<><><><><> 56:171 Operations Research <><><><><>
Final Exam - December 14, 1993
<><><><><> Instructor: D.L. Bricker <><><><><>

Do Part One, one from Part Two, and three from Part Three (20 pts. each)

Score:			
	Part One:	I. ________	Multiple Choice
	Part Two:	II. ________	LINDO Analysis
		III. ________	Simplex Algorithm
	Part Three:	IV. ________	Decision trees
		V. ________	Markov chain model: Student Progress
		VI. ________	Markov chain model: Inventory System
		VII. ________	Continuous-time Markov chain
		VIII. ________	Dynamic programming
	TOTAL:	________	(of 100 possible)

<><><><><> PART ONE <><><><><>

I. Multiple Choice

____ 1. If you make a mistake in choosing the pivot row in the simplex method, the solution in the next tableau
a. will be nonbasic
b. will be nonfeasible
c. will have a worse objective value
d. will be degenerate

Consider a discrete-time Markov chain with transition probability matrix :

$$\begin{bmatrix} .2 & .8 \\ .9 & .1 \end{bmatrix}$$

____ 2. If this Markov chain is initially in state #1, the probability that the system will be in state 2 after exactly **one** step is:
a. 0.1
b. 0.64
c. 0.8
d. 0.24
e. none of the above

____ 3. If the Markov chain in the previous problem was initially in state #1, the probability that the system will still be in state 1 after **two** steps is
a. 0.64
b. 0.24
c. 0
d. 0.26
e. 0.76
f. none of the above

____ 4. A transient state of a Markov chain is one in which the probability of
a. returning to that state is zero.
b. returning to that state is 1.
c. returning to that state is less than 1.
d. none of the above.

____ 5. A recurrent state of a Markov chain is one
a. which is not transient.
b. which is not absorbing.
c. which communicates with an absorbing state.
d. none of the above.

____ 6. The steady-state probability vector π of a discrete Markov chain with transition probability matrix P satisfies the matrix equation
a. $P^t \pi = 0$
b. $P\pi = 0$
c. $P\pi = \pi$
d. $\pi P = \pi$
e. none of the above

____ 7. For a continuous-time Markov chain, let Λ be the matrix of transition rates. The sum of each...
a. column is 1
b. column is 0
c. row is 1
d. row is 0
e. none of the above

____ 8. For a discrete-time Markov chain, let P be the matrix of transition probabilities. The sum of each...
a. column is 1
b. column is 0
c. row is 1
d. row is 0
e. none of the above

____ 9. To compute the steady state distribution π of a continuous-time Markov chain, one must solve (in addition to sum of components 0f π equal to 1) the matrix equation (where Λ^t is the transpose of Λ):

a. $\pi\Lambda = 1$ c. $\Lambda^t\pi = \pi$ e. $\pi\Lambda = \mathbf{0}$

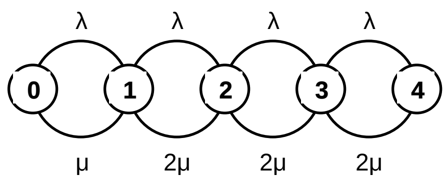
b. $\Lambda^t\pi = 1$ d. $\pi\Lambda = \pi$ f. none of the above

____ 10. In PERT, the completion time for the project is assumed to
a. have the Beta distribution c. be constant e. none of the above
b. have the Normal distribution d. have the exponential distribution

____ 11. If you make a mistake in choosing the pivot column in the simplex method, the solution in the next tableau
a. will be nonbasic c. will have a worse objective value
b. will be nonfeasible d. will be degenerate
e. None of the above

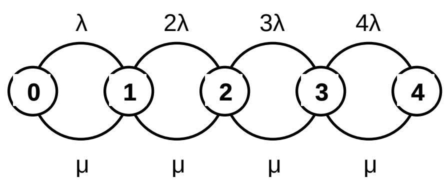
____ 12. If, in the optimal <u>dual</u> solution of an LP problem (min cx st $Ax \leq b$, $x \geq 0$), dual variable #3 is zero, then in the optimal primal solution,
a. primal variable #3 must be zero c. slack variable for primal constraint #3 must be zero
b. primal variable #3 must be positive d. primal constraint #3 must be slack
e. None of the above

