

## FLOW DURATION CURVES II: A REVIEW OF APPLICATIONS IN WATER RESOURCES PLANNING<sup>1</sup>

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**ABSTRACT:** A streamflow duration curve illustrates the relationship between the frequency and magnitude of streamflow. Flow duration curves have a long history in the field of water-resource engineering and have been used to solve problems in water-quality management, hydropower, instream flow methodologies, water-use planning, flood control, and river and reservoir sedimentation, and for scientific comparisons of streamflow characteristics across watersheds. This paper reviews traditional applications and provides extensions to some new applications, including water allocation, wasteload allocation, river and wetland inundation mapping, and the economic selection of a water-resource project.

(**KEY TERMS:** hydrology; hydraulics; water resources engineering; streamflow; flow duration curves; hydropower; sedimentation; instream flow; floods; water quality; habitat suitability; water allocation; wetlands; rivers.)

### INTRODUCTION

A streamflow duration curve illustrates the percentage of time a given streamflow was equalled or exceeded during a specified period of time. Historically, streamflow duration curves have been used in hydrologic studies including hydropower engineering, flood control, water-quality management, river sedimentation, and water-use engineering. Given their widespread applications and long history, the literature on flow duration curves (FDCs) is surprisingly sparse. This is the first comprehensive review of applications of FDCs since Searcy (1959). A companion paper by Vogel and Fennessey (1994) introduces some new nonparametric methods for constructing and interpreting an FDC and associated confidence intervals.

This study reviews the application of FDCs to river and wetland inundation mapping; river, reservoir and lake sedimentation studies; instream flow assessments; hydropower feasibility analysis; water quality management; wasteload allocation; water-resource allocation; flood frequency analysis; flood damage assessment; and the selection of an optimal water-resource project.

### THE GRAPHICAL INFORMATION CONTENT OF FLOW DURATION CURVES

Flow duration curves exemplify the old Chinese proverb "one picture is worth a thousand words" through their ability to condense a wealth of hydrologic information into a single graphic image. FDCs are used to summarize the results of detailed and complex water-resource studies. FDCs are often used to summarize the impacts of potential climate change scenarios on water-resource systems (see, for example, Schwarz, 1977, Figure 7.1). FDCs are often used to graphically illustrate the impact of regional differences in geology, climate, and physiography on the hydrologic response of river basins (Searcy, 1959; Pearce, 1990; also see references in Fennessey and Vogel, 1990). Fennessey and Vogel (1990) review regional FDC models developed in the U.S. and elsewhere.

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## THE INTERPRETATION OF A FLOW DURATION CURVE

A flow duration curve is the complement of the cumulative distribution function for streamflow. In an FDC, discharge ( $Q$ ) is plotted against exceedance probability ( $p$ ). This section reviews the traditional "period-of-record" FDC and the annual-based FDC introduced by Vogel and Fennessey (1994), as well as the differences between the two.

An FDC represents the relationship between the magnitude and frequency of daily, weekly, monthly streamflow (or some other time interval) for a particular river basin. Searcy (1959, Figure 2) provides a comparison of the daily, monthly, and annual FDCs for a river basin, although most applications of FDCs use daily streamflows. If average daily streamflows for the complete period-of-record are used and if the data are stationary, then the resulting FDC represents the steady-state or long-term exceedance probability for a station. Given the usual anthropogenic modifications due to reservoir operations, diversions, transfers, land-use changes, and other changes, the "steady-state" FDC is really just a concept. Yet such a "period-of-record" or "steady-state" FDC is useful for describing the likelihood of daily streamflows over a long planning horizon. Vogel and Fennessey (1994) document that the lower tail of daily FDCs are highly sensitive to the particular period-of-record used. Therefore the interpretation of a "period-of-record" FDC depends upon the particular period-of-record used. This fact led Vogel and Fennessey (1994) to define annual-based daily FDCs which allow one to define the average return period and to construct confidence intervals for an FDC, useful information in hydrologic planning and design.

Annual-based daily FDCs consist of  $n$  daily FDCs constructed for each year of record. To summarize the resulting year-to-year variability among the  $n$  annual FDCs, Vogel and Fennessey (1994) suggest using the median of the annual-based FDCs to represent the frequency and magnitude of streamflow in a typical (but hypothetical) year. Vogel and Fennessey (1994) term this the median annual FDC. The median annual FDC is computed as the median value of streamflow (across the  $n$  years of streamflow) for each exceedance probability  $p$  (there are 365 exceedance probabilities associated with the 365 days in each year), and it represents the distribution of daily streamflow in a "typical" or median hypothetical year. Unlike the traditional "period-of-record" FDC, the interpretation of the median annual FDC is minimally affected by the observation of abnormally wet or dry periods during the period of record.

LeBoutillier and Waylen (1993) introduced an annual interpretation of FDCs for the purpose of

selecting a suitable probability density function for daily streamflow. Searcy (1959) suggested an annual interpretation of FDCs for examining the year-to-year variations in streamflow. Searcy (1959) suggested using climatic years beginning in April 1 when constructing annual FDCs in order to avoid the arbitrary division of low-flow periods. Vogel and Fennessey (1994) describe how to associate confidence intervals, average recurrence intervals, and annual reliabilities with an annual-based FDC.

## THE PROBABILITY DISTRIBUTION OF DAILY STREAMFLOW

Since sequences of daily streamflow often contain thousands of observations, one expects it to be easy to discern the statistics of daily streamflow. Interestingly, Vogel and Fennessey (1993) use the theory of L-moments to prove that ordinary product-moment ratios such as the coefficient of variation and the coefficient of skewness provide almost no information about the probability distribution of daily streamflow. This is due to the fact that daily streamflows originate from highly skewed populations and product-moment ratios contain remarkable bias under those circumstances even for sample sizes in the tens of thousands. Vogel and Fennessey (1993) and Fennessey and Vogel (1994) use L-moment diagrams to document that the generalized Pareto distribution and the three-parameter lognormal distributions provide a good approximation to the distribution of daily streamflow at 166 basins in the northeastern United States.

