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RIVER MEANDERS AND CHANNEL SIZE

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ABSTRACT

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This study uses an enlarged data set to (1) compare measured meander geometry to that predicted by the Langbein and Leopold (1966) theory, (2) examine the frequency distribution of the ratio radius of curvature/channel width, and (3) derive 40 empirical equations (31 of which are original) involving meander and channel size features. The data set, part of which comes from publications by other authors, consists of 194 sites from a large variety of physiographic environments in various countries. The Langbein–Leopold sine-generated-curve theory for predicting radius of curvature agrees very well with the field data (78 sites). The ratio radius of curvature/channel width has a modal value in the range of 2 to 3, in accordance with earlier work; about one third of the 79 values is less than 2.0. The 40 empirical relations, most of which include only two variables, involve channel cross-section dimensions (bankfull area, width, and mean depth) and meander features (wavelength, bend length, radius of curvature, and belt width). These relations have very high correlation coefficients, most being in the range of 0.95–0.99. Although channel width traditionally has served as a scale indicator, bankfull cross-sectional area and mean depth also can be used for this purpose.

INTRODUCTION

The two general approaches to analyzing river-meander patterns are: (1) the traditional approach, which assumes and emphasizes an underlying regularity of meander geometry (e.g. Inglis, 1947; Leopold and Wolman, 1960), and (2) the series approach, which seeks to account for varying degrees of irregularity or quasi-randomness using a detailed analysis of the meander trace (e.g. Ferguson, 1976). This paper deals with the traditional approach. That is, meanders in this investigation are treated in the simplistic, symmetrical, idealized sense. The study consists of using a large amount of empirical data to determine the extent to which theory predicts observed relations, to examine the distribution of values of the ratio bend radius of curvature/channel width, and to derive new equations involving meander geometry and channel size.

NOMENCLATURE

The channel cross-sectional dimensions studied here are the bankfull width, W; bankfull cross-sectional area, A; and bankfull mean depth, D, defined as

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A/W. Meander features of interest (Fig. 1) are the wavelength, L_m; bend length, L_b; belt width, B; radius of curvature, R_c; and arc angle, θ.

The symmetrical meander and constant-width channel of Fig. 1 of course represent the idealized case. Nearly all natural meanders lack such geometrical perfection (e.g. Carson and Lapointe, 1983). However, general underlying relations occur in spite of varying departures from symmetry.

PREVIOUS WORK

A comprehensive review of the voluminous literature on river meanders and channel dimensions is beyond the scope of this paper. Much attention in other studies logically has been devoted to the influence of water discharge — obviously the critical governing variable but not a subject of this investigation. Some of the noteworthy papers that have related meander features to channel variables are Fergusson (1863), Inglis (1947), Leopold and Wolman (1957, 1960), Zeller (1967), Leeder (1973), Ferguson (1975), Dury (1976), and Hey (1976).

The three empirical Leopold and Wolman (1960) equations give meander wavelength as a function of channel width, wavelength as a function of meander radius of curvature, and meander amplitude as a function of channel width. These and other equations will be discussed below.

THEORIES

Viable theories or models relating meander features to one another or to channel dimensions are scarce. (Many hypotheses deal with closely-related topics, such as the origin of meanders, flow in curved bends, bank erosion, and migration rates of meanders.) Most theories and models designed to predict a typical meander characteristic require special flow variables as input (e.g. Ikeda et al., 1981; Chang, 1984; Howard, 1984), and this group is not treated in this study.

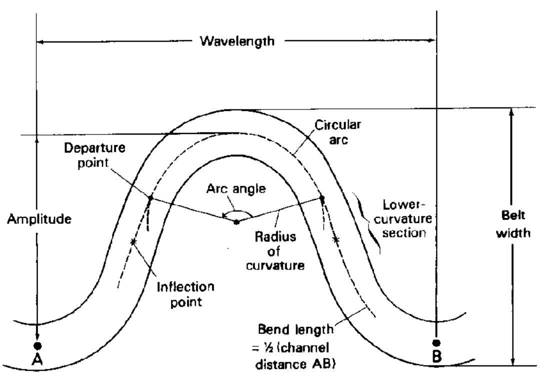

Fig. 1. Plan-view sketch of idealized river meander.

Langbein and Leopold (1966) suggested that a sine-generated curve describes symmetrical meander paths. From this basis, they derived the relation:

$$R_c = \frac{L_m K^{1.5}}{13(K - 1)^{0.5}} \tag{1}$$

in which K is channel sinuosity (ratio of channel distance to downvalley distance). This relation does not require flow variables and will be compared to field observations in this study.

The ratio of bend curvature to channel width (R_c/W) plays a key role in several hypotheses that deal with flow resistance and bend migration rates. Bagnold's (1960) separation-collapse theory, Hickin's (1978) concave-bank flow-separation theory, and Begin's (1981) flow-momentum bank-erosion model all involve or are closely interrelated with the range and distribution of naturally-occurring R_c/W values. The involvement of R_c/W in these theories and models is based on the data of Leopold and Wolman (1960), who implied (p. 787) that further work would be useful in confirming the indication that the modal value of R_c/W is in the range 2–3. The data set compiled for the present study provides a broader base for evaluating the distribution of R_c/W values.