____ 13. Consider the following queueing model:



The notation for this type of queue is:
a. M/M/1 c. M/M/4 e. M/M/1/4
b. M/M/2 d. M/M/2/4 f. none of the above

____ 14. Consider the following queueing model:



The notation for this type of queue is:
a. M/M/1 c. M/M/1/4 e. M/M/1/4/4
b. M/M/2 d. M/M/4 f. none of the above

____ 15. If there is a "tie" in the Minimum-Ratio Test in the simplex method, the solution in the next tableau
a. will be nonbasic c. will have a non-improved objective value
b. will be optimal d. will be degenerate e. None of the above

____ 16. In an M/M/1 queue, if $\lambda > \mu$,

a. $\pi_0 = 1$ in steady state c. $\pi_i > 0$ for all i

b. no steady state exists d. $\pi_0 = 0$ in steady state e. None of the above

____ 17. In an M/M/1/N queue, if $\lambda > \mu$,

a. $\pi_0 = 1$ in steady state c. $\pi_i > 0$ for all i

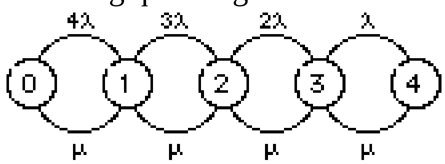
b. no steady state exists d. $\pi_0 = 0$ in steady state e. None of the above

____ 18. The probabilities in the transition probability matrix P of a discrete-time Markov chain are actually

a. simple probabilities b. joint probabilities c. conditional probabilities
d. more than one of a,b,c are correct e. None of a,b,c are correct

____ 19. An absorbing state in a Markov chain is one in which the probability of
a. moving into that state is zero b. moving out of that state is 1
c. moving out of that state is zero d. None of the above

____ 20. Consider the following queueing model:



The notation for this type of queue is:
a. M/M/1 c. M/M/1/4 e. M/M/1/4/4
b. M/M/2 d. M/M/4 f. none of the above

◇◇◇◇◇ PART TWO ◇◇◇◇◇

II. LINDO analysis

Problem Statement: The Classic Stone Cutter Company produces four types of stone sculptures: figures, figurines, free forms, and statues. Each product requires the following hours of work for cutting and chiseling stone and polishing the final product:

Operation	FIGURES	FIGURINES	FREE FORMS	STATUES
Cutting	30	5	45	60
Chiseling	20	8	60	30
Polishing	0	20	0	120
Profit ($/unit)	280	40	500	510

The company's current work force has production capacity sufficient to allocate 300 hours to cutting, 180 hours to chiseling, and 300 hours to polishing each week.

Define the variables:

FIGURES = # of figures to be produced each week,
FIGURINES = # figurines to be produced each week,
etc.

The LINDO output for solving this problem follows:

```
MAX     280 FIGURE + 40 FIGURINE + 500 FREEFORM + 510 STATUE
SUBJECT TO
  2)    30 FIGURE + 5 FIGURINE + 45 FREEFORM + 60 STATUE  <=  300
  3)    20 FIGURE + 8 FIGURINE + 60 FREEFORM + 30 STATUE  <=  180
  4)    20 FIGURINE + 120 STATUE                          <=  300
END

      OBJECTIVE FUNCTION VALUE
 1)       2700.00000

    VARIABLE        VALUE          REDUCED COST
    FIGURE        6.000000           0.000000
    FIGURINE      0.000000          30.000000
    FREEFORM      0.000000          70.000000
    STATUE        2.000000           0.000000

  ROW         SLACK OR SURPLUS       DUAL PRICES
    2)           0.000000             6.000000
    3)           0.000000             5.000000
    4)          60.000000             0.000000
```

