbf{q}^T \quad \boldsymbol{\lambda}^T]^T$. By defining a new vector $\mathbf{z} = [\mathbf{q}^T \quad \boldsymbol{\lambda}^T]^T$ (input parameters), the augmented coordinates can be partitioned as

$$\mathbf{q}^* = \left[\mathbf{x}^T \quad \mathbf{z}^T\right]^T \tag{10}$$

The set defined by $\mathbf{G}(\mathbf{q}^*)$ is the totality of points in the workspace that can be touched by the end-effector of a serial robot manipulator. The objective of this work is to obtain a better understanding of this set, to determine its exact boundary, and to visualize it.

The input Jacobian of $\mathbf{G}(\mathbf{q}^*)$ is obtained by differentiating **G** with respect to **z** as

$$\mathbf{G}_\mathbf{z} = \begin{bmatrix} \mathbf{x}_\mathbf{q} & \mathbf{0} \\ \mathbf{I} & \boldsymbol{\Psi}_\lambda \end{bmatrix} \tag{11}$$

which is an $(n+3)\times(2n)$ matrix, where $\mathbf{x}_\mathbf{q} = \partial x_i / \partial q_j$ is a $(3\times n)$ matrix, **I** is the $(n\times n)$ identify matrix, and $\boldsymbol{\Psi}_\lambda = \partial \Psi_i / \partial \lambda_j$ is an $(n\times n)$ diagonal matrix with diagonal elements as $\Psi_\lambda)_{ii} = b_i \cos\lambda_i$. We define $\mathbf{G}_\mathbf{z}$ as the augmented Jacobian matrix.

Since this Jacobian inherently combines information about the position, orientation, and joint limits of the end-effector, it is a viable measure of dexterity. Furthermore, because of the simplicity in determining an analytical expression of $\mathbf{G}_\mathbf{z}$, it is by far a simpler approach in comparison with the widely used manipulability ellipsoid in the field of robotics (Yoshikawa 1984). We define the dexterity measure as

$$W_d = \sqrt{\left|\mathbf{G}_\mathbf{z}\mathbf{G}_\mathbf{z}^{\;T}\right|} \tag{12}$$

Note that the measure characterized by Eq. (3.5.15) takes into consideration all joint limits (ranges of motion). Although it is perhaps the most difficult to measure, it is also the most accurate.

Effort

The effort needed to reach a target point is defined herein as the displacement required for each joint from its initial position to complete the task (Fig. 10). A quantification of effort may aid designers of assembly lines and processes to setup the tasks required for an individual to achieve. This is especially important when the effort involves repetitive tasks.

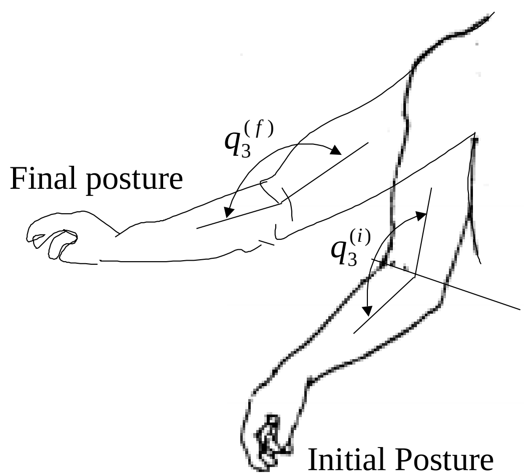

Fig. 10 Definition of the effort needed to perform a given task

$$E = \sum_{i=1}^{n} w_i Abs[q_i^{(f)} - q_i^{(i)}] \tag{13}$$

where w_i is a weight value specified for each degree of freedom, $q_i^{(i)}$ is the initial value of the variable, $q_i^{(f)}$ is the final value of variable, and *n* is the number of degrees of freedom.