DATA SOURCES

Data were obtained from published sources and from new measurements. The three requirements were that (1) channels were alluvial, (2) sinuosities were $\geqslant 1.20$, and (3) the same measuring technique (described below) was used. Under these criteria, published data were compiled from:

Leopold and Wolman (1957, appendix E — 9 reaches: their sites 21, 23, 25, 27, 32, 68, 71, 261b, and 261d);

Leopold and Wolman (1960, appendix — 31 reaches: sites 1–2, 4, 7–13, 15–20, 22–24, 27–28, 30, 32, 36–39, 41, and 46–48; bend lengths were divided by two to be compatible with present definition);

Carlston (1965, his table 1 — 31 reaches);

Schumm (1968, 29 reaches — his table 1, sites 2, 4–5; table 6, sites 1–9 and 16–32);

Ackers and Charlton (1970, their table 1 — 10 data sets: average of runs 6–14, 20/II);

Chitale (1970, his table 1 — 22 reaches: sites 1–6, 8–13, 17–18, 20–23, and 28–31);

Kellerhals et al. (1972, their table 1 — 14 reaches: reaches 5, 8, 17, 19–23, 25, 58, 95, 97, 108, and 116).

Other publications gave some channel dimensions for a meandering river and also included a plan-view map or aerial photograph of the reach, with scale. In these instances I was able to measure meander characteristics to complement the channel-size data. Reports in this second category were Brice (1964 — 3 reaches), Schumm (1968 — 2 reaches), Kellerhals et al. (1972 — 7 reaches), Leopold (1973 — 1 reach), and Andrews (1979 — 2 reaches).

A third group of data consisted of reaches for which I measured both the channel- and meander features, either in the field or from published maps and diagrams. This group included six western U.S. rivers, 17 rivers in Sweden (Williams, 1984), Friedkin's (1945) laboratory stream (one run), seven reaches of the Mississippi River (Fisk, 1947), and two Soviet rivers (Rozovskii, 1957). Table 1 lists the data for these second and third groups.

The entire data set for the study amounted to 194 reaches, although not all variables were available for each reach. The set contains a large variety of environments and countries, including the United States (114 sites), India and Pakistan (21 sites), Canada (21 Albertan sites), Sweden (17 sites) and Australia (5 sites).

MEASURING TECHNIQUES

For channel cross-sectional size, bankfull values specifically designated as bankfull in the published reports were preferred. However, widths labelled "channel widths" and widths measured from topographic maps also were accepted, on the assumption that these are not significantly different from bankfull widths. "Channel depths" were excluded if, insofar as could be determined, authors did not calculate them as A/W.

Cross-sectional data probably were not measured at the same relative location around or near a meander bend. Some published cross-sectional data are average values, calculated from as many as ten cross sections; in other instances, authors did not indicate whether their data are from one cross section or are averages from many cross sections. My own cross-sectional measurements are described below.

Meander features (Fig. 1) were measured from maps or aerial photographs. A single wavelength was assigned to each meander. With this method, a short lower-curvature section of channel might be included within a wavelength (e.g. Fig. 1). Similarly, circular arcs of known radius are superimposed on a meander loop, and the arc that best seems to fit the channel centerline around the bend of the loop is subjectively chosen to get radius of curvature. Other features were measured according to the definitions indicated in Fig. 1.

For the six western U.S. rivers (Table 1, this study), I measured three channel cross sections in the field and computed the arithmetic average bankfull width, bankfull cross-sectional area, and bankfull mean depth; meanders were measured on standard U.S. Geological Survey topographic maps (7-1/2 min, in most cases). For the 17 meandering rivers in Sweden (Williams, 1984), I measured meander features and channel widths from topographic maps enlarged to a scale of 1:2000. The number of wavelengths, bend lengths and belt widths I measured for each river ranged from 1 to 19 with an average of 8. The number of radius-of-curvature measurements for a reach ranged from 1 to 56, averaging 11; from 1 to 27 arc angles were measured, with an average of 14. In the cases of Friedkin (1945), Fisk (1947), and Rozovskii (1957), cross section and meander features were measured from the published cross sections and plan views.

