Expected Annual Damages and Uncertainties in Flood Frequency Estimation

By Nigel W. Arnell[1]

Abstract: The expected annual damge is the most frequently used index of the impact of flooding at a site. However, estimates of expected annual damages are very uncertain as a result of uncertainties in both the estimation of the flood frequency relationship from limited data and the relationships between magnitude and damage. Computer simulation experiments using synthetic flood peak data and fixed magnitude-damage functions have shown that the sampling distribution of estimates of expected annual damages is highly skewed to a degree depending on the form of the damage function, and most importantly, that bias in the estimates is most closely related to error in the estimated probability at which damage begins. The use of expected probability leads to a very significant increase in bias in the estimation of expected annual damages.

Introduction

When designing a scheme to alleviate flooding, planners and engineers need an estimate of the costs of flood damage. The most commonly used measure is the expected annual damage, which is best understood as the average of flood damages computed over many years. One way of calculating this is simply to add up a long time series of annual damages and divide by the number of years. However, this is rarely possible in practice; a very long record would be necessary because damage would be zero in most years, and in any case exposure to damage would have changed considerably over time.

Expected annual damages are therefore calculated by first fitting a frequency distribution to flood magnitudes. A function relating flood magnitude to damage is then used to derive a relationship between flood damage and the probability of incurring that damage in any one year. All of these stages include unknowns and uncertainties—the relationship between flood discharge and depth may be poorly defined, as might the function relating depth to damage—but it is the objective of this paper to examine the effects of the uncertainties associated with the estimation of the flood frequency relationship. In particular, there is uncertainty about both the appropriate form of the statistical model of flood frequencies, and the value of model parameters. These uncertainties are primarily due to the problems caused by making inferences from small samples of flood peaks. In this paper, emphasis is placed on parameter uncertainty—the form of the model is assumed known—and three alternative procedures for estimating expected annual damages are compared. Practical implications of bias and variability are also considered.

Estimation of Expected Annual Damages

At its simplest, the mean of a random variable x such as annual flood damage is

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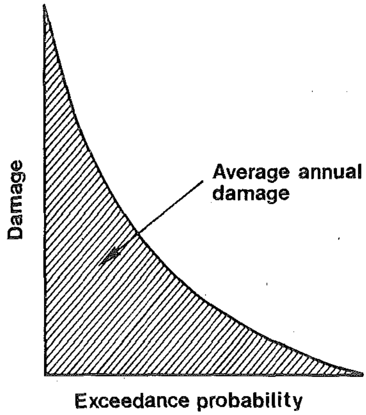

FIG. 1. Expected Annual Damages as Area under Damage-Probability Curve

$$E(x) = \int x f(x) dx \quad (1)$$

where $f(x)$ = the probability density function of that variable. Flood frequency analysts are more used to working with exceedance or non-exceedance probabilities, defined by the cumulative distribution

$$E(x) = \int_0^1 x df \quad (2)$$

which shows that expected annual damages are equal to the area under the graph of damage against non-exceedance (or exceedance) probability (Fig. 1). This is well known to analysts, who routinely calculate expected annual damages by computing damage associated with several return period floods, drawing up a graph similar to Fig. 1 and measuring the area under the curve.

Several authors (Hardison and Jennings 1972; Beard 1978; Tai 1987) have maintained, however, that "conventional" flood frequency estimation procedures such as the methods of moments or maximum likelihood underestimate the true frequency of flooding and thus the value of expected annual damages (Arnell 1988). Beard (1960) illustrated the problem by considering a large number of independent but identical rivers, each with the same record length. If the flood with an exceedance probability of 0.01 was estimated from each sample and the true exceedance probabilities were determined for each estimate, it would be found that the average true exceedance probability would be greater than 0.01 even if the average magnitude was equal to the true magnitude (because the relationship between flood magnitude and frequency is not linear). Over all sites, events would therefore occur in the future with an average frequency greater than 0.01. Beard (1960) called the mean true exceedance probability of estimates of the magnitude of the p probability event the *expected probability* of that flood, and urged that the design flood be taken as the flood with an expected probability equal to p. If not, he argued, the risk of future flooding and hence expected annual damages would be underestimated. Hardison and Jennings (1972) and Tai (1987) showed that use of expected probability resulted in an increase in

