Signal-to-Noise Ratio

Signal noise is always with us in medical imaging. In the case of MRI, the noise is distributed uniformly throughout the image. One way of measuring the effect of noise is to calculate the signal-to-noise ratio (SNR). In the case of MRI, the SNR can be measured by computing the mean signal intensity over a certain region of interest (ROI) and dividing this by the standard deviation of the signal from a region outside the image. In other modalities, this is not true; the noise is not uniformly distributed over the image. As a consequence, other method must be used to estimate the SNR.

Signal Averaging

There are a number of alternate ways of dealing with the noise. One way is to assume that the SNR can be increased by averaging the signals from a number of images if the signals from successive images are coherent and the noise is at least partially incoherent. The measured signal $\hat{x}$ can be represented as

$$\hat{x} = x + \xi \tag{1.1}$$

where x is the true signal and ξ is the noise component with mean zero and standard deviation σ_ξ. The SNR is then given as

$$\mathrm{SNR}_{\hat{x}} = \frac{|x|}{\sigma_\xi} \tag{1.2}$$

If N measurements are acquired, the average signal is given by

$$\hat{y} = x + \frac{1}{N}\sum_{n=1}^{N}\xi_n \tag{1.3}$$

Assuming that the noise for each measurement is uncorrelated, then the SNR for the averaged scans is given by

$$\mathrm{SNR}_{\hat{y}} = \frac{|x|}{\sqrt{\mathrm{var}\left\{\left(1/N\right)\sum_{n=1}^{N}\left(x+\xi_n\right)\right\}}} = \sqrt{N}\frac{|x|}{\xi_n} = \mathrm{SNR}_{\hat{x}}\sqrt{N} \tag{1.4}$$

Equation (1.4) shows that the SNR increases as the square root of the number of averaged images.

In ultrasound imaging, however, the noise contribution from speckle is coherent, and so simple signal averaging does not increase the SNR.

The Poisson Distribution

In X-ray imaging, X-ray based CT, and nuclear medicine it is usually true that the image SNR is determined solely by the Poisson statistics of the X-ray beam and the radioactive source, respectively. If the mean number of incident X-rays per unit area of detector is denoted by μ, then the probability $P(N)$ that there are N incident X-rays per unit area is shown in Figure 1 and is described as

$$P(N) = \frac{\mu^N e^{-\mu}}{N!} \tag{2.1}$$

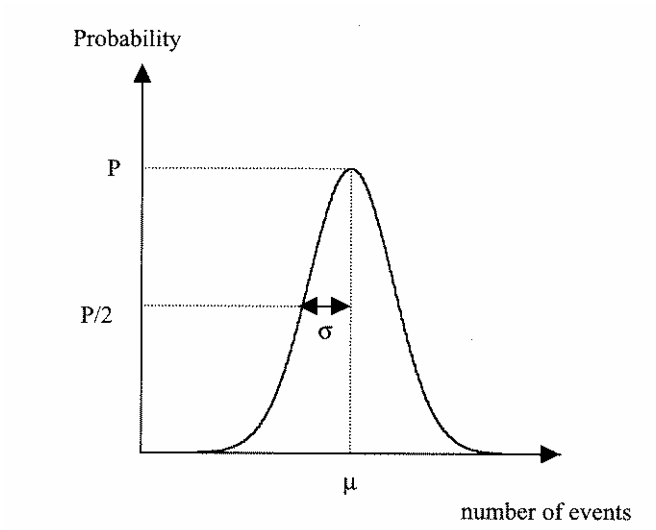

Figure 1 – The Poisson distribution describes the probability of a particular number of events happening for a random process.

A characteristic of the Poisson distribution is that the value of σ is related to μ by

$$\sigma = \sqrt{\mu} \tag{2.2}$$

For large values of *N*, μ can be well-approximated by *N*. In this case, the SNR, defined as N/σ, is given by

$$\text{SNR} \propto \sqrt{N} \tag{2.3}$$

Contrast-to-Noise Ratio

Even if the image has a high signal-to-noise ratio, it is not useful unless there is a high enough CNR to be able to distinguish among different tissues and tissue types, and in particular between healthy and pathological tissue. Various definitions of image contrast exist, but the most common is

$$C_{AB} = |S_A - S_B| \tag{3.1}$$

where C_{AB} is the contrast between tissues A and B, and S_A and S_B are the signals from tissues A and B. The CNR between tissues A and B is defined in terms of the respective SNRs of the two tissues.

$$\text{CNR}_{AB} = \frac{C_{AB}}{\sigma_N} = \frac{|S_A - S_B|}{\sigma_N} = |\text{SNR}_A - \text{SNR}_B| \tag{3.2}$$

where σ_N is the standard deviation of the noise.

Receiver Operating Curve

There are four possibilities for a practitioner making a diagnosis: a true positive (where true refers to a correct diagnosis and positive refers to a tumor being present), a true negative, a false positive, and a false negative. These are shown in Figure 2.

	Actual situation	
Diagnosis	*Tumor present*	*Tumor absent*
Tumor present	True positive	False positive
Tumor absent	False negative	True negative

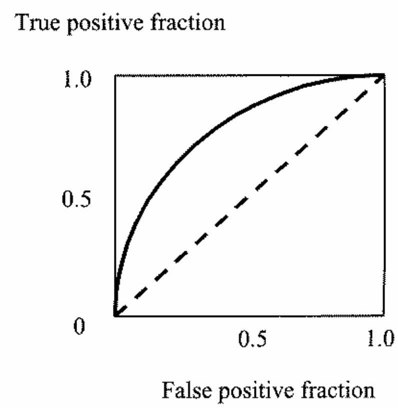

Figure 2 – (Left) A table showing the four possible outcomes of a tumor diagnosis. (Right) The ROC represented by the dashed line represents a random diagnosis. The upper curve represents an improved diagnosis. The better the diagnosis, the larger is the integrated area under the ROC.

There are three measures commonly used in ROC analysis

1. The **accuracy** is the correct number of diagnoses divided by the total number of diagnoses
2. The **sensitivity** is the number of true positives divided by the sum of the true positives and false negatives
3. The **specificity** is the number of true negatives divided by the number of true negatives and false positives

The ROC plots the fraction of true positives versus the fraction of false positives for a series of images acquired under different conditions, or with a different value of some parameter, or with different SNRs, or with different practitioners, for example. The area under the ROC is a measure of the effectiveness of the imaging system and/or the practitioner: the greater the area under the curve the more effective is the diagnosis.