TABLE 1

Data on channel cross-sectional- and river-meander dimensions. Measurements are conservatively rounded. Dash means no data

Site number	Site	Channel cross-sectional features			Meander features					
		bankfull area (m^2)	bankfull width (m)	bankfull mean depth (m)	wavelength (m)	bend length (m)	belt width (m)	radius of curvature (m)	sinuosity	arc angle (degrees)
Friedkin (1945, plates 40–41)										
1	Laboratory model	0.044	1.5	0.03	10.5	6.8	5.2	2.6	1.31	140
Fisk (1947, plates 10, 13, 15, 18, 19, 20, 22, 28, 34, 40, 46–48, 50)										
2	Mississippi River near Blytheville, Ark.	8,090	670	12.0	11,720	12,780	10,040	2,270	2.05	163
3	Mississippi River near Tunica, Miss.	12,350	932	13.2	13,580	13,300	8,480	2,470	2.08	159
4	Mississippi River near Arkansas City, Ark.	20,900	1,975	10.6	15,480	12,940	9,290	2,640	1.68	177
5	Mississippi River near Grand Gulf, Miss.	16,500	1,275	13.0	11,720	8,610	7,490	2,420	1.47	136
6	Mississippi River near Natchez, Miss.	15,100	980	15.4	12,040	12,800	9,500	3,430	2.05	165
7	Mississippi River near Black Hawk, La.	17,200	1,200	14.4	16,520	12,460	9,000	3,580	1.56	141
8	Mississippi River near Baton Rouge, La.	17,350	986	17.6	11,500	11,380	9,850	2,430	1.98	171
Rozovskii (1957, pp. 162, 166–171)										
9	Desna River near Chernigov, U.S.S.R.	510	133	3.8	1,790	2,040	1,590	324	2.28	177
10	Snov River near Chernigov, U.S.S.R.	10.0	27.7	0.37	—	—	—	74	—	90
Brice (1964, p. D30)										
11	Calamus River near Harrop, Nebr.	—	20.4	—	232	190	156	59	1.78	163
12	South Loup River near Cumro, Nebr.	—	34.1	—	350	286	228	87	2.28	148
13	South Loup River at St. Michael, Nebr.	—	63.5	—	792	698	548	204	1.58	179

TABLE 1 (continued)

Site number	Site	Channel cross-sectional features			Meander features					
		bankfull area (m^2)	bankfull width (m)	bankfull mean depth (m)	wavelength (m)	bend length (m)	belt width (m)	radius of curvature (m)	sinuo-sity	arc angle (degrees)
Schumm (1968, pp. 13, 25–26)										
14	Murrumbidgee River at Hay, Australia	498	74.5	6.7	634	540	438	144	2.35	152
15	Murrumbidgee River near Maude, Australia	318	50.5	6.3	710	410	418	150	1.89	153
Kellerhals et al. (1972, pp. 8–19, 29–30, 32–34, 37–38)[a]										
16	Little Smoky river near Guy, Alta., Canada	428	111	3.8	1,930	1,640	1,450	448	1.70	152
17	Notikewin River at Manning, Alta., Canada	256	74.0	3.5	1,450	1,520	1,290	228	2.10	149
18	Lesser Slave River at Highway 2, Alta., Canada	156	59.5	2.6	966	966	804	116	2.00	134
19	Swan River near Kinuso, Alta., Canada	158	42.1	3.8	804	684	402	150	1.70	165
20	Clearwater River at Draper, Alta., Canada	632	141	4.5	1,930	1,450	966	390	1.50	139
21	North Saskatchewan River at Edmonton, Alta., Canada	1,860	244	7.6	3,220	2,250	2,410	542	1.40	135
22	Vermilion River near Mannville, Alta., Canada	20.0	18.6	1.1	194	212	112	28	2.20	171
Leopold (1973, p. 1847)										
23	Watts Branch near Rockville, Md.	6.8	9.2	0.73	32	20.5	24.0	10.0	1.61	158
Andrews (1979, pp. 73, 86)										
24	Muddy Creek near Pinedale, Wyo.	3.0	5.0	0.61	36	34	23.5	8.0	1.92	174
25	East Fork River near Pinedale, Wyo.	21.0	18.0	1.2	182	112	94	43	2.47	162
Williams (1984, p. 93)										
26	Pengån near Pengsjön Nedre, Sweden	—	8.5	—	132	142	120	27.5	2.27	140

27	Öreälven near Torrböle, Sweden	—	67.0	—	680	660	546	160	2.21	130
28	Lögdeälven near Fällfors, Sweden	—	28.0	—	306	250	196	66	1.56	136
29	Lögdeälven near Norrfors, Sweden	—	32.9	—	372	310	234	75	1.67	146
30	Moälven near Kubbe, Sweden	—	11.0	—	106	88	70	24	1.70	134
31	Svågan near Svedjebo, Sweden	—	20.1	—	208	164	124	37	1.85	120
32	Ljusnan near Ljusnedal Övre, Sweden	—	13.1	—	112	106	83	27.5	2.29	148
33	Voxnan near Nybro, Sweden	—	38.1	—	274	226	180	81	2.60	113
34	Örekilsälven near Gunnarsbo, Sweden	—	13.1	—	278	246	184	67	1.95	121
35	Emån near Järnforsen, Sweden	—	20.1	—	204	200	152	50	2.35	121
36	Lagan near Värnamo, Sweden	—	25.0	—	244	282	232	69	2.10	130
37	Häråan near Granstorp, Sweden	—	17.1	—	122	110	95	28.5	2.43	128
38	Nissan near Färgebro, Sweden	—	27.1	—	318	258	202	59	2.01	113
39	Ätran near Hillared, Sweden	—	24.1	—	186	154	138	41	1.60	118
40	Assman near Assmebro, Sweden	—	21.0	—	192	146	116	44	1.43	118
41	Bivarödsån near Bivarödsmölla, Sweden	—	11.0	—	96	81	68	20	1.91	127
42	Skräbeån near Näsum, Sweden	—	18.0	—	132	120	112	32	2.33	126
Williams (this study)										
43	Tomichi Creek at Gunnison, Colo.	16.0	23.5	0.70	176	152	106	29	1.93	132
44	Blacks Fork at Little America, Wyo.	104	58.0	1.8	698	628	446	144	1.90	171
45	Dry Piney Creek near Big Piney, Wyo.[b]	2.5	3.4	0.76	46	34	21.5	10.5	1.66	122
46	White River near Soldier Summit, Utah	9.7	13.7	0.70	70	55	37	14.0	1.93	139
47	Sevier River near Lynndyl, Utah	29.0	21.9	1.3	606	506	338	126	2.19	137
48	Thomas Creek near Scio, Oreg.	18.0	24.7	0.70	626	390	234	172	1.32	120