## SOME ADVANTAGES AND LIMITATIONS OF FLOW DURATION CURVES

FDCs apply to a variety of water resource problems, are easy to use, explain, and understand, and, as graphical displays, express a wealth of hydrologic information. Their widespread usage is in part due to the fact that FDCs can convey complex hydrologic information to decision makers who may not have a background in hydrology. FDCs have a long history in water resource engineering, and recent innovations which allow one to compute average return periods and confidence intervals for FDCs (Vogel and Fennessey, 1994) provide additional flexibility for their application. FDCs are attractive because they tend to simplify water resource problems and allow easy explanations for them, but their primary limitation is that they tend to oversimplify.

There are further limitations associated with the use of FDCs in water resource engineering. Since the serial structure of streamflow is ignored, FDCs should not be applied to any problem in which the timing of streamflow is important. For example, one cannot use FDCs to describe the behavior of storage reservoirs or detention ponds because the timing of streamflow is so important in those applications. When FDCs are based on average daily streamflow, as is usually the case, they should not be used in flood frequency applications which require instantaneous peak streamflows. Also, the interpretation of an FDC will depend upon the particular "period-of-record" used unless annual FDCs are employed. Probably the most significant limitation associated with FDCs relates to their use with rating curves, as is discussed in the following section.

### WATER RESOURCE INDEX DURATION CURVES

A water resource index duration curve is defined as the relationship which describes the exceedance probability of any appropriate water resource index such as hydropower energy output, river sediment load, turbidity, habitat suitability, river stage, etc. Figure 1 illustrates the construction of a water-resource index duration curve. A rating curve, describing the relationship between streamflow and the water resource index, is combined with an FDC to produce the water-resource index duration curve. This analysis makes sense for any water resource index which can be uniquely related to streamflow. The accuracy of the resulting water resource index duration curve depends upon the accuracy of the FDC and the rating curve. Such rating curves can be both misleading and incorrect because many water resource indices are not related to streamflow alone, but rather are complex multivariate functions of other variables in addition to streamflow. For example, many river cross-sections do not exhibit unique stage-discharge relationships due to varied hydraulic controls which are exerted either upstream or downstream of the location of interest. Similar limitations exist for all rating curves.

Procedures for developing rating curves which describe the relationship between river stage and discharge are well documented in Linsley *et al.* (1982, pp. 107-112). Other rating curves which describe the relationship between streamflow and turbidity, sediment load, or say habitat suitability are not nearly as well defined or well documented as the classical stage discharge rating curve; nevertheless, such rating curves can be useful for deriving duration curves for water quality, sediment, fish management,

hydropower, and other water resource applications. Unfortunately, all such rating curves are gross simplifications which may or may not be accurate descriptions of reality.

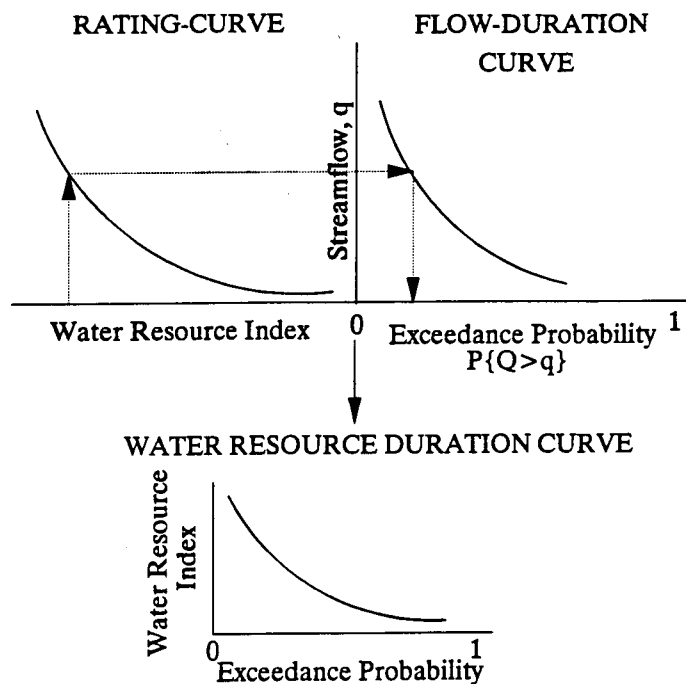


Figure 1. Illustration of the Development of a Water Resource Index Duration Curve.

The interpretation of the resulting water resource index duration curve depends upon the method used to construct it. For example, if a median annual FDC is used to construct the index duration curve, then the resulting index duration curve represents the exceedance probability of that index during a typical or median year.

### WATER RESOURCE INDEX DURATION CURVES FOR A WIDE RANGE OF APPLICATIONS

This section discusses the use of rating curves and water resource index duration curves in various water resource applications.

#### *River and Wetland Inundation Mapping Using a Stage Duration Curve*

The stage discharge rating curve can be combined with an FDC to produce a stage duration curve. Here

stage becomes the water resource index in Figure 1. The stage duration curve represents the likelihood that a given water surface elevation will be exceeded over a prespecified period. If the median annual FDC is used, then the resulting stage duration curve represents the probability of exceedance of a given water surface elevation (stage) during a median or typical (but hypothetical) year.