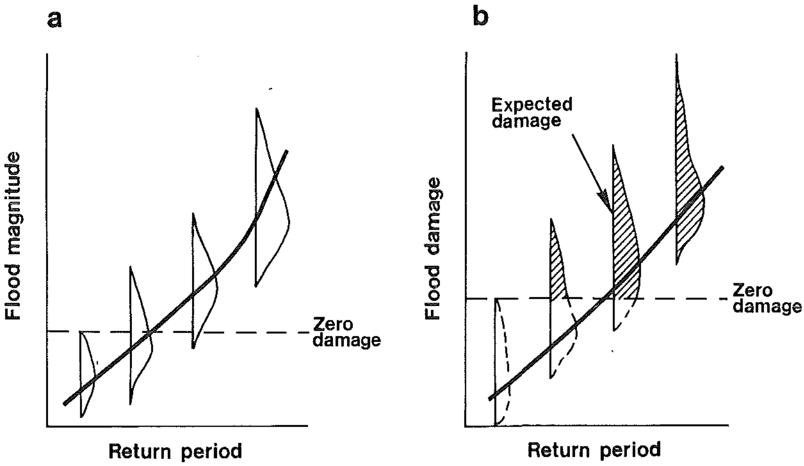

FIG. 2. Sampling Distribution of Flood Magnitudes and Damages

expected annual damages (because converting to expected probability increases the probability assigned to a particular magnitude event), and inferred that bias in the estimation of expected annual damages was therefore reduced. However, a method which gives an unbiased estimate of flood risk does not necessarily give an unbiased estimate of expected annual damages, and Gould (1973a, 1973b) argued that rather than eliminating bias, the use of expected probability increased it. He showed that bias in the estimation of expected annual damages using "conventional" methods was small and of the opposite direction to that implied by Hardison and Jennings (1972). Doran and Irish (1980) subsequently supported Gould's (1973a) conclusions using computer simulation experiments. The present investigations were designed to further clarify this issue.

A second refinement to the conventional procedure for estimating expected annual damages has been presented by James and Hall (1986), followed by Tung (1987) and Bao et al. (1987). The method is based on the recognition that uncertainty in the parameters of the flood frequency distribution can be expressed by sampling distributions for given flood quantile estimates, as shown in Fig. 2, from which confidence limits can be determined. The sampling distribution of the magnitudes of a given frequency flood can then be converted to a sampling distribution of flood damage using the magnitude-damage function [Fig. 2(*b*)], and the expected value of this sampling distribution can be taken as the appropriate estimate of damage for that frequency. This can be expressed as

$$E(D) = \int_0^1 [Dh(D)]dF \qquad (3)$$

where $h(D)$ = the probability density function of the estimate of damage D for a given frequency event. James and Hall (1986), Tung (1987), and Bao et al. (1987) all found that the effect of this refinement was to increase the estimate of expected annual damages, although the magnitude of this effect

J. Water Resour. Plann. Manage. 1989.115:94-107.

depends of course on the sampling distribution of estimates of flood quantiles (which is strongly influenced by record length) and the shape of the function relating flood magnitude to damage. This procedure, too, was examined in the current study.

Experimental Design

The relative performances at the preceding conventional procedure for estimating expected annual damages and the two refinements were assessed using computer simulation experiments. A general analytical approach is not feasible. Gould (1973b) developed a theoretical expression for bias in expected annual damages, but was forced to assume a normal distribution of flood depths and a linear depth-damage function, and hence a normal distribution of damages. In essence, the simulation experiments involved: (1) Generating a synthetic sample of flood depths from a pre-defined parent distribution; (2) estimating the form of the depth-probability relationship from the sample; (3) converting depth to damage using a depth-damage function; and (4) computing the area under the depth-probability curve. By generating synthetic flood depths it is assumed that the relationship between flood discharge and depth is known with complete certainty; this will not of course be true in practice. Similarly, step (4) of the procedure neglects uncertainties in the relationship between flood depth and flood damage.