[a] Bend lengths for all sites of this author computed as 1/2 (sinuosity × wavelength) by present author.
[b] Ephemeral.

Chitale (1970) and Ackers and Charlton (1970) do not mention whether their values are for a single selected meander or are reach averages. Leopold and Wolman (1957, 1960) for some rivers measured only one "reasonably symmetrical, representative" meander from a reach (they are the only authors within this study known to have used this method), and for other rivers they measured as many as four or five bends and gave the median value. Carlston (1965), Schumm (1968), and Kellerhals et al. (1972) gave reach averages, presumably arithmetic averages. My own measurements also are arithmetic averages. Most of the meander data in this study therefore are reach averages. Visual inspection of plotted data gave no indication of significant differences between single-meander- and reach-averaged data.

Of the potential sources of error associated with this traditional way of measuring meander geometry (Hooke, 1984), the only one of some significance probably is the subjectivity involved. This in turn is largely due to differences in delineating the meander features (wavelength endpoints, arc that best fits a loop, etc.). Some approximate maximum percentage differences due to this subjectivity, based on having two people analyze the same meanders, are: wavelength, bend lengths and belt width, 15% (well-defined meanders with no intervening straight reaches); radius of curvature, 25% in the R_c of any single arc and 6% in the average R_c for the arcs in a river reach; arc angle, 13% if the two investigators have chosen the same R_c and 40% if they have not.

THEORY PREDICTIONS

Seventy eight observations were available for use in eqn. (1), the Langbein and Leopold (1966) equation for bend radius of curvature. The data cover three log cycles of R_c. Predicted versus observed R_c-values are shown in Fig. 2. The points plot about the line of perfect agreement with a standard error of estimate of 0.0869 $\log_{10}$unit or about 20%. In view of the variety of conditions and investigators represented in the data set, this degree of agreement probably would be considered quite satisfactory by most observers.

RADIUS OF CURVATURE/CHANNEL WIDTH RATIO

The frequency distribution of the 79 available R_c/W values (Fig. 3) is asymmetric, regardless of whether arithmetic or geometric class intervals are used; of the two scales, it is more nearly symmetric on the geometric basis (Fig. 3). The distribution is slightly different from that based on just Leopold and Wolman's (1960) data, due to both the additional data and to the exclusion here of channels with sinuosities < 1.20. (31 of the 79 values in the present data set are from the 1960 paper.) However, there are no radical departures from Leopold and Wolman's results.

The computed geometric mean value of R_c/W is 2.43. The range is from 1.02 to 6.97 or about from 1 to 7. The data of Leopold and Wolman (1960), which include sinuosities < 1.20, have a wider range, namely from 0.84 to 9.7. The central two-thirds of the distribution lies between values of 1.6 and 3.4, whereas

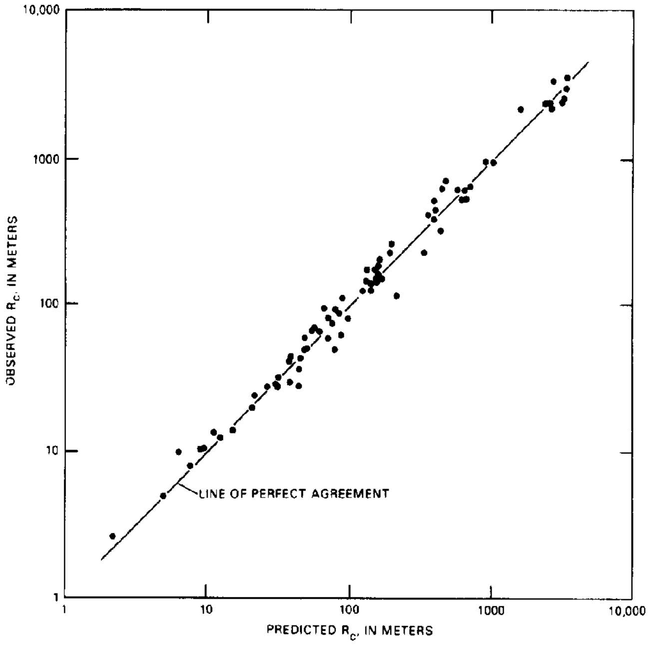

Fig. 2. Observed values of bend radius of curvature versus values predicted by the equation of Langbein and Leopold (1966).

