Transport Phenomena 052:217
Confidence Builder I
February 17, 2005
Closed Book Exam

1.(30) Let $\vec{u} = \vec{\delta}_i + \vec{\delta}_j + \vec{\delta}_k$, $\vec{v} = -2xy\vec{\delta}_i + 2zy\vec{\delta}_j$ and $\vec{\vec{D}} = xy\vec{\delta}_i\vec{\delta}_k + xz\vec{\delta}_j\vec{\delta}_k$. Prove or disprove $\nabla \cdot (\vec{u}\vec{v}) = \vec{u}(\nabla \cdot \vec{v}) + \vec{v}(\nabla \cdot \vec{u})$ for arbitrary values of x, y, z.

2.(30) The gradient of a function, f, that describes a surface is often used to calculate the normal to that surface. This is true since the gradient of f is a vector oriented perpendicularly to the $f(x, y, z) = a^2$ surface with contains the position vector, and which points toward larger f.

Letting $f(x, y, z) \equiv x^2 + (y-a)^2 + z^2 = a^2$. Determine the normal, $\vec{n}$, of the surface at the point $\left(a\frac{\sqrt{3}}{2}, \frac{a}{2}, 0\right)$. Remember, the normal has a magnitude of one.

3.(40) Starting with the total derivative of a scalar $f(r, \theta, \phi)$ in spherical coordinates and the position vector, derive ∇f for spherical coordinates.

<u>Spherical Coordinates</u>

$$\frac{\partial}{\partial \theta}\vec{\delta}_\theta = -\vec{\delta}_r \qquad \frac{\partial}{\partial \theta}\vec{\delta}_r = \vec{\delta}_\theta \qquad \frac{\partial}{\partial \phi}\vec{\delta}_\theta = \vec{\delta}_\phi \cos\theta \qquad \frac{\partial}{\partial \phi}\vec{\delta}_r = \vec{\delta}_\phi \sin\theta$$

$$\frac{\partial}{\partial \phi}\vec{\delta}_\phi = -\vec{\delta}_r \sin\theta - \vec{\delta}_\theta \cos\theta$$