RANGES IN WHICH THE BASIS IS UNCHANGED

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
FIGURE	280.000000	60.000000	9.333333
FIGURINE	40.000000	30.000000	INFINITY
FREEFORM	500.000000	70.000000	INFINITY
STATUE	510.000000	23.333336	89.999992

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	300.000000	7.500000	30.000000
3	180.000000	20.000000	5.000000
4	300.000000	INFINITY	60.000000

THE TABLEAU

ROW	(BASIS)	FIGURE	FIGURINE	FREEFORM	STATUE	SLK 2
1	ART	0.000	30.000	70.000	0.000	6.000
2	FIGURE	1.000	1.100	7.500	0.000	-0.100
3	STATUE	0.000	-0.467	-3.000	1.000	0.067
4	SLK 4	0.000	76.000	360.000	0.000	-8.000

ROW	SLK 3	SLK 4	RHS
1	5.000	0.000	2700.000
2	0.200	0.000	6.000
3	-0.100	0.000	2.000
4	12.000	1.000	60.000

Ignoring the restriction that the numbers of items produced per week must be integer, answer the following questions:

1. The optimal solution above is *(check as many as apply):*
___ basic ___ degenerate ___ unique

____ 2. The number of basic variables in this optimal solution (not including z, the objective value) is
a. one b. two c. three
d. four e. none of the above

____ 3. In every basic solution of this problem
a. not every product will be included b. exactly two products will be included
c. at least one slack variable will be >0 d. none of the above

____ 4. If it were required to make one figurine as a salesman's sample, the profit will decrease by *(choose the nearest value)*
a. zero b. $5 c. $6
d. $30 e. $40 f. $50
g. cannot be determined h. none of the above

____ 5. If it were required to make one figurine as a salesman's sample, the production of statues would
a. be unchanged b. increase by less than 1 c. decrease by less than 1
d. increase by more than 1 e. decrease by more than 1 f. cannot be determined
g. none of the above

____ 6. If it were required to make one additional statue, the profit will decrease by *(choose the nearest value)*
a. zero b. $5 c. $6
d. $25 e. $100 f. $510
g. cannot be determined h. none of the above

____ 7. If the profit of free forms were to be \$550 per unit,
a. the profit would be unchanged b. the profit would increase by \$100
c. the production of free forms should increase d. none of the above

____ 8. If five additional hours of cutting were available, the profit would increase by *(choose the nearest value)*
a. less than \$10 b. \$10 c. \$20
d. \$30 e. more than \$40 f. cannot be determined

____ 9. If five additional hours of cutting were available, the number of figures would
a. be unchanged b. increase by 0.5 c. decrease by 0.5
d. increase by 1 e. decrease by 1 f. none of the above

____ 10. The value of the second variable in the optimal dual solution
a. is zero b. is positive c. is negative
d. cannot be determined e. none of the above

____ 11. The value of the optimal objective value of the dual problem is
a. zero b. 2700 c. -2700
d. cannot be determined e. none of the above

III. Simplex Algorithm for LP: At an intermediate step of the simplex algorithm, in which the objective is to be **minimized**, the tableau is:

-z	X_1	X_2	X_3	X_4	X_5	X_6	RHS
1	0	0	0	-2	0	6	-10
0	2	0	1	-4	0	1	4
0	0	1	0	1	0	-1	3
0	-2	0	0	2	1	3	1

1. What are the basic variables for this tableau? *(circle):*
-Z X_1 X_2 X_3 X_4 X_5 X_6

2. The current value of the cost for this basic solution is (*circle:* +10 or -10)

_____ 3. The current value of X_1 for this basic solution is
a. 0 b. 1 c. 3 d. 4 e. 10

_____ 4. The current value of X_2 for this basic solution is
a. 0 b. 1 c. 3 d. 4 e. 10

_____ 5. The current value of X_3 for this basic solution is
a. 0 b. 1 c. 3 d. 4 e. 10

_____ 6. Increasing X_4 would (*circle:* increase / decrease) the objective function.

_____ 7. What is the substitution rate of X_4 for X_5?
a. 0 b. 1 c. -1 d. 2 e. -2

_____ 8. If X_4 were increased by 2 units, the value of X_5 will
a. not change b. increase by 2 c. decrease by 2
d. increase by 4 e. decrease by 4 f. none of the above