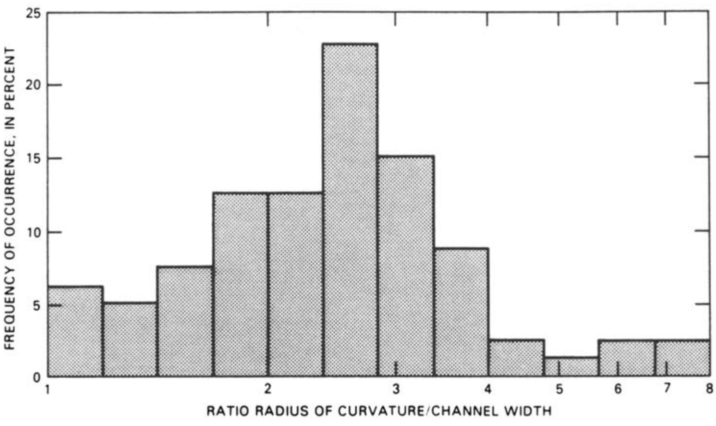

Fig. 3. Frequency distribution of R_c/W values for 79 streams.

the corresponding Leopold and Wolman values are 1.5 and 4.3. About 42% of the values are between 2.0 and 3.0, compared to 25% reported by Leopold and Wolman (1960, p. 774). Thus the present data have a somewhat better sorting or stronger central tendency than the Leopold and Wolman data, at least in part due to the required minimum sinuosity of 1.20. About one-third of the values is less than 2.0; this suggests perhaps a more common occurrence of such lower values than might heretofore have been anticipated.

NEW EMPIRICAL RELATIONS

Plots of the data showed that a power law describes the relation between any two variables, as expected. A reduced major axis line (Imbrie, 1956; Hirsch and Gilroy, 1984, their line of organic correlation; Troutman and Williams, 1986) was fitted to 38 two-variable relations, using logarithms and then de-transforming from logs (arithmetic laws) to power laws. No reduced major axis program was available for the two additional cases having two independent variables; a least-squares multiple regression equation was derived in these instances. (Structural relations, of which the reduced major axis is a special case, estimate the true or actual relation between variables. The reduced major axis assumes approximately equal percentage errors in the two variables.) Figure 4 shows some typical plots.

Table 2 lists the 40 equations derived. Twelve of these (eqns. 2–13) relate meander features to one another, 12 (eqns. 14–25) give channel size as a function of a meander feature, 12 (eqns. 26–37) give a meander feature as a function of some measure of channel size, and the remaining four (eqns. 38–41) involve channel width, channel depth, and sinuosity. All correlations are significant at the 0.01% level.

In the initial line fitting, the several equations having a common dependent or independent variable turned out to have nearly the same exponent (a difference of only a few percent — typically 2–6% — from the average exponent for the group). For example, the four equations relating channel width to meander features had exponents ranging from 0.86 to 0.94. The originally-computed exponents in the 12 equations relating meander features to one another ranged from 0.97 to 1.03, with 10 of the 12 falling between 0.98 and 1.02. The average exponent was adopted for the several equations of each subgroup, on the basis of my assumption that this average exponent was the best approximation to the true exponent. Intercepts then were adjusted accordingly.

Nine of the 40 equations (discussed in the following section) are of a type similar to equations proposed by other authors; the remaining 31 equations do not seem to have been proposed previously. A feature of potential significance that emerges from these new equations is that channel cross-sectional area and mean depth can serve as scale indicators — in some instances as well as channel width. (Width traditionally has been the only channel feature used for this purpose, probably because it is the easiest of the three to measure.)

COMPARISON WITH EARLIER EQUATIONS

Interrelations between meander features

Of the 12 equations relating meander features to one another (eqns. 2–13), the only one having a prominent counterpart in previous studies is eqn. (4):