Stage-duration curves for selected cross-sections along a river or wetland can be transformed into inundation maps analogous to the way in which floodplain delineations are created by using flood stages derived from hydraulic step-backwater computations (Urbonas and Roesner, 1993). The use of stage duration curves for inundation mapping has distinct advantages over traditional floodplain management procedures which tend to focus only on the most severe annual maximum floods with recurrence intervals of 100- and 500-years. Stage duration curves can provide information about all discharge events; hence, they are particularly well suited to ecologic, wetland, and other habitat investigations which seek to define the frequency of inundation for all stages, not just the extreme floods. However, the stages and inundation maps derived from a FDC which is based on average daily streamflow cannot substitute for traditional (peak flow) flood frequency analysis which is (and should be) based on instantaneous peak stages and discharges.

#### *River, Reservoir, and Lake Sedimentation Studies Using Sediment Duration Curves*

All reservoirs and lakes suffer varying degrees of sedimentation. An understanding of river sediment loads is needed to evaluate impacts of urbanization on runoff, for quantifying pesticide loads, heavy metal loads, and other forms of pollutant transport, and to ensure that sedimentation does not interfere with reservoir operations.

Since the primary mechanism for the movement of both suspended sediment and bedload is extreme flood events (Meade and Parker, 1984), many investigators have focused exclusively on the distribution of the annual maximum suspended sediment load (Parker and Troutman, 1989). This reasoning is similar to the reasoning in flood frequency investigations which usually leads investigators to concentrate on the distribution of annual maximum floodflows. Annual maximum sediment transport studies combine a suspended sediment discharge rating curve with the cumulative distribution of annual maximum discharge to obtain the cumulative distribution of annual maximum suspended sediment load. Such analyses are useful, yet they ignore the sediment

loads corresponding to all events less than the annual maximum floodflow.

The U.S. Bureau of Reclamation takes a different approach. For quantifying total sediment load into reservoirs, the U.S. Bureau of Reclamation (Strand and Pemberton, 1982) advocates the use of the period-of-record (or steady state) FDC in combination with the suspended sediment-discharge rating curve to produce a suspended sediment-duration curve. Now suspended sediment load becomes the water resource index in Figure 1. These procedures quantify the magnitude and frequency of the complete history of suspended sediment load; hence, they may be more suitable than the procedures described by Parker and Troutman (1989) and others for estimating both average annual sediment loads and the year-to-year variability in sediment load.

Extrapolation of rating curves can lead to serious errors since sediment load rating curves often contain significant retransformation bias (Thomas, 1988). In some cases retransformation bias can lead to underestimation of constituent loads by as much as 50 percent (Cohn *et al.*, 1989). Hirsch *et al.* (1993, Section 17.4.4) review several procedures for reducing and/or eliminating retransformation bias. Sediment load rating curves often exhibit lack of fit due to missing variables and non-normality of residuals (Thomas, 1988). Cohn *et al.* (1989) describe a minimum variance unbiased estimator of individual sediment loads which should perform well even when sediment-load rating curves exhibit lack of fit, retransformation bias, and non-normal residuals.

#### *Assessment of Instream Flow Requirements Using Habitat Duration Curves*

Recently, environmental concerns about the impacts of urbanization on aquatic ecosystems have increased. Maintaining instream flows for a single purpose such as recreation, navigation, hydropower peaking operations, maintenance of endangered species, or esthetics is no longer possible. The engineer's perspective is now more holistic, viewing rivers as balanced ecosystems. Engineers are often asked to make recommendations for instream flows to assure adequate fish passage, temperature levels, dissolved oxygen, turbidity, and sediment concentrations, and to maintain existing aquatic habitats ranging from fish species to the flora and fauna in the river under consideration.

In order to protect and insure instream flows, the volume of water available, under all circumstances, must be quantified. Estes and Orsborn (1986) and Gordon *et al.* (1992) review a variety of methods for determining in-stream flow requirements ranging

from "rule-of-thumb" methods to computer simulation models. Perhaps the most widely accepted approach in the U.S. is the Instream Flow Incremental Method (IFIM), introduced by the Cooperative Instream Flow Group of the U.S. Fish and Wildlife Service (Milhous *et al.*, 1990) and summarized by Nestler *et al.* (1989). IFIM is a conceptual framework for the assessment of river habitats. It consists of a collection of computer modules which can predict changes in fish and other habitats due to modifications in river flow regimes.

A primary component of IFIM is the Physical Habitat Simulation System termed PHABSIM (Milhous *et al.*, 1990). PHABSIM can be used to develop a rating curve which relates total habitat area to river discharge for a particular species during a particular stage in its life. This rating curve is then combined with an FDC to produce a habitat-duration curve as shown in Figure 2.

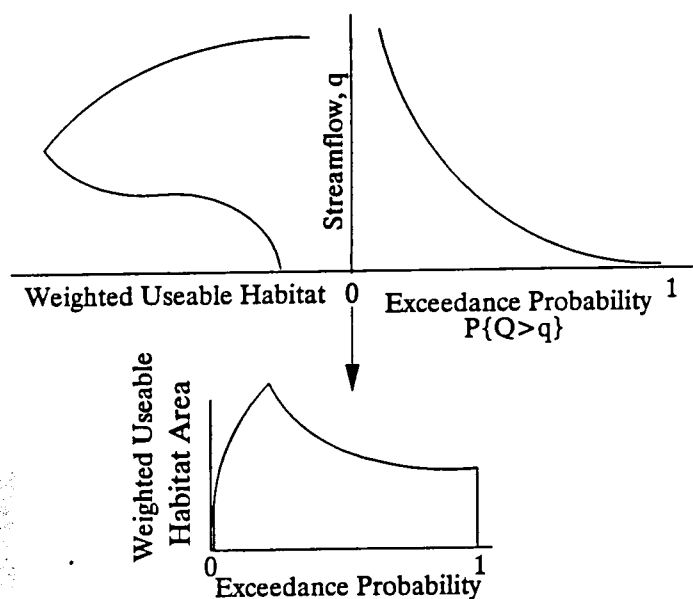


Figure 2. Illustration of the Development of a Habitat Duration Curve.

the implementation of FDCs in hydropower feasibility studies (for example, see Salembier and Isambert, 1991; and HYDUR, 1985).