_____ 9. If the original constraints were all of type "≤" and X_4, X_5, and X_6 are slack variables, the value of the first dual variable π_1 corresponding to the tableau given above is
a. 0 b. 1 c. -1 d. 2 e. -2
f. none of the above g. cannot be determined

_____ 10. If the original constraints were of type "≥" and X_4, X_5, and X_6 are surplus variables, the value of the second dual variable π_2 corresponding to the tableau above is
a. 0 b. 1 c. -1 d. 2 e. -2
f. none of the above g. cannot be determined

11. Perform a pivot to improve the objective function, and complete the blank entries in the tableau below:

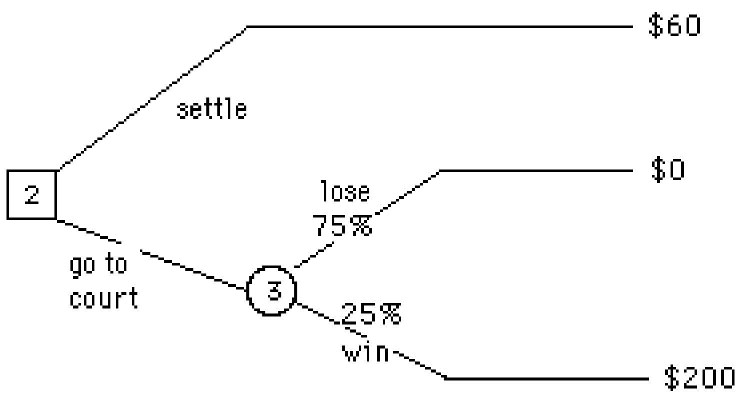
-z	X_1	X_2	X_3	X_4	X_5	X_6	RHS

_____ 12. The improvement in the objective resulting from the pivot in (11) is *(choose the nearest value)*

a. zero b. 1 c. 2
d. 3 e. 4 f. ≥5

◇◇◇◇◇ PART THREE ◇◇◇◇◇

IV. Decision Theory & Decision Trees: General Custard Corporation is being sued by Sue Smith. Sue can settle out of court and win $60,000, or she can go to court. If she goes to court, there is a 25% chance that she will win the case *(event W)* and a 75% chance she will lose *(event L)*. If she wins, she will receive $200,000, and if she loses, she will net $0. A decision tree representing her situation appears below, where payoffs are in thousands of dollars:



_____ 1. What is the decision which maximizes the expected value? a. settle b. go to court

For $20,000, Sue can hire a consultant who will predict the outcome of the trial, i.e., either he predicts a loss of the suit *(event PL)*, or he predicts a win *(event PW)*. The consultant is correct 80% of the time.