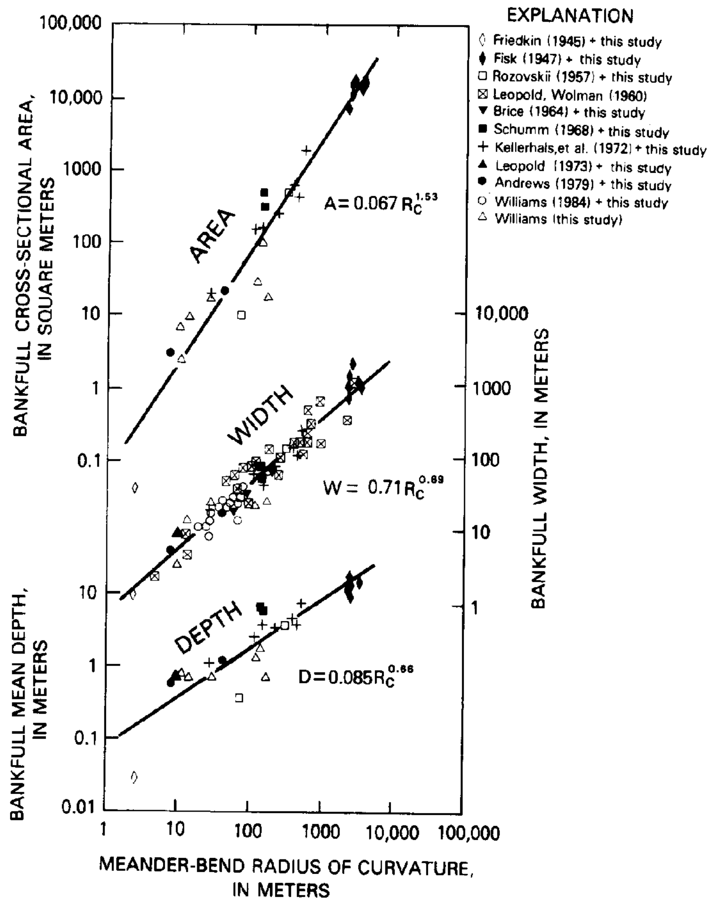

Fig. 4. Graph of typical data for bankfull cross-sectional area, width, and mean depth related to meander-bend radius of curvature.

$L_m = 4.53R_c$. In metric units, Leopold and Wolman's (1960) counterpart is $L_m = 4.6R_c^{0.98}$. These relations are very similar to one another.

Meander features related to channel size

Equations proposed by other authors for meander wavelength versus channel width, both in meters, are: $L_m = 11.0W^{1.01}$ (Leopold and Wolman, 1960); $L_m = 10.0W^{1.025}$ (Zeller, 1967); $L_m = 6.28W$ (Yalin, 1971); and $L_m = 12.34W$ (Richards, 1982). The relation for the 191 observations in the present data set is $L_m = 7.5W^{1.12}$ (eqn. 30). Thus the new intercept of 7.5 is slightly larger than Yalin's but about 25 to 40% smaller than those of the other equations. Moreover, the new exponent of 1.12 is slightly larger than the 1.0 (or thereabouts) of the earlier equations.

TABLE 2

Derived empirical equations for river-meander and channel-size features (A = bankfull cross-sectional area, W = bankfull width, D = bankfull mean depth, L_m = meander wavelength, L_b = along-channel bend length, B = meander belt width, R_c = loop radius of curvature, K = channel sinuosity, m = meters)

Equation number	Equation	Standard deviation of residuals, in percent +	Standard deviation of residuals, in percent −	Sample correlation coefficient r	Number of data points	Applicable range
Interrelations between meander features						
2	$L_m = 1.25L_b$	32	24	0.99	102	$5.5 \leqslant L_b \leqslant 13{,}300$ m
3	$L_m = 1.63B$	31	24	0.99	155	$3.7 \leqslant B \leqslant 13{,}700$ m
4	$L_m = 4.53R_c$	21	17	0.99	78	$2.6 \leqslant R_c \leqslant 3{,}600$ m
5	$L_b = 0.80L_m$	32	24	0.99	102	$8 \leqslant L_m \leqslant 16{,}500$ m
6	$L_b = 1.29B$	31	24	0.99	102	$3.7 \leqslant B \leqslant 10{,}000$ m
7	$L_b = 3.77R_c$	35	26	0.98	78	$2.6 \leqslant R_c \leqslant 3{,}600$ m
8	$B = 0.61L_m$	31	24	0.99	155	$8 \leqslant L_m \leqslant 23{,}200$ m
9	$B = 0.78L_b$	31	24	0.99	102	$5.5 \leqslant L_b \leqslant 13{,}300$ m
10	$B = 2.88R_c$	42	29	0.98	78	$2.6 \leqslant R_c \leqslant 3{,}600$ m
11	$R_c = 0.22L_m$	21	17	0.99	78	$10 \leqslant L_m \leqslant 16{,}500$ m
12	$R_c = 0.26L_b$	35	26	0.98	78	$6.8 \leqslant L_b \leqslant 13{,}300$ m
13	$R_c = 0.35B$	42	29	0.98	78	$5 \leqslant B \leqslant 10{,}000$ m
Relations of channel size to meander features						
14	$A = 0.0054L_m^{1.53}$	103	51	0.96	66	$10 \leqslant L_m \leqslant 23{,}200$ m
15	$A = 0.0085L_b^{1.53}$	140	58	0.95	41	$6 \leqslant L_b \leqslant 13{,}300$ m
16	$A = 0.012B^{1.53}$	97	49	0.97	63	$5 \leqslant B \leqslant 11{,}600$ m
17	$A = 0.067R_c^{1.53}$	138	58	0.97	28	$2 \leqslant R_c \leqslant 3{,}600$ m
18	$W = 0.17L_m^{0.89}$	56	36	0.96	191	$8 \leqslant L_m \leqslant 23{,}200$ m
19	$W = 0.23L_b^{0.89}$	56	36	0.97	102	$5 \leqslant L_b \leqslant 13{,}300$ m
20	$W = 0.27B^{0.89}$	63	39	0.96	153	$3 \leqslant B \leqslant 13{,}700$ m
21	$W = 0.71R_c^{0.89}$	48	32	0.97	79	$2.6 \leqslant R_c \leqslant 3{,}600$ m