For example, a person has his/her arm in the initial configuration of $\mathbf{q}^{(i)} = \begin{bmatrix} 0 & 0 & 45^o & 0 & 0 & 90^o & 30^o \end{bmatrix}^T$ and would like to pick up a glass whereby the joints would require the following configuration $\mathbf{q}^{(f)} = \begin{bmatrix} 10^o & 20^o & 40^o & 10^o & 10^o & 70^o & 40^o \end{bmatrix}^T$. Then the effort expended during this motion is

$$E = w_1(10) + w_2(20) + w_3(5) + w_4(10) + w_5(10) + w_6(20) + w_7(10)$$

where the weights are specified according to the importance of the joint.

Joint Stress

We first define the neutral position of a given joint regardless of the motion. It is well known that a joint experiences significant amount of stress as it is displaced from its most neutral position. Although different people have different neutral positions for the same joint, this approximate measure provides a viable measure for qualitatively identifying stress. Consider for example the wrist joint. The most neutral position is noted when the hand is parallel to the arm. The hand in the configuration shown in Fig. 11 (with an angle $(q_i^N - q_i)$)

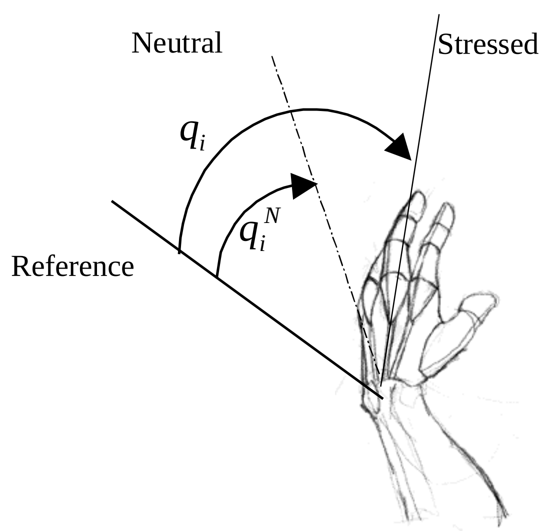

Fig. 11 Definition of stress as displacement from neutral position

To quantify this displacement over all degrees of freedom, we sum the variation from neutral position for all joints. Again, we use q weight value for each joint to emphasize that some joints' stress is deemed higher than others.

$$S = \sum_{i=1}^{n} w_i \left| q_i^N - q_i \right| \tag{14}$$

Energy

A measure of energy is needed to evaluate a certain task. For example, what is the energy that will be exerted by an operator to perform a given task? Energy, from a mechanical point of view, is defined in two parts: kinetic and potential. The energy consumed in performing a given task can be calculated based on a mathematical formulation similar to that used in robotics.

Potential Energy

The potential energy of an object of mass m is measured as $P = mgh$, where g is the gravitational acceleration and h is the height from the reference (ground). The potential energy is an important measure in ergonomics because it is encountered in our everyday living. Every time a person lifts an object from one location to another, the potential energy needed to lift the object is expended by the person. Moreover, consider the lifting of the arm from one location to another. Although no objects are carried, each body part has a mass and a significant amount of energy is needed to perform this motion. Therefore, in this section, we address the computation of the potential energy of a limb.

Consider the arm shown in Fig. 12, where each anatomical part of the arm is marked by a center of mass (it is assumed that the general location of the center of mass and the approximate mass of each part are well known).