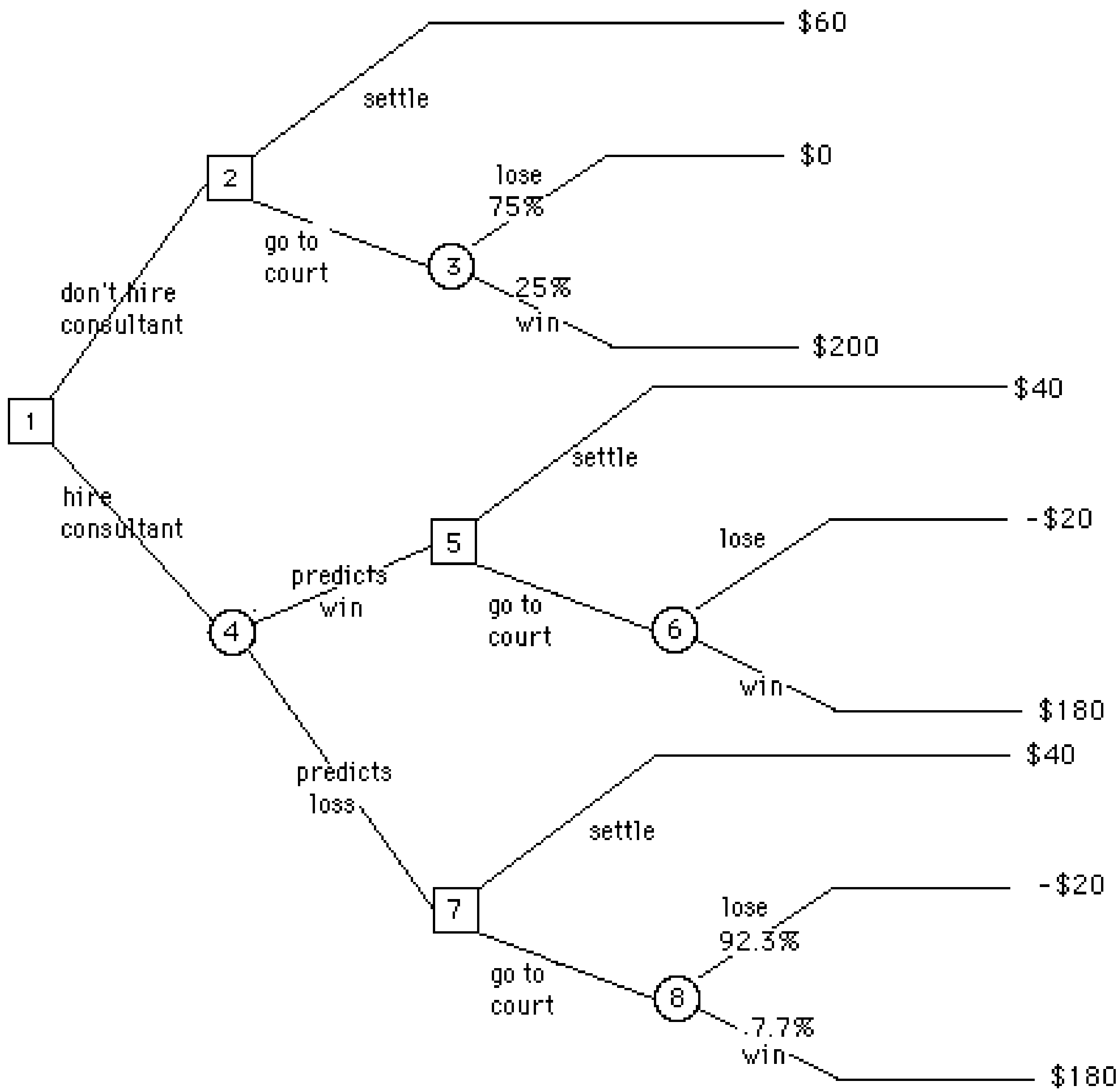
_____ 2. The probability that the consultant will predict a win, i.e. P{PW} is *(choose nearest value)*

a. ≤25% b. 30% c. 35%
d. 40% e. 45% f. ≥ 50%

_____ 3. According to Bayes' theorem, the probability that, *if* the consultant predicts a win, then in fact Sue *will* win, i.e. P{W êPW} is *(choose nearest value)*

a. ≤25% b. 30% c. 35%
d. 40% e. 45% f. ≥ 50%

The decision tree below includes Sue's decision as to whether or not to hire the consultant.



4. "Fold back" nodes 2 through 8, and write the value of each node below:

Node	Value	Node	Value	Node	Value
8	______	5	94.286	2	60
7	40	4	______	1	______
6	94.286	3	50		