22	$D = 0.027L_m^{0.66}$	79	44	0.86	66	$10 \leqslant L_m \leqslant 23{,}200\,m$
23	$D = 0.036L_b^{0.66}$	72	42	0.90	41	$7 \leqslant L_b \leqslant 13{,}300\,m$
24	$D = 0.037B^{0.66}$	66	40	0.90	63	$5 \leqslant B \leqslant 11{,}600\,m$
25	$D = 0.085R_c^{0.66}$	90	47	0.90	28	$2.6 \leqslant R_c \leqslant 3{,}600\,m$
Relations of meander features to channel size						
26	$L_m = 30A^{0.65}$	59	37	0.96	66	$0.04 \leqslant A \leqslant 20{,}900\,m^2$
27	$L_b = 22A^{0.65}$	77	43	0.95	41	$0.04 \leqslant A \leqslant 20{,}900\,m^2$
28	$B = 18A^{0.65}$	56	36	0.97	63	$0.04 \leqslant A \leqslant 20{,}900\,m^2$
29	$R_c = 5.8A^{0.65}$	76	43	0.97	28	$0.04 \leqslant A \leqslant 20{,}900\,m^2$
30	$L_m = 7.5W^{1.12}$	65	39	0.96	191	$1.5 \leqslant W \leqslant 4{,}000\,m$
31	$L_b = 5.1W^{1.12}$	65	39	0.97	102	$1.5 \leqslant W \leqslant 2{,}000\,m$
32	$B = 4.3W^{1.12}$	74	42	0.96	153	$1.5 \leqslant W \leqslant 4{,}000\,m$
33	$R_c = 1.5W^{1.12}$	55	35	0.97	79	$1.5 \leqslant W \leqslant 2{,}000\,m$
34	$L_m = 240D^{1.52}$	142	59	0.86	66	$0.03 \leqslant D \leqslant 18\,m$
35	$L_b = 160D^{1.52}$	128	56	0.90	41	$0.03 \leqslant D \leqslant 17.6\,m$
36	$B = 148D^{1.52}$	115	53	0.90	63	$0.03 \leqslant D \leqslant 18\,m$
37	$R_c = 42D^{1.52}$	165	62	0.90	28	$0.03 \leqslant D \leqslant 17.6\,m$
Relations between channel width, channel depth, and channel sinuosity						
38	$W = 21.3D^{1.45}$	160	62	0.81	67	$0.03 \leqslant D \leqslant 18\,m$
39	$D = 0.12W^{0.69}$	94	48	0.81	67	$1.5 \leqslant W \leqslant 4{,}000\,m$
40	$W = 96D^{1.23}K^{-2.35}$	121	55	0.87	66	$0.03 \leqslant D \leqslant 18\,m$ and $1.20 \leqslant K \leqslant 2.60$
41	$D = 0.09W^{0.59}K^{1.46}$	73	42	0.86	66	$1.5 \leqslant W \leqslant 4{,}000\,m$ and $1.20 \leqslant K \leqslant 2.60$

Besides the differing data sets, a substantial part of the differences between the equations of Leopold and Wolman (1960), Zeller (1967) and the one derived here could be due to the way in which the lines were fitted. The Leopold and Wolman, and Zeller papers do not say how the lines were fitted. The different data sets, different fitting methods, and lack of associated statistics prevent the testing of constants in different equations for significant differences.

By algebraic manipulation of the Leopold and Wolman (1960) equations, Hey (1976, 1984) arrived at $L_b = 6.28W$. The corresponding equation from Table 2 is $L_b = 5.1W^{1.12}$ (eqn. 31).

Zeller (1967) listed "meander width" $= 4.5W^{1.00}$. It is not clear whether this is meander amplitude or belt width (Fig. 1). Leopold and Wolman's (1960) relation, in metric units, gives amplitude (not belt width) as $3.0W^{1.1}$. For the present data, belt width $B = 4.3W^{1.12}$ (eqn. 32).

Hey (1976, 1984), again by algebraic manipulation of the Leopold and Wolman (1960) relations, arrived at $R_c = 2.4W$. The equation of Carson and Lapointe (1983, p. 54) implies that R_c varies with $W^{1.0}$ but that the constant of proportionality varies with several other meander variables. Phelps (1984) gave $R_c = 4.59W$ but measured R_c to the outer edge of the bend rather than to the centerline. The equation of Table 2 is $R_c = 1.5W^{1.12}$ (eqn. 33). Again, therefore, the exponent derived here is slightly greater than the 1.0 usually adopted, and the intercept is smaller.

