

Lesson 9: Frequency Analysis Extreme Value Type I (EV1) Distribution -- Gumbel

Probability Distribution Function

The probability distribution function for an EV1 distribution is:

$$F(x) = \exp\left[-\exp\left(-\frac{x-b}{a}\right)\right],$$

where a and b are parameters. The parameters are related to the moments:

$$a = \frac{\sqrt{6}\sigma}{\pi},$$

$$b = \mu - 0.5772a,$$

where μ is the mean of X , and σ is the standard deviation of X .

Quantile Function

In terms of the exceedance probability p , the quantile function is:

$$x_p = b - a \ln(-\ln(1-p)).$$

In terms of the return period T , the quantile function is:

$$x_T = b - a \ln(-\ln(1-1/T)).$$

Substituting for the parameters a and b :

$$\begin{aligned} x_T &= \mu - \sigma \frac{\sqrt{6}}{\pi} [0.5772 + \ln(-\ln(1-1/T))] \\ &= \mu - \sigma \frac{\sqrt{6}}{\pi} \left[0.5772 + \ln\left(-\ln\left(\frac{T-1}{T}\right)\right) \right] \\ &= \mu - \sigma \frac{\sqrt{6}}{\pi} \left[0.5772 + \ln\left(\ln\left(\frac{T}{T-1}\right)\right) \right] \end{aligned}$$

Therefore:

$$x_T = \mu + K_T \sigma,$$

$$K_T = -\left(\frac{\sqrt{6}}{\pi}\right) \left(0.5772 + \ln\left(\ln\left(\frac{T}{T-1}\right)\right) \right).$$