The two-parameter lognormal distribution was used as the parent distribution, with parameters selected such that the difference between the true 10- and 100-year flood depths was equal to 1 "synthetic" meter. This distribution was selected because it is possible to apply relatively easily the three alternative ways of estimating expected annual damages. The first, conventional, approach involves the estimation of the lognormal parameters from the sample data. Using both the method of moments and the method of maximum likelihood, the parameters can be estimated from

$$\hat{\mu} = \bar{x} = \frac{\sum_{i=1}^{N} x}{N} \quad (4)$$

and

$$\hat{\sigma} = s = \left[\frac{\sum_{i=1}^{N} (x - \bar{x})^2}{N - 1}\right]^{1/2} \quad (5)$$

where x_i = the natural logarithm of flood magnitude; and N = sample size. The logged depth corresponding to a specified frequency can then be calculated from

$$\hat{x}_p = \bar{x} + s z_p \quad (6)$$

where z_p = the standard normal deviate with exceedance probability p. An estimate of the logarithm of the flood which will be exceeded with an *expected probability* equal to p can be computed from (Beard 1960; Stedinger 1983a):

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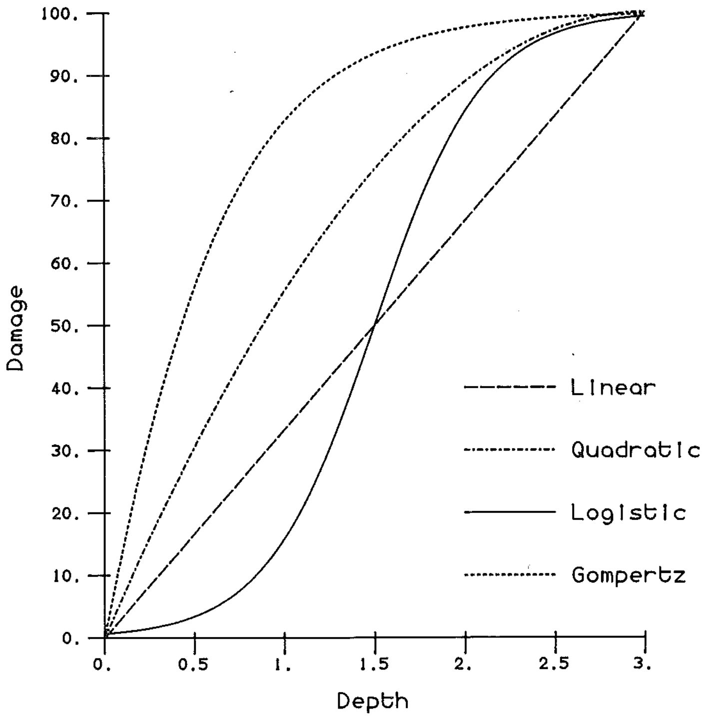

FIG. 3. Depth-Damage Functions

$$\hat{x}_{ep} = \bar{x} + s\left[1 + \frac{1}{N}\right]^{1/2} t_{p,N-1} \qquad (7)$$

t_p, υ = the quantile with exceedance probability p from a student's t distribution with υ degrees of freedom.

The third approach, involving sampling distributions of quantile estimates, is rather more complicated. Stedinger (1983b) showed that if floods (or their logarithms) were normally distributed, the sampling distribution of a quantile estimate could be derived using the non-central t distribution. The random variable $\sqrt{N}\xi(p)$, where