FDCs are usually applied to hydropower feasibility studies for run-of-the-river operations; however FDCs can also be applied to hydropower applications when a modest daily storage impoundment is available for storing surplus water during low output periods (off peak) to provide a reserve of water to draw upon during times of peak demand (Palmer and Duder, 1990). This section describes the use of FDCs in both run-of-the-river and daily-storage hydropower applications. A daily-storage hydropower facility has storage facilities which can only accommodate enough water to balance energy output between off-peak and peak requirements in a given day.

A rating-curve which represents the relationship between output power  $P$  and discharge  $Q$  is illustrated in Figure 3. Hydraulic computations are performed which summarize the relationship between the effective plant head  $H$ ,  $Q$ , and  $P$ . These computations involve the application of the energy equation from the plant intake, through the penstock, turbine intake, turbine, draft tube, and tailwater regions. Such computations are shown elsewhere (Warnick, 1984, pp. 24-30). Also required is the relationship between the turbine efficiency, discharge, and head, which is usually provided by the turbine manufacturer.

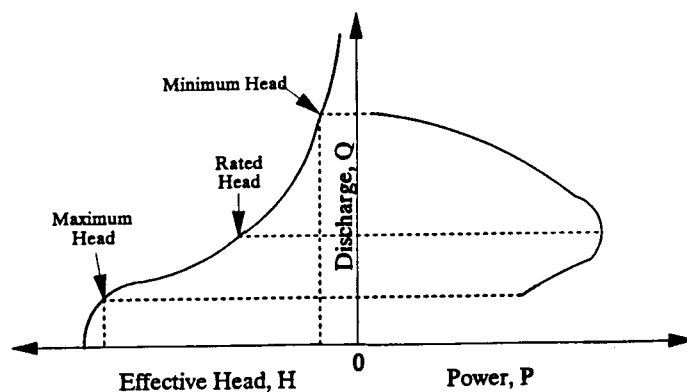


Figure 3. Illustration of the Development of a Power Discharge Rating Curve for a Hydropower Project.

### Hydropower Feasibility Analysis Using Energy Duration Curves

Perhaps the oldest and most widely documented use of flow duration curves is for the economic evaluation of hydropower facilities (Hickox and Wessenauer, 1933; Searcy, 1959, pp. 26-29; Warnick, 1984, pp. 57-73). Since the application of an FDC in hydropower studies is discussed elsewhere, we only provide a brief introduction here. Computer software is available for

Figure 4 illustrates how the power discharge rating curve is combined with the FDC to produce a power-duration curve. The power duration curve reflects the likelihood of various power output levels. If the period of record FDC is employed to construct the power-duration curve, then the area under the power

duration curve is the average annual energy. This is useful in feasibility investigations seeking to determine the economic feasibility of a variety of potential turbine/dam/penstock configurations. If the median annual FDC is employed, then the area under the power duration curve reflects the annual energy in a typical or median (but hypothetical) year. Similarly, construction of the energy duration curve using the annual FDC with a 20-year average return period allows examination of annual energy revenues during a particularly wet or dry hypothetical year.

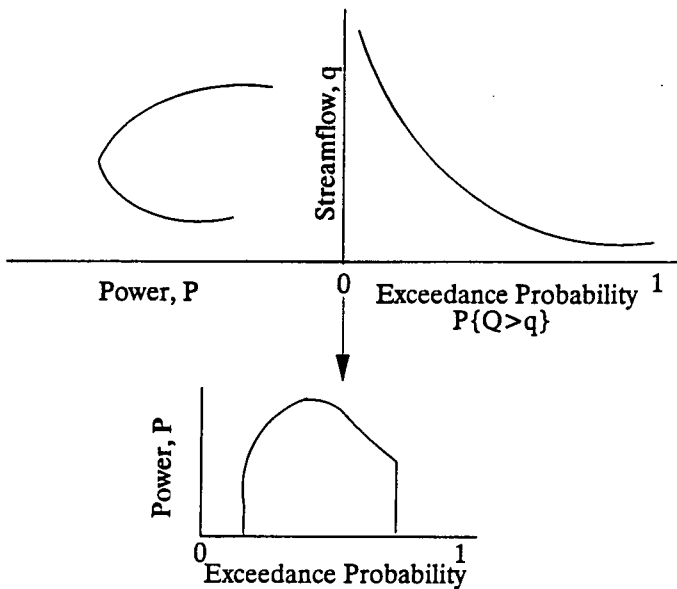


Figure 4. Illustration of the Development of a Power Duration Curve for a Hydropower Project.

including sediment, turbidity, and hardness. This can be expanded to include organic pesticides, metals, chlorophyll, BOD, and other parameters. The first step in constructing a water quality index duration curve is to construct a rating curve of the water quality index of interest versus stream discharge. The rating curve is combined with the FDC, in Figure 5, to provide a water quality index duration curve. Such curves are useful for determining the frequency with which a water quality standard will be violated. In Figure 5 the probability of exceedance of streamflow becomes the probability that the water quality standard will not be violated (exceeded).