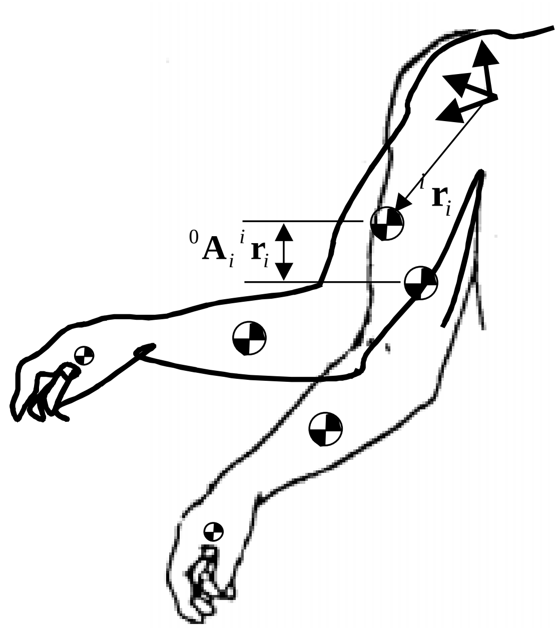

Fig. 12 Defining the potential energy for a limb

In order to determine the position and orientation of any one part of the arm, we shall use the transformation matrices $^{(i-1)}A_i$ that relates one part to another (see Appendix B). Let the vector $^{i}\overline{\mathbf{r}}_i$ denote the position of the center of mass of a body part from the origin of its own coordinate system and let $\mathbf{g}$ be the gravity vector (Fig. 13). Then for the first body part in the chain, the potential energy is $P_1 = m_1\mathbf{g}\,^{0}\mathbf{A}_1\,^{1}\overline{\mathbf{r}}_1$. However, for the second body part in the chain, we must compute the previous result in addition to the energy contribution by the second body part in the chain. We use a second transformation matrix in order to keep track of the second joint variable as $P_2 = m_2\mathbf{g}\,^{0}\mathbf{A}_1\,^{1}\mathbf{A}_2\,^{2}\overline{\mathbf{r}}_2 + P_1$. For a complete chain (e.g., a 9 degree of freedom arm), the total potential energy is given by

$$P = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n}\left(-m_i\mathbf{g}(^{0}\mathbf{A}_i\,^{i}\overline{\mathbf{r}}_i)\right) \tag{15}$$

Kinetic Energy

The kinetic energy for an arm represented as a center of mass (modeled only as a 1DOF system) rotating at an angular velocity ω is given by $K = (1/2)m(r\omega)^2$, where r is the radius of rotation.

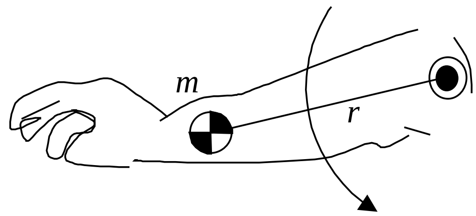

Fig. 13 Rotation of the arm modeled as a 1dof system

To generalize the kinetic energy for a serial chain, we write the kinetic energy of a particle with differential mass dm in link i as

$$dK_i = \tfrac{1}{2}(\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2)dm \tag{16}$$

The position of a point ${}^i\mathbf{r}$ with respect to point i is

$$\mathbf{r} = \mathbf{A}_i\,{}^i\mathbf{r} \tag{17}$$

The velocity is

$$\frac{d\mathbf{r}}{dt} = \left(\sum_{j=1}^{i} \frac{\partial \mathbf{A}_{ii}}{\partial q_j} \dot{q}_j \right) {}^i\mathbf{r} \tag{18}$$

The velocity squared is

$$\left(\frac{d\mathbf{r}}{dt}\right)^2 = \mathbf{r}.\mathbf{r} = Trace\left(\mathbf{r}\mathbf{r}^T\right) \tag{19}$$

Substitute from equation

$$\begin{aligned}\left(\frac{d\mathbf{r}}{dt}\right)^2 &= Trace\left[\sum_{j=1}^{i} \frac{\partial \mathbf{A}_i}{\partial q_j} \dot{q}_j . {}^i\mathbf{r} \sum_{k=1}^{i} \left(\frac{\partial \mathbf{A}_i}{\partial q_k} \dot{q}_k\, {}^i\mathbf{r} \right)^T \right] \\ &= Trace \sum_{j=1}^{i} \sum_{k=1}^{i} \left[\frac{\partial \mathbf{A}_i}{\partial q_j} {}^i\mathbf{r}\, {}^i\mathbf{r}^T \frac{\partial \mathbf{A}_i^T}{\partial q_k} \dot{q}_j \dot{q}_k \right]\end{aligned} \tag{20}$$