$$\xi(p) = \left(\frac{x_p - \bar{x}}{S}\right) \qquad (8)$$

has a non-central t distribution with non-centrality parameter $\delta = z_p\sqrt{N}$ and $\upsilon = N - 1$ degrees of freedom, and it is therefore possible to determine the value of $\xi(p)$ exceeded with probability α. Eq. 8 can then be rearranged to give the estimate of the p-probability flood $x_p(\alpha)$, which would be exceeded in samples of size N with probability α: in other words, of all samples of size N from a lognormal distribution with parameters μ and σ, a proportion α would yield estimates of x_p greater than $x_p(\alpha)$. The expected damage associated with exceedance probability p (Eq. 3) is computed by converting log depth $x_p(\alpha)$ to damage D_p using the depth-damage function, and calculating the area under the damage-α curve. This was done using Simpson's Rule, and the approximation to the non-central t distribution presented by Abramowitz and Stegun (1965) was used to compute $x_p(\alpha)$ for a given probability α. It is clear that this approach is much more consuming of computer simulation time than the other two.

Four depth-damage functions were defined as shown in Fig. 3. The quadratic has the form:

$$D = 100\left[\frac{1 - (3 - \text{depth})^2}{9}\right] \quad (9)$$

the Gompertz has the form (Ouellette et al. 1985):

$$D = \frac{100\{e^{[1-e^{-\text{depth}}]e^{-\alpha}} - 1\}}{\{e^{e^{-\alpha}} - 1\}} \quad (10)$$

with $\gamma = 2.0$ and $\alpha = 0.5$, and the logistic function is defined as

$$D = 100\left[1 + \exp\left(\frac{-(\text{depth} - u)}{a}\right)\right] - 1 \quad (11)$$

where $u = 1.5$ and $a = 0.3$ All the damage functions give a damage of zero at zero depth and 100 at a depth of 3 "synthetic" meters.

The three procedures yield an array of pairs of damage and associated exceedance probabilities, which are used to construct a damage-probability curve. The area under this curve was calculated for all three methods using the "mid-range probability" method

$$EAD = \sum_{i=1}^{M-1} (p_i - p_{i+1}) \frac{[D_{i+1} + D_i]}{2} \quad (12)$$

(where M = the number of pairs; p = exceedance probability; and D = damage), rather than by the more accurate Simpson's Rule, for two reasons. First, it is more often used in practice, since there are rarely enough pairs of damage and probability available to justify the use of Simpson's Rule, and secondly, use of Simpson's Rule with the third method—which requires numerical integration for each exceedance probability—would be very costly in computer resources. Simulation experiments were undertaken with samples of size 10, 20 and 40; 500 repetitions were used for each experiment. Expected annual damages were calculated for situations where damage begins at the levels of the true 5, 10, 25, 50 and 100 year floods.

Results

Tables 1, 2, and 3 show, for the quadratic and logistic damage functions, the mean, standard deviation, and skewness of estimates of expected annual damages. Similar results were found with the other damage functions. In general, it is clear that all the methods overestimate expected annual damages, particularly when damage commences in infrequent events, but that the conventional method is least biased. This supports Gould's (1973a) and Doran and Irish's (1980) conclusions and conflicts with Hardison and Jennings' (1972) and Beard's (1978) inferences. Although the degree of dif-

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TABLE 1. Bias in Estimates of Expected Annual Damage, Expressed as Percentage of True Value, for Different True Probabilities at which Damage Begins

	DAMAGE FUNCTION					
	Quadratic			Logistic		
Probabilities (1)	10[a] (2)	20[a] (3)	40[a] (4)	10[a] (5)	20[a] (6)	40[a] (7)
Threshold probability = 0.2						
conventional method	7.6	0.4	1.7	33.8	12.0	8.7
expected probability method	38.5	16.0	9.6	120.9	52.6	28.5
expected damage method	41.4	17.5	10.5	143.5	64.4	35.5
Threshold probability = 0.04						
conventional method	53.3	20.9	14.1	169.3	68.6	41.8
expected probability method	86.5	80.0	42.6	622.9	232.0	108.5
expected damage method	200.9	84.6	44.5	846.4	340.0	168.0
Threshold probability = 0.01						
conventional method	179.3	72.7	43.3	568.4	194.7	105.3
expected probability method	644.0	246.7	118.0	2,647.3	742.1	278.9
expected damage method	714.7	273.3	129.3	4,163.6	1,400.0	589.5