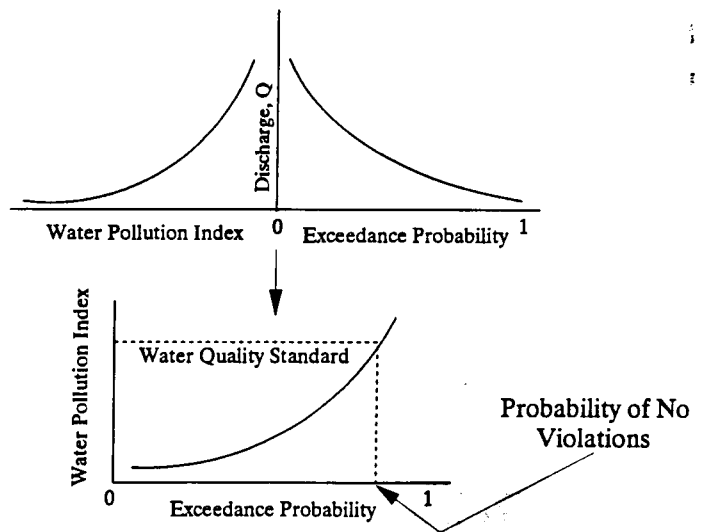


Figure 5. Illustration of the Use of a Flow Duration Curve for the Determination of the Probability of Violating a Water Quality Standard.

### WATER QUALITY MANAGEMENT

The goal of most water quality management programs is to maintain a specified water quality standard for a fixed percentage of the time. FDCs provide a simple and elegant approach to the implementation of water quality management programs. In the following sections, we discuss the application of FDCs to three general water quality problems: (1) construction of water quality index duration curves, (2) evaluation of a wastewater treatment plant design, and (3) determination of wasteload allocations.

#### Water Quality Index Duration Curves

Searcy (1959) suggests the construction of duration curves for a variety of water quality parameters,

#### Tradeoffs Among Variables in the Design of Wastewater Treatment Plants

In water quality management programs, the water quality standards and the associated probability of violating the standards are fixed by either state or federal law. At the policy level too, there is a need to evaluate the tradeoffs among water quality management variables so that site specific water quality standards can be considered. Simple methods are needed for explaining these tradeoffs. For example, in designing a wastewater treatment plant, the policy analyst must consider the tradeoffs between plant cost, water quality standards, and the probability of violating water quality standards. Male and Ogawa (1984) suggest the use of composite diagrams like Figure 6 for examining the tradeoffs among these three decision variables.

In Figure 6, the water quality standard is dissolved oxygen (DO) in the receiving water. The treatment plant efficiency is the percentage of BOD removed by the plant. If the exceedance probability of not violating the standard is fixed, then increasing the DO from 3 to 5 mg/l requires an increase in plant efficiency and project cost. Figure 6 could also be used to evaluate tradeoffs between plant efficiency and probability of water quality standard violations. Figure 6 shows FDCs corresponding to daily streamflow (1-day flow) and 7-day flow. Using FDCs based on different streamflow durations allows one to further evaluate the consequences of changes in the definition of the design streamflow event in terms of the water quality standard, and treatment plant efficiency.

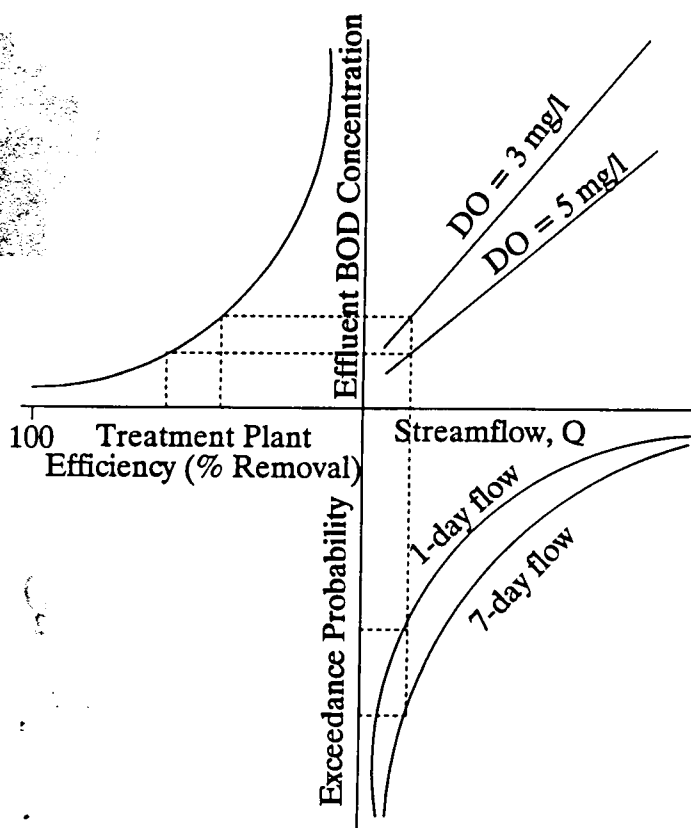


Figure 6. A Composite Diagram Which Uses a Flow Duration Curve to Illustrate the Tradeoffs Among Variables Involved in the Selection of a Wastewater Treatment Plant (adapted from Male and Ogawa, 1984).

tool for implementing water quality management programs in the U.S. One of two approaches is usually taken to evaluate the level of treatment required. Either the minimum treatment requirements specified by EPA are used, or the minimum level of treatment required to meet instream water quality standards are used.