Hey (1976, 1984) also concluded that arc angle should be incorporated as a second independent variable in the relations between any two of wavelength, radius of curvature and channel width. In multiple regressions of the present data, with $\log_{10}$ of arc angle as the second independent variable, inclusion of arc angle did not add substantial improvement to the simpler two-variable relations. Possible explanations are that (1) arc angle is not relevant, (2) the range of arc angles I had (about 90–180°) is not wide enough, and their distribution is not sufficiently uniform within this range, to reveal any statistical significance of arc angle, and (3) arc angles were measured between inflection points in Hey's studies but between departure points (Fig. 1) in this study.

Channel width and depth

Leeder (1973) derived an empirical equation between bankfull width and channel maximum depth D_{max}, where D_{max} is the elevation difference between the lower banktop and the thalweg. His relation is $W = 6.8D_{max}^{1.54}$, for 57 channels with sinuosity $K > 1.70$. For my data, $W = 15.5D^{1.40}$ (30 sites; $K > 1.70$). The differences between the two equations apparently are due to the different data sets and to the different definitions of channel depth (maximum depth versus hydraulic mean depth). (In a purely mathematical sense, it is spurious to relate W to A/W; in a physical or conceptual sense, however, bankfull width is a distinctly different variable from bankfull mean depth.) For all of the present bankfull width–depth data (67 sites), regardless of sinuosity, $W = 21.3D^{1.45}$ (eqn. 38).

Improved relations between channel width and depth are obtained by including sinuosity as a second independent variable. Multiple regression (least squares) on the 66 available stations produces $W = 96D^{1.23}K^{-2.35}$ (eqn. 40). Using the same data, $D = 0.09W^{0.59}K^{1.46}$ (eqn. 41). If one chooses to use in the model those variables that are statistically significant at the 0.01% level, then K should be included in the model. On the other hand, merely to estimate a value of W or D (see discussion of prediction, below), knowledge of K probably will mean knowledge of at least one of L_m, L_b, B, or R_c, and the simpler and more accurate relations between these meander features and W or D (eqns. 18–25) can be used.

ESTIMATED TRUE RELATIONS AND PREDICTION

The counterpart ordinary least squares (OLS) relations (not shown) for eqns. 2–37 were computed to see if the reduced major axis relations were significantly worse for purposes of predicting values of a dependent variable. Despite differences in constants, the standard errors (in percent) showed surprising agreement. (The standard deviation of the residuals of the Table 2 equation was compared to the standard error of the OLS equation.) The errors above the fitted line, in percent, were within two percentage points (much of which probably is rounding error) for 28 of the 36 pairs of equations; in seven other instances the OLS equation was 3–5 percentage points better than the Table 2 equation; and in the final instance (predicting R_c from D), the OLS equation was 16 percentage points better. For computed errors below the fitted line, percentage errors in all 36 instances were within two percentage points.

Based on this comparison, the Table 2 equations (eqns. 2–37) statistically can serve about as well as OLS equations for prediction or estimation, at least for the conditions reflected in the data. Estimates of a meander feature from another meander feature (eqns. 2–13) have the lowest errors; most estimates statistically can be expected to fall within about -20% to $+40\%$ of true values (Table 2). In using a meander feature to estimate a channel-size variable (eqns. 14–25), the expectable errors are within about -50% to $+140\%$ in estimating A, -35% to $+60\%$ in estimating W, and -40% to $+90\%$ in estimating D. Using channel dimensions to estimate a meander characteristic (eqns. 26–37), most estimates are likely to fall within about -40% to $+75\%$ if either A or W is given and -60% to $+160\%$ if D is given.

Some possible applications of eqns. 2–37 for estimation purposes might be in the restoration of disturbed streams on strip-mined landscapes (Rechard and Schaefer, 1984), the estimation of channel size from maps or aerial photographs, the design of sinuous canals, and the paleohydrologic postdiction of the characteristics of former streams from a surviving remnant of a channel.

CONCLUSIONS

Data on channel sinuosity, meander wavelength, and bend radius of curvature agree well with the R_c-values predicted by Langbein and Leopold's

(1966) sine-generated curve theory. The frequency distribution of 79 naturally-occurring R_c/W values has a geometric mean value of 2.43; the central two thirds of the distribution falls between 1.6 and 3.4; nearly one third of the values is less than 2.0.

Of 40 empirical equations involving meander and channel-size features, 9 are of the same form as equations already in use, and the remaining 31 (especially those involving bankfull cross-sectional area and depth) are new. Channel sinuosity showed some potential in relations between bankfull width and mean depth, for these meandering streams. The equations probably approximate the true relations between the variables and are about as good as least-squares equations for prediction; accuracy of such predictions ranges from about 20 to 160% in standard error, depending on the particular equation used.

Natural differences in meanders, along with the techniques used for measurement and analysis, are such that the equations represent broad generalities only; local variability (departures or noise) is not accounted for. Nevertheless, the correlations suggest identifiable underlying tendencies and a general orderliness in the plan morphology and related cross-sectional size of natural meandering channels. Incorporation of these results into a unifying rational theory remains to be done; in this sense, the equations represent problems more than they do conclusions (Mackin, 1963).

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