[a]Sample size.
Note: Simulation results from 500 repetitions.

ference varies with damage function, the results clearly show that use of either expected probabilities or the "expected damage" method would produce very biased estimates of expected annual damages. These two methods yield very similar results (when using a lognormal distribution), which re-

TABLE 2. Standard Deviation of Expected Annual Damage Estimates, Divided by True Value, for Different True Probabilities at which Damage Begins

	DAMAGE FUNCTION					
	Quadratic			Logistic		
Probabilities (1)	10[a] (2)	20[a] (3)	40[a] (4)	10[a] (5)	20[a] (6)	40[a] (7)
Threshold probability = 0.2						
conventional method	0.80	0.53	0.40	1.48	0.89	0.66
expected probability method	0.86	0.56	0.41	1.86	1.05	0.72
expected damage method	0.86	0.56	0.42	1.84	1.05	0.72
Threshold probability = 0.04						
conventional method	2.03	1.17	0.86	5.07	2.35	1.57
expected probability method	2.69	1.44	0.96	8.58	3.60	2.04
expected damage method	2.71	1.44	0.96	8.71	3.69	2.10
Threshold probability = 0.01						
conventional method	5.35	2.54	1.75	18.11	6.26	3.58
expected probability method	440.11	3.79	2.22	40.11	12.68	5.63
expected damage method	8.92	3.80	2.20	43.11	14.21	6.47

[a]Sample size.
Note: Simulation results from 500 repetitions.

TABLE 3. Skewness of Expected Annual Damage Estimates, for Different True Probabilities at which Damage Begins

Probabilities (1)	DAMAGE FUNCTION					
	Quadratic			Logistic		
	10[a] (2)	20[a] (3)	40[a] (4)	10[a] (5)	20[a] (6)	40[a] (7)
Threshold probability = 0.2						
conventional method	1.08	0.77	0.70	2.07	1.50	1.31
expected probability method	0.85	0.67	0.66	1.48	1.22	1.18
expected damage method	0.85	0.68	0.68	1.47	1.22	1.20
Threshold probability = 0.04						
conventional method	2.37	1.76	1.51	3.63	2.88	2.53
expected probability method	1.66	1.39	1.36	2.25	2.08	2.16
expected damage method	1.66	1.43	1.36	2.12	1.93	2.02
Threshold probability = 0.01						
conventional method	3.48	2.75	2.36	5.15	4.36	3.84
expected probability method	2.21	1.99	2.01	2.83	2.91	3.27
expected damage method	2.12	1.99	2.01	2.34	2.34	2.58

[a]Sample size.
Note: Simulation results from 500 repetitions.

flects similarities in their derivation. Both are based on the t-distribution (Stedinger 1983a, 1983b), and the expected value of the estimate of the p probability flood $\int x_p f(x) dx_p$ is very close to the estimate computed from Eq. 7. The actual difference between the two methods depends on the shape of the damage function (the expected damage is not equal to the damage associated with the expected magnitude, except with a linear damage function) and, to a lesser extent, the numerical approximation.

The contrasts in the degree of bias between the different damage functions depends on the rate of change of damage with magnitude, particularly at low magnitudes. With the logistic curve, damage is limited for floods just above the damage threshold (Fig. 3) but increases significantly at higher depths. The frequency with which floods reach this depth is estimated with greater bias and uncertainty than the frequency with which damage begins. For a given flood frequency relationship and threshold at which damage begins, therefore, the greater the proportion of damage which occurs in small floods, the less the bias and variability in estimate of expected annual damage.

As sample sizes increase, all the methods become less biased (bias falls from over 50% to just over 15% for the conventional method, with damage occurring with a true probability of 0.04, for example). The expected probability and expected damage methods improve the most and with very large samples all three methods would give the same results. Sample variability of estimates also falls as sample sites increase (Table 2) and, for high damage thresholds at least, there is less difference in variability than bias between the three methods. The coefficient of skew, given in Table 3, shows the high asymmetry in the sampling distribution of expected annual damages, due to the occasional very large estimates.