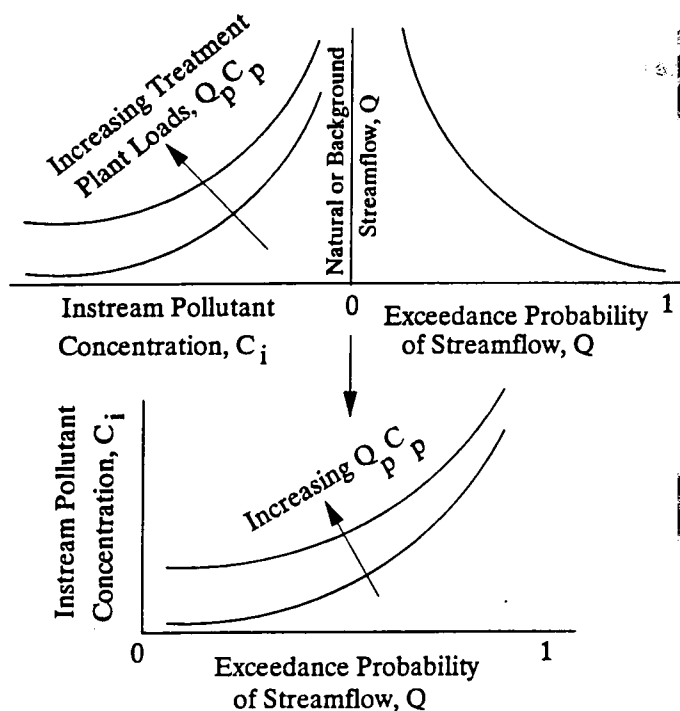


Figure 7. Illustration of the Determination of the Distribution of Instream Pollutant Concentration Using a Flow Duration Curve.

Traditionally, water quality standards and allowable discharge concentrations are based on critical low flow conditions specified by some low flow statistic. The most widely used index of low flow in the United States is the seven-day, ten-year low flow. Allowable plant discharge concentrations and associated wasteload allocations for an arbitrary pollutant, based on a fixed threshold of this type, are computed using

$$C_i = \frac{C_p Q_p + C Q}{Q_p + Q} \tag{1}$$

where  $Q$  = natural (background) stream discharge;  $C$  = pollutant concentration associated with  $Q$ ;  $Q_p$  = treatment plant discharge;  $C_p$  = pollutant concentration associated with  $Q_p$ ; and  $C_i$  = fully mixed instream

Wasteload Allocation Using Flow Duration Curves

The Federal Water Pollution Control Act requires that all point source discharges have a National Pollutant Discharge Elimination System (NPDES) permit. As a result, NPDES permits are the primary

pollutant concentration downstream of plant discharge.

Normally Equation (1) is used to determine the values of  $Q_p$  and  $C_p$  needed to assure that  $C_i$  is lower than some target level. Equation (1) can be summarized graphically using an FDC. Figure 7 combines Equation (1) with an FDC to examine the relationships between treatment plant pollutant load,  $C_p Q_p$ , instream pollutant concentration downstream of treatment plant discharge,  $C_i$ , and the exceedance probability of streamflow,  $p$ . Here  $p$  is equivalent to the non-exceedance probability of pollutant concentrations. Figure 7 illustrates that, for a fixed  $p$ , instream pollutant concentration  $C_i$  increases as plant loads  $C_p Q_p$  increase. For a fixed pollutant concentration  $C_i$ , increases in  $p$  result in decreases in treatment plant loads  $C_p Q_p$ .

Equation (1) summarizes an approach for determining treatment plant loads which meet a single threshold type water quality standard for a single point discharge on a river. Other procedures exist for determining acceptable treatment plant discharges such as seasonal wasteload allocation procedures (Rossman, 1989) and the numerous methods available for proportioning restrictions on wasteloads among multiple dischargers (Chadderton and Kropp, 1985).

### WATER-RESOURCE ALLOCATION

The problem of how best to allocate water resources is a continuing problem which will likely become more and more difficult to solve. The equitable allocation of water resources has emerged as an international crisis (Clarke, 1993). In this section, we concentrate on problems of water allocation for both regulated and unregulated rivers.

#### *Water-Resource Allocation Using Flow Duration Curves*

Alaouze (1991) suggested the use of a FDC for determining the optimal release (allocation) schedule of water from a reservoir system. His approach is elegant, simple to explain, graphical, and extendable for use in both regulated and unregulated river systems. Alaouze's approach is to use properties of the cumulative distribution function to determine the proportion of water in a river which may be allocated to each use, or user, given prespecified statements regarding the reliability of the desired withdrawals. Although Alaouze only recommended his approach for determining the optimal release schedule from reservoir

systems, we extend his approach for use in both unregulated and regulated rivers.

Figure 8 illustrates the application of an FDC for determining how two water withdrawals,  $q_1$  and  $q_2$ , are proportioned given that they are to be withdrawn with reliabilities  $p_1$  and  $p_2$ , respectively. The solid line represents the FDC for a particular point on a river. If a single withdrawal were required with reliability  $p_1$ , that withdrawal could equal  $q_{p_1}$ . As soon as another withdrawal is required, with a reliability equal or greater than  $p_1$ , all subsequent withdrawals are found from