The Kinetic Energy of link *i* is:

$$K_i = \int_i dK_i = \frac{1}{2} Trace\left[\sum_{j=1}^{i} \sum_{k=1}^{i} \frac{\partial \mathbf{A}_i}{\partial q_j} \left(\int_i {}^i\mathbf{r}^i \mathbf{r}^T dm \right) \frac{\partial \mathbf{A}_i}{\partial q_k}^T \dot{q}_j \dot{q}_k \right] \quad (21)$$

The inertia matrix of link i is given by:

$$J_i = \int_i {}^i\mathbf{r}^i \mathbf{r}^T dm = \begin{bmatrix} \frac{-I_{ixx} + I_{iyy} + I_{izz}}{2} & I_{ixy} & I_{ixz} & m_i \overline{x_i} \\ I_{ixy} & \frac{I_{ixx} - I_{iyy} + I_{izz}}{2} & I_{iyz} & m_i \overline{y_i} \\ I_{ixz} & I_{iyz} & \frac{I_{ixx} + I_{iyy} - I_{izz}}{2} & m_i \overline{z_i} \\ m_i \overline{x_i} & m_i \overline{y_i} & m_i \overline{z_i} & m_i \end{bmatrix} \quad (22)$$

where,

$$I_{xx} = \int (y^2 + z^2) dm, \quad I_{xy} = \int xy\, dm \quad (23)$$

$$\int x^2\, dm = -\frac{1}{2}\int (y^2 + z^2) dm + \frac{1}{2}\int (x^2 + z^2) dm + \frac{1}{2}\int (x^2 + y^2) dm$$

$$= (-I_{xx} + I_{yy} + I_{zz}) / 2 \quad (24)$$

The total kinetic energy is

$$K_i = \sum_{i=1}^{6} K_i = \frac{1}{2} \sum_{i=1}^{6} Trace\left[\sum_{j=1}^{i} \sum_{k=1}^{i} \frac{\partial \mathbf{A}_i}{\partial q_j} J_i \frac{\partial \mathbf{A}_i}{\partial q_k}^T \dot{q}_j \dot{q}_k \right] \quad (25)$$

Force and Strength

Human strength has been described in terms of the maximum force one could exert under isometric conditions. In this section, we address the calculation of a force measure needed to move (with or without an object) from one location to another. Because the human body is indeed a comprised of a number of serial linkages, it is best addressed using Lagrangian mechanics (Greenwood 19. The force exerted at the hand (i.e., at the end of the serial chain), can be derived by first calculating the Lagrangian quantity

$$L = K - P$$

The forces (or torques) are then defined by the Lagrangian equations as

$$F_i = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} \tag{26}$$

$$L = \frac{1}{2}\sum_{i=1}^{6}\sum_{j=1}^{i}\sum_{k=1}^{i} Trace\left(\frac{\partial \mathbf{T}_i}{\partial q_j} J_i \frac{\partial \mathbf{T}_i}{\partial q_k}^T\right)\dot{q}_j \dot{q}_k + \frac{1}{2}\sum_{i=1}^{6} Ia_i \dot{q}_k^2 + \sum_{i=1}^{6} m_i \mathbf{g}^T \mathbf{T}_i {}^i\mathbf{r}_{ci} \tag{27}$$

$$\frac{\partial L}{\partial \dot{q}_p} = \frac{1}{2}\sum_{i=1}^{6}\sum_{k=1}^{i} Trace\left(\frac{\partial \mathbf{T}_i}{\partial q_p} J_i \frac{\partial \mathbf{T}_i}{\partial q_k}^T\right)\dot{q}_k$$

$$+\frac{1}{2}\sum_{i=1}^{6}\sum_{j=1}^{i} Trace\left(\frac{\partial \mathbf{T}_i}{\partial q_j} J_i \frac{\partial \mathbf{T}_i}{\partial q_p}^T\right)\dot{q}_j + Ia_p \dot{q}_p \tag{28}$$