The magnitude of estimated expected annual damages depends partly on the estimated slope of the depth-frequency curve but much more closely on

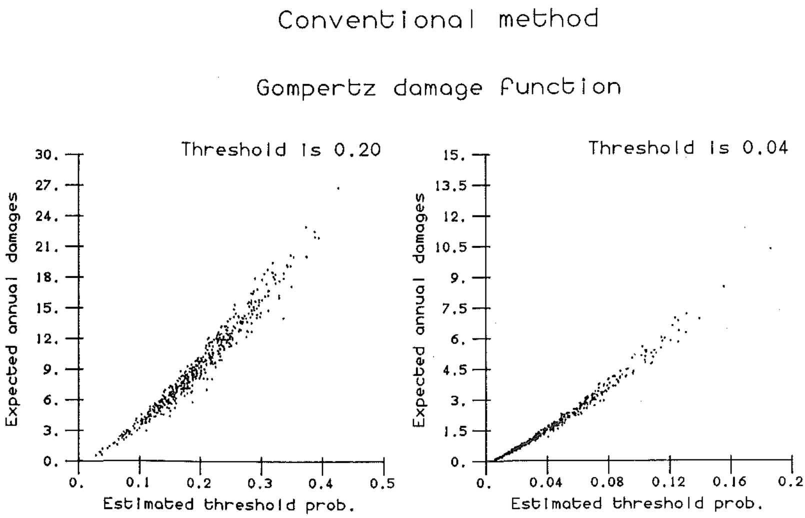

FIG. 4. Variation of Estimated Expected Annual Damages with Estimated Threshold Probability

the estimated probability at which damage begins. Fig. 4 shows the strong relationship between estimated threshold probability and computed expected annual damages (for the Gompertz damage function and a sample size of 20). The bias and variability in expected annual damages is clearly related to the bias and variability in the estimated threshold probability. The reason for the difference in bias between the conventional and expected probability methods can be seen in Table 4, which shows the mean estimated probability at which damage begins. The conventional estimator provides a good estimate of the threshold probability, but the expected probability method produces a very biased estimate of the risk of damage. This arises because a

TABLE 4. Mean Estimated Threshold Probability: Conventional and Expected Probability Estimators

Estimators (1)	True Threshold Probability 0.2 (2)	0.1 (3)	0.04 (4)	0.02 (5)	0.01 (6)
$N = 10$					
conventional method	0.199	0.103	0.047	0.027	0.016
expected probability	0.219	0.126	0.068	0.045	0.031
$N = 20$					
conventional method	0.194	0.098	0.042	0.023	0.013
expected probability	0.205	0.111	0.053	0.031	0.019
$N = 40$					
conventional method	0.199	0.101	0.042	0.022	0.012
expected probability	0.205	0.107	0.047	0.026	0.015

Note: Averaged over 500 repetitions.

method that gives an estimate of the flood exceeded on average with the desired risk p (i.e., unbiased); it does not produce an unbiased estimate of the risk of a specified magnitude (such as a floor level) being exceeded.

Implications of Uncertainty in Estimation of Expected Annual Damages

The results of the previous section have emphasized the potentially very large sampling variability in the estimation of expected annual damages due solely to sampling variability in the observed flood data. In current practice only a single "best" estimate of expected annual damages is used, derived from the "best" estimate of the flood frequency curve, but it may be useful to have information on the precision of this estimate. Some workers, for example Grigg (1978), have attempted to derive confidence limits for an estimate of expected annual damages directly from confidence intervals on flood magnitude estimates, as shown in Fig. 5(a)–5(c). This, however, is incorrect due to a misinterpretation of the meaning of confidence intervals for flood quantiles. These confidence limits should be interpreted solely as intervals for the range of magnitudes for a *specified* exceedance probability; the locus of 90% confidence interval values (i.e., 90% of estimates of magnitude for that probability are greater) does not define the frequency curve which will be exceeded over all probabilities 90% of the time. One sample curve may yield an estimate of the 10-year flood outside the 90% interval for that return period, for example, while yielding a 100-year flood estimate close to the mean value [Fig. 5(d)]. An approach such as this would overestimate confidence intervals and give an unduly pessimistic impression of precision.