$$q_i = w_i q_{p_i} \quad i = 1, 2, \dots, n \quad (2)$$

where

$$\sum_{i=1}^n w_i = 1.$$

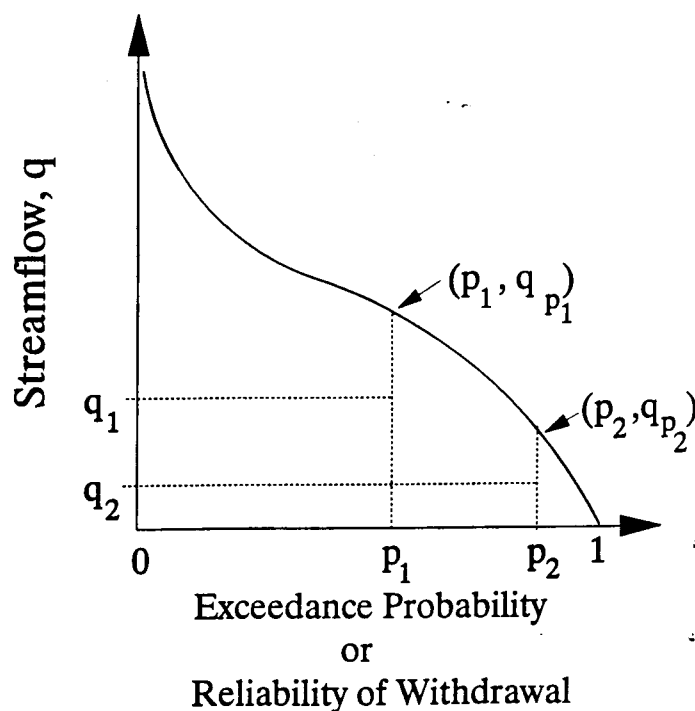


Figure 8. Illustration of the Use of a Flow Duration Curve for Determining the Proportion of Water Allocated to Two Uses as a Function of the Reliability Associated With Each Use of Water.

In Figure 8 the two withdrawals are found from Equation (2) using the simultaneous solution of the three equations  $q_1 = w_1 q_{p_1}$ ,  $q_2 = w_2 q_{p_2}$ , and  $w_1 + w_2 = 1$ . In general, the FDC provides the relationship



between each desired reliability  $p_i$  and each maximum withdrawal  $q_{pi}$ . For the  $n$ -withdrawal problem, Equation (2) yields  $n+1$  equations, with  $n$  unknown withdrawals  $q_i$ ,  $n$  unknown weights  $w_i$ , and  $n$  unknown reliabilities  $p_i$  for a total of  $3n$  unknowns. Therefore, a unique solution is obtained when  $3n - (n+1) = 2n-1$  variables are specified *a priori*. For example, in the two-withdrawal problem, there are six unknowns  $q_1, q_2, w_1, w_2, p_1,$  and  $p_2$  with three equations, hence one needs to specify the values of three variables, *a priori*. Usually, the weights  $w_i$  are unknown, and the problem either amounts to computing the one remaining withdrawal  $q_i$  or the one remaining reliability,  $p_i$ . Alaouze (1991) shows that Equation (2) ensures, under all conditions, that each desired release  $q_i$  will be available with reliability (or exceedance probability) equal to  $p_i$ .

For an example of the application of this procedure in river basin management, consider the following situation. Suppose a river basin authority requires the maintenance of an instream flow equal to  $q_2$  with reliability  $p_2$ . The FDC for the point in question is given in Figure 8, and the weight  $w_2$  is computed from  $w_2 = q_2/q_{p2}$ , since both  $q_2$  and  $q_{p2}$  are given. Now, suppose a water-resource agency wishes to determine the maximum amount of water it can withdraw from the same location with reliability  $p_1$ . Given  $p_1$ , and the FDC, and that  $w_1 = 1 - w_2$ , one can compute the maximum amount of water available to the water agency by using  $q_1 = (1 - w_2)q_{p1}$ . This procedure ensures the instream flow requirements are met. This procedure is particularly powerful when there are a variety of water resource uses and users who among them, are willing to withdraw water with differing reliabilities.

### FLOOD FREQUENCY ANALYSIS USING DAMAGE DURATION CURVES

Beard (1943) first suggested the use of FDCs in flood frequency analysis. Beard provides values of the design exceedance probability,  $p_{design}$ , appropriate to assure that during the lifetime of a flood control structure, the design flood will only be exceeded, on average, once. He suggests selecting the design discharge as that discharge with an exceedance probability  $p_{design} = 1 - 0.5^{1/N}$ , using the period of record FDC, with  $N$  equal to the design lifetime. For example, a design life equal to 100 years implies selection of the design discharge as that discharge which is exceeded with a probability of 0.0069. Actually, Beard (1943) provided a table of values of  $p_{design}$  and  $N$  since calculators were not readily available when he published his paper.

Many investigations seek to estimate average annual flood damages or annual average benefits of flood control. Such studies normally employ sequences of annual maximum floodflows to characterize the frequency of extreme flood events. Those studies only focus upon flood damages due to the most extreme flood events; however, damaging flood events are not always rare events, and it may not make sense to ignore the damages caused by floods smaller than the annual maximum event. FDCs allow us to focus on all flood events, regardless of their magnitudes.

Figure 9 illustrates how one may use an annual FDC to determine the frequency and magnitude of damage for a particular location. In Figure 9 a stage discharge relationship shown in quadrant (A) is combined with a stage damage curve in quadrant (B) and the median annual flow duration curve in (C) to produce the damage duration curve in quadrant (D).

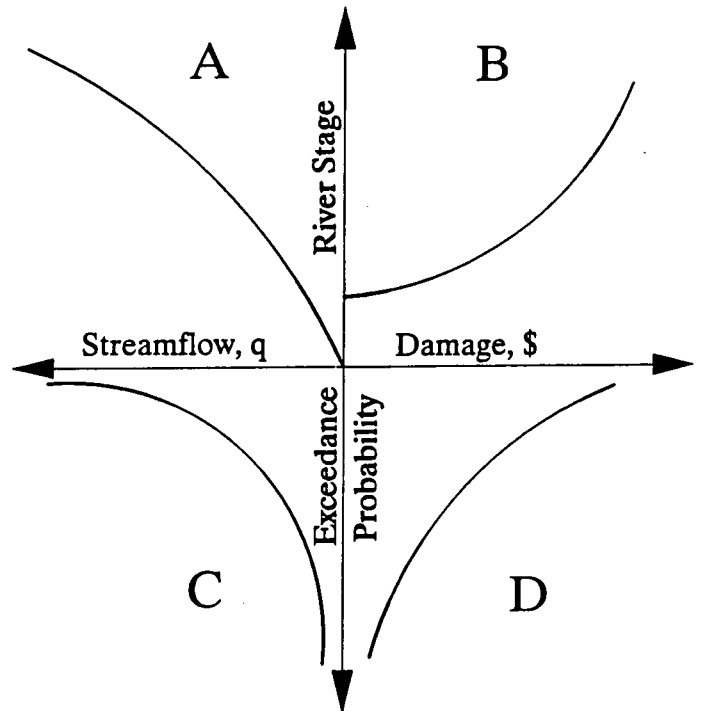


Figure 9. Illustration of the Use of (A) Stage Discharge Curve, (B) Stage Damage, and (C) Flow Duration Curve for Constructing the (D) Damage-Duration Curve.