With some manipulation

$$\frac{\partial L}{\partial \dot{q}_p} = \frac{1}{2}\sum_{i=p}^{6}\sum_{k=1}^{i} Trace\left(\frac{\partial \mathbf{T}_i}{\partial q_p} J_i \frac{\partial \mathbf{T}_i}{\partial q_k}^T\right)\dot{q}_k + Ia_p \dot{q}_p \tag{29}$$

Now differentiate the above equation with respect to time,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_p} = \sum_{i=p}^{6}\sum_{k=1}^{i} Trace\left(\frac{\partial \mathbf{T}_i}{\partial q_k} J_i \frac{\partial \mathbf{T}_i}{\partial q_p}^T\right)\ddot{q}_k + Ia_p \ddot{q}_p$$

$$+\sum_{i=p}^{6}\sum_{k=1}^{i}\sum_{m=1}^{i} Trace\left(\frac{\partial^2 \mathbf{T}_i}{\partial q_k \partial q_m} J_i \frac{\partial \mathbf{T}_i}{\partial q_p}^T\right)\dot{q}_k \dot{q}_m$$

$$+\sum_{i=p}^{6}\sum_{k=1}^{i}\sum_{m=1}^{i} Trace\left(\frac{\partial^2 \mathbf{T}_i}{\partial q_p \partial q_m} J_i \frac{\partial \mathbf{T}_i}{\partial q_k}^T\right)\dot{q}_k \dot{q}_m \tag{30}$$

The last term in the Lagrange equation

$$\frac{\partial L}{\partial q_p} = \frac{1}{2}\sum_{i=p}^{6}\sum_{j=1}^{i}\sum_{k=1}^{i} Trace\left(\frac{\partial^2 \mathbf{T}_i}{\partial q_j \partial q_p} J_i \frac{\partial \mathbf{T}_i}{\partial q_k}^T\right)\dot{q}_j \dot{q}_k$$

$$+\frac{1}{2}\sum_{i=p}^{6}\sum_{j=1}^{i}\sum_{k=1}^{i} Trace\left(\frac{\partial^2 \mathbf{T}_i}{\partial q_k \partial q_p} J_i \frac{\partial \mathbf{T}_i}{\partial q_j}^T\right)\dot{q}_j \dot{q}_k + \sum_{i=p}^{6} m_i \mathbf{g}^T \frac{\partial \mathbf{T}_i}{\partial q_p} {}^i\mathbf{r}_{ci}$$

(31)

Interchange the dummy indices of summation j and k in the second term of the above equation

$$\frac{\partial L}{\partial q_p} = \sum_{i=p}^{6}\sum_{j=1}^{i}\sum_{k=1}^{i} Trace\left(\frac{\partial^2 \mathbf{T}_i}{\partial q_p \partial q_j} J_i \frac{\partial \mathbf{T}_i^{\,T}}{\partial q_k}\right)\dot{q}_j \dot{q}_k + \sum_{i=p}^{6} m_i \mathbf{g}^T \frac{\partial \mathbf{T}_i}{\partial q_p} {}^i\mathbf{r}_{ci} \tag{32}$$

Substituting in the Lagrange equation all the derived terms,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_p} - \frac{\partial L}{\partial q_p} = \sum_{i=p}^{6}\sum_{k=1}^{i} Trace\left(\frac{\partial \mathbf{T}_i}{\partial q_k} J_i \frac{\partial \mathbf{T}_i^{\,T}}{\partial q_p}\right)\ddot{q}_k + Ia_p \ddot{q}_p$$

$$+\sum_{i=p}^{6}\sum_{k=1}^{i}\sum_{m=1}^{i} Trace\left(\frac{\partial^2 \mathbf{T}_i}{\partial q_k \partial q_m} J_i \frac{\partial \mathbf{T}_i^{\,T}}{\partial q_p}\right)\dot{q}_k \dot{q}_m - \sum_{i=p}^{6} m_i \mathbf{g}^T \frac{\partial \mathbf{T}_i^{\,T}}{\partial q_p} {}^i\mathbf{r}_{ci} \tag{33}$$