It is well known that the standard deviation of the sampling distribution of the mean of a random variable is equal to the standard error, or the standard deviation of the variable divided by the square root of the sample size:

$$\text{standard error } (\bar{x}) = \frac{\text{s.d.}(x)}{\sqrt{N}} \quad (13)$$

It is therefore possible to estimate the standard error of the sampling distribution of expected annual damages by computing the standard deviation of annual damages using

$$\text{s.d.}(D) = [E(D^2) - E^2(D)]^{1/2} \quad (14)$$

where $E(D)$ = expected annual damages and $E(D^2)$ = the area under the "damage-squared"-probability curve. Table 5 compares the average standard error of expected annual damages (computed using Eq. 14), with the observed standard deviation of estimates of expected annual damages. It can be seen that, for cases where damage occurs in frequent events at least, the standard error provides a good estimate of sample standard deviation. However, the high skew of the sampling distribution (Table 3) means that confidence limits cannot be based on just expected annual damages and standard error, and, although it is possible to estimate the skew of annual damages using the area under the "damage-cubed"-probability curve, sample skewness estimates are notoriously unreliable.

J. Water Resour. Plann. Manage. 1989.115:94-107.

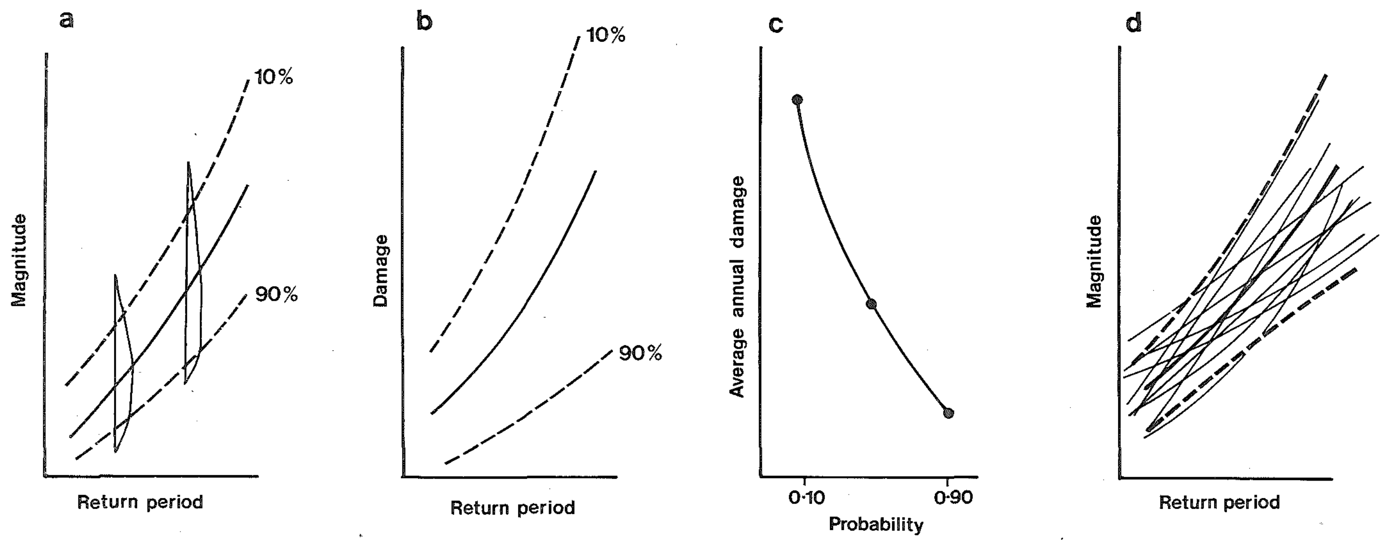

FIG. 5. Calculation of Sampling Distribution of Expected Annual Damages from Confidence Limits on Flood Magnitude—and Why It Is Incorrect