### Selection of an Optimal Water Resources Project

Figure 9 can be repeated for a variety of different flood control strategies. Each time one constructs a damage risk curve as shown in quadrant D of Figure 9, the rating curves in quadrants A and B and the

FDC in quadrant C assume a particular project setting. For example, if a levee is constructed, the rating-curves in quadrants A and B will change, leading to a different damage risk curve and correspondingly to a different expected damage (the area under the damage-risk curve). If one repeats Figure 9 for a range of alternative flood control strategies, the results may be illustrated as in Figure 10, which plots the reductions in expected damage costs which should result from increasing the scale of the project. Also shown in Figure 10 are the increasing project capital and operating maintenance costs associated with larger flood control schemes. The sum of these two curves is shown as the total cost curve. The optimal project is chosen as that project which minimizes the sum of the expected damage costs and the project capital costs.

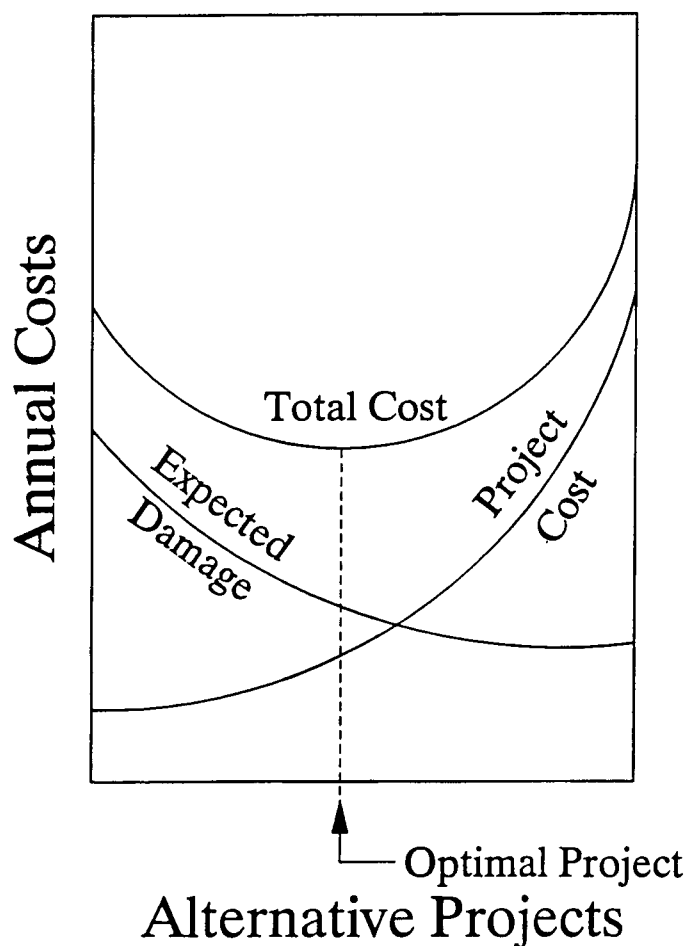


Figure 10. Illustration of the Selection of the Optimal Project as That Project Which Minimizes the Sum of the Expected Damage Costs and Project Costs.

The procedures described in Figures 9 and 10 for determining the optimal scale of a flood control project can be generalized to any water resource project. Similar procedures could be employed for any water resource project for which one can compute a water resource index duration curve. Therefore, an FDC is useful for combining the hydrology, hydraulics, and economics of projects in such a way that the optimal project can be selected from a range of alternatives. This procedure provides a simple, graphical, and elegant alternative to the use of more complex systems methodologies such as linear and nonlinear programming procedures.

### SUMMARY

Streamflow duration curves are a useful graphical and analytical tool for illustrating and evaluating the relationship between the magnitude and frequency of daily streamflow. Flow duration curves provide a graphical yet compact representation of streamflow data. The graphical representation of hydrologic information is not just helpful and useful; in many cases it is essential. Graphical displays often force us to notice features of problems and data we never expected to see. Some statisticians consider graphical displays as the single most effective and robust statistical tool. Most hydrologists would not make engineering decisions without reference to a graphical display of the frequency and magnitude of streamflow data. Most importantly, flow duration curves are an effective graphical instrument for conveying information regarding a wide range of water resource problems; hence, they provide an effective medium for communication between water resource engineers, lawyers, managers, planners, politicians, and others. As the demands on our water resource systems become increasingly complex, the need for simple-to-understand graphical displays of hydrologic information (such as FDCs) becomes correspondingly more important.

Any user of an FDC is warned to consult the section titled "Some Advantages and Limitations of FDCs," because there are many circumstances cited in which either FDCs or their combined use with rating-curves can lead to meaningless and/or incorrect results.

The purpose of this study was to compile and review recent innovations associated with the construction of FDCs along with their varied applications. Hopefully this study has elucidated those situations in which the relatively simple and elegant FDC procedures provide an effective alternative to more complex simulation alternatives.

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