Exchange dummy summation indices p and i for i and j, to get the simplified equations:

$$F_i = \sum_{j=i}^{6}\sum_{k=1}^{j} Trace\left(\frac{\partial \mathbf{T}_j}{\partial q_k} J_j \frac{\partial \mathbf{T}_j^{\,T}}{\partial q_i}\right)\ddot{q}_k + Ia_i \ddot{q}_i$$

$$+\sum_{j=i}^{6}\sum_{k=1}^{j}\sum_{m=1}^{j} Trace\left(\frac{\partial^2 \mathbf{T}_j}{\partial q_k \partial q_m} J_j \frac{\partial \mathbf{T}_j^{\,T}}{\partial q_i}\right)\dot{q}_k \dot{q}_m - \sum_{j=i}^{6} m_j \mathbf{g}^T \frac{\partial \mathbf{T}_j^{\,T}}{\partial q_i} {}^j\mathbf{r}_{cj} \tag{34}$$

The equations are rewritten as:

$$F_i = \sum_{j=1}^{6} D_{ij}\ddot{q}_j + Ia_i \ddot{q}_j + \sum_{j=1}^{6}\sum_{k=1}^{6} D_{ijk}\dot{q}_j \dot{q}_k + D_i \tag{35}$$

where,

D_{ii} The effective inertia at joint *i*

D_{ij} Coupling inertia between joints *i* and *j*

D_{ijj} Centripetal forces at joint *i* due to velocity at joint *j*

D_{ijk} Coriolis forces at joint *i* due to velocities at joints *j* and *k*

D_i Gravity loading at joint *i*

It is important to note that the mass and inertia properties of human anatomy are well documented.

Work

Work done by a force on an object is the product of the magnitude of the force and the distance moved by the object.

$$W_{0,1} = \int_{\mathbf{r}_o}^{\mathbf{r}_1} \mathbf{f} \cdot d\mathbf{r} \tag{36}$$

where $\mathbf{r}_o$ is the starting point of motion and $\mathbf{r}_1$ is the terminating point of motion.

Power

The average power is the time rate at which work is done.

$$P = \frac{W_{0,1}}{t} \tag{37}$$

Conclusions

Broadly applicable functions characterizing measures of human performance have been developed. The main contribution of this paper is a sound mathematical approach to the modeling and evaluation of performance. This is the first step towards automating the ergonomic design process.

In mechanical design, an engineer calculates the best thickness of a beam by first defining the design variables (those that can be varied) a cost function (the goal for example is to maintain a high stiffness with a low weight). Then, in an iterative manner through an optimization algorithm, the engineer arrives at the best optimum design. The ergonomic design process has only depended on empirical data and therefore, has not been able to perform rigorous design. This effort is aimed at establishing the fundamentals of such a rigorous formulation by first developing the necessary cost functions.

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Appendix A (Parametrization)

A convenient parameterization of constraints imposed on $\mathbf{q}$ was presented such that joint inequality constraints $q_i^L \le q_i \le q_i^U$ are given by

$$\mathbf{q}(\boldsymbol{\lambda}) = (\mathbf{q}^U + \mathbf{q}^L)/2 + [(\mathbf{q}^U - \mathbf{q}^L)/2]\sin\boldsymbol{\lambda} \qquad \text{(A.1)}$$

where $\mathbf{q}^U = [q_1^U \quad \ldots \quad q_n^U]^T$ and $\mathbf{q}^L = [q_1^L \quad \ldots \quad q_n^L]^T$ are the upper and lower joint limits, respectively, and $\boldsymbol{\lambda} = [\lambda_1 \quad \ldots \quad \lambda_n]^T$ are the new variables that have been introduced by adding as many equations as the number of variables without reducing the dimensionality of the problem ($\boldsymbol{\lambda}$ are usually called slack variables in the field of optimization).