57:020 Mechanics of Fluids and Transfer Processes
Laboratory Experiment #4

ENERGY LOSS IN A HYDRAULIC JUMP

Principle

When a fast and shallow (supercritical) channel flow changes to a slow and deep (subcritical) flow, there is energy loss through an abrupt flow feature known as the hydraulic jump.

Introduction

When a supercritical flow in a channel is forced to become subcritical by a downstream obstruction, an abrupt change in depth usually occurs, and considerable energy loss accompanies the process (see Figure 1). The change in depth can be forced by a sill in the downstream part of the channel or just by the prevailing depth in the stream further downstream. This flow phenomenon is called the *hydraulic jump*, and is analogous to a normal shock wave in compressible gas.

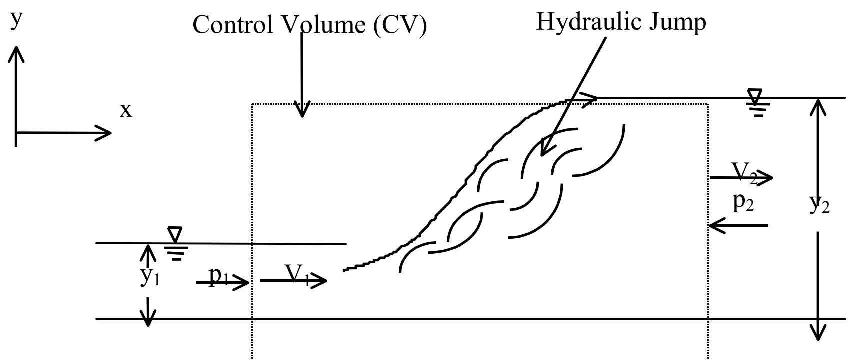

Figure 1. Profile of the hydraulic jump

By applying mass conservation and momentum equation to the control volume shown in Figure 1, we have

$$V_1 A_1 = V_2 A_2 \tag{1}$$

$$\sum F_x = \int_{cv} V \rho \, V \cdot dA \tag{2}$$

where V_1 and V_2 represent the velocities at inlet and outlet of the control volume, respectively, and $A_1 = Wy_1$ and $A_2 = Wy_2$ (where W denotes the width of the channel) are the cross section areas at inlet and outlet, respectively. Solving equations (1) and (2) for y_1/y_2 and V_2/V_1 yields

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) \tag{3}$$

$$\frac{V_2}{V_1} = \frac{2}{\sqrt{1+8Fr_1^2} - 1} \tag{4}$$

where $Fr_1 = V_1 \big/ \sqrt{g\, y_1}$ is the Froude number (> 1 for supercritical flow) at the inlet.

The energy loss in the jump is characterized by the head loss, $\Delta H_L = H_1 - H_2$, or the dimensionless head loss, $\Delta H_L/y_1$, where $H_1 = y_1 + V_1^2/2g$ and $H_2 = y_2 + V_2^2/2g$. Using the above expressions, we can write this in terms of Fr_1, or in terms of depths

$$\frac{\Delta H_L}{y_1} = 1 + \frac{1}{2}Fr_1^2 - \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) - \frac{2\,Fr_1^2}{\left(\sqrt{1+8Fr_1^2} - 1\right)^2} \tag{5.a}$$

$$\frac{\Delta H_L}{y_1} = \frac{1}{4}\frac{y_1}{y_2}\left(\frac{y_2}{y_1} - 1\right)^3 \tag{5.b}$$

Apparatus

The experiment is conducted in the small hydraulic flume located in the Fluids Laboratory. The glass-walled flume is 1 ft wide and 12 ft long. Water is circulated in the channel using a pump. The discharge is measured using an elbow-meter mounted in the water supply line. The high speed shallow flow is created via a sluice gate and the hydraulic jump is created by raising a gate located at the downstream end of the flume. For the measurement of water depths, a depth gage is attached to the instrument carriage mounted on top of the flume.

Procedures

1. With water running through the channel, adjust the inflow to a low rate.
2. Determine the flow rate using the indications of the differential manometer connected to the elbow-meter, and the calibration chart attached to the flume.
3. Slowly raise the gate at the end of the flume until the jump starts to form; this jump may not be steady at first. Continue to raise the downstream gate until the jump becomes stationary at the midpoint of the channel.
4. Measure the depths of water upstream, y_1, and downstream, y_2, of the jump with the vertical ruler.
5. Change the inflow to a medium and a higher rate value. Lower the gate at the end of the flume to stabilize the hydraulic jump.
6. Repeat steps 1- 4 for these flow rates.

Measurements

Flume width, w (ft) = ________________

Case	Q (ft^3/s)	y_1 (ft)	y_2 (ft)
1			
2			
3			

Data Analysis

1. Calculate y_2/y_1 for each of the three flow rates.
2. Plot y_2/y_1 vs. Froude number, Fr_1, and compare with the theoretical result. Use the limit $0 < Fr_1 < 10$ for the horizontal axis and $0 < y_2/y_1 < 12$ for vertical axis. Discuss the agreement between experiment and theory.
3. Calculate the theoretical head loss from equation (5.a) for a range of Froude numbers from 1

to 10.0. Plot these values. On the same graph, plot experimental head loss against Froude number for each flow rate. Discuss the agreement between experiment and theory. Use the limit $0 < Fr_1 < 10$ for the horizontal axis and $0 < y_2/y_1 < 24$ for vertical axis.

Case	$V_1=Q/Wy_1$	$V_2=Q/Wy_2$	Fr_1	y_2/y_1	H_1	H_2	$\Delta H_L/y_1$
1							
2							
3							

Further Considerations

1. What qualitative changes were observed in the flow as the hydraulic jump became less pronounced?
2. Why is the hydraulic jump impossible for $Fr < 1$?
3. Derive equations (3) - (5).

References

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