TABLE 5. Mean Standard Error of Expected Annual Damages and Observed Standard Deviation of Expected Annual Damages Estimates (Logistic Damage Function)

Expected annual damages (*EAD*) (1)	True Threshold Probability 0.2 (2)	0.1 (3)	0.04 (4)	0.02 (5)	0.01 (6)
$N = 10$					
average standard error of *EAD*[a]	2.301	1.410	0.778	0.510	0.343
standard deviation of estimated *EAD*[b]	2.332	1.410	0.776	0.511	0.344
$N = 20$					
average standard error of *EAD*[a]	1.553	0.883	0.423	0.244	0.144
standard deviation of estimated *EAD*[b]	1.398	0.765	0.359	0.206	0.119
$N = 40$					
average standard error of *EAD*[a]	1.114	0.612	0.271	0.143	0.077
standard deviation of estimated *EAD*[b]	1.036	0.548	0.240	0.128	0.068

[a]The "average standard error" is the average of 500 estimates of the standard error of *EAD*.

[b]The "standard deviation of estimated *EAD*" is the standard deviation of the 500 estimates of *EAD*.

To estimate the sampling distribution and confidence intervals for expected annual damages in practice, it would therefore be necessary to resort to the use of computer simulation. Such an approach would follow the form of the experiments reported here, with the parent distribution defined by the parameters as estimated at the site of interest. The computer experiments would allow the construction of a sampling distribution of expected annual damages and the identification of desired confidence intervals, but the mean of this distribution (the statistical "best" estimate of expected annual damages) would be different to—and greater than—the value of expected annual damages derived from the best estimate of the frequency curve. The analyst would have to insure that the parent used for the simulation experiments yielded "realistically variable" estimates of flood frequencies. This can be done by selecting a sample size which produces synthetic sampling distributions of flood quantiles consistent with previously defined confidence intervals calculated from the original site data. The synthetic sample size need not be the same as the observed sample size; additional (for example regional) information has an equivalent effect to providing extra years of data.

Conclusions

This paper has presented results of a series of computer simulation experiments into the effects of uncertainties in flood frequency estimation on the bias and variability of estimates of expected annual damages. It has been shown that the "conventional" approach (using a method such as moments or maximum likelihood to attempt to obtain unbiased estimates of flood magnitudes) slightly overestimates expected annual damages where damage begins in frequent events, with greater overestimation where damage begins in rare events. These results conflict with the assertion of Hardison and Jennings (1972) and Beard (1978) that conventional estimators underestimate expected annual damages, and it has been shown that their proposed approach—to use expected probabilities—gives even greater bias. This is because while the expected probability method gives an estimated magnitude exceeded on average with the specified risk, it does not give an unbiased estimate of the risk of a specified magnitude (such as the level at which damage begins) being exceeded. The third method considered, which computes the expected damage for each probability flood by averaging across the sampling distribution of that flood estimate, also gives higher estimates of damage for a given flood probability and hence also leads to very significant overestimation of expected annual damages. For all three methods, bias reduced rapidly as sample sizes increased.

The experiments have shown that estimates of expected annual damages are highly variable, particularly where damage begins in low-frequency events. The sampling distribution of expected annual damages is also very highly skewed. It has been shown that the bias and variability in the estimate of expected annual damages is closely linked to the bias and variability in the estimation of the probability at which damage begins, emphasizing again the importance of using as good an estimate of this threshold probability as possible.

The exact form of the flood magnitude-damage relationship determines the degree of bias in estimated expected annual damage. Bias is least if damages increase rapidly once the damage threshold is reached; conversely, it is higher the greater the magnitude that "significant" damage begins.

A simulation based method has been briefly described for deriving confidence intervals for an estimate of expected annual damages in practice.

Finally, it is important to note that the results show only the effect of uncertainties in flood magnitude-frequency estimation. In practice, the bias and variability that these produce are compounded by uncertainties in the relationships linking flood magnitude with damages.

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