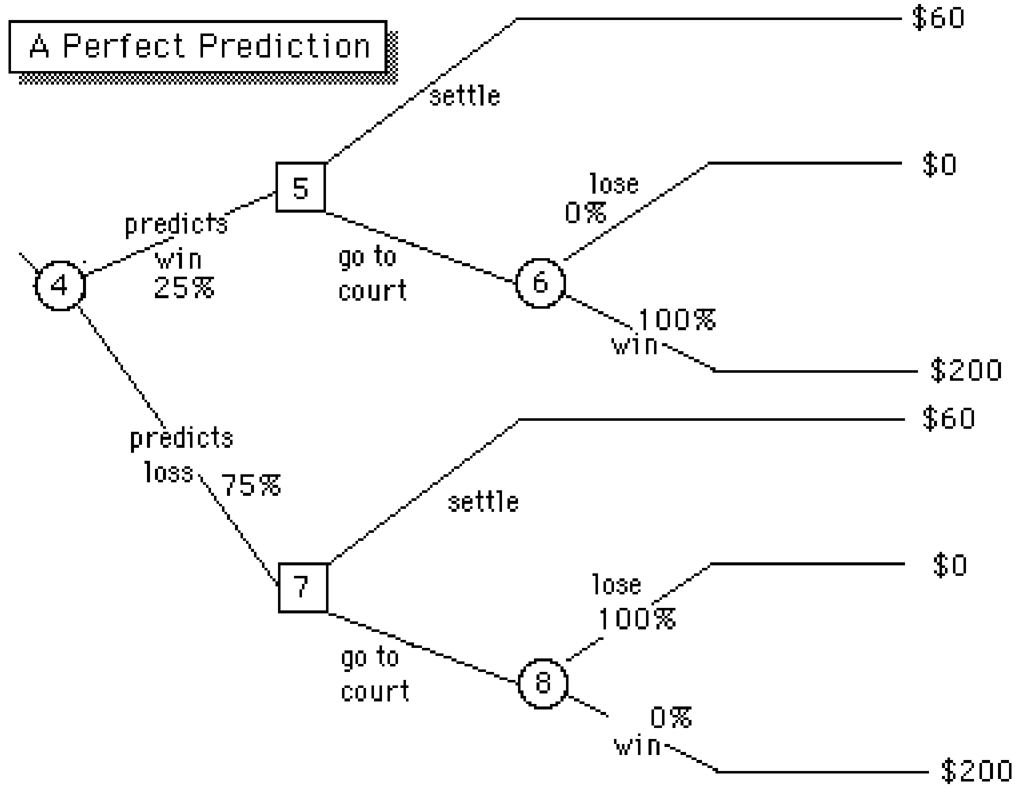
5. Should Sue hire the consultant? *Circle:* Yes No

____ 6. The expected value of the consultant's opinion is (in thousands of $) *(Choose nearest value)*:

a. ≤16 b. 17 c. 18 d. 19
e. 20 f. 21 g. 22 h. ≥23

_____ 7. What would be the expected value of "perfect information" which is *given* to Sue, i.e., a prediction which is 100% accurate, so that the portion of the tree containing nodes 4, 5, 6, 7, etc., would appear as below? *(Choose nearest value, in thousands of $)*

a. ≤15 b. 20 c. 25 d. 30
e. 35 f. 40 g. 45 h. ≥50



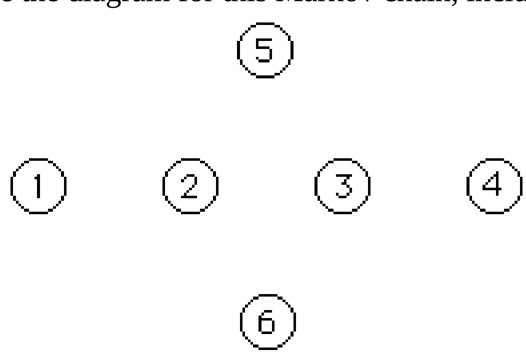
V. Absorption Analysis of Markov Chain. The college admissions officer has modeled the path of a student through the Engineering College as a Markov chain:

Each student's state is observed at the beginning of each fall semester. For example, if a student is a junior at the beginning of the current fall semester, there is an 80% chance that he will be a senior at the beginning of the next fall semester, a 15% chance that he will still be a junior, and a 5% chance that he will have quit. (Assume that once a student quits, he never re-enrolls.)

State	Description
1	Freshman
2	Sophomore
3	Junior
4	Senior
5	Drop-out
6	Graduate

Consult the computer output below to answer the questions that follow.

1. Complete the diagram for this Markov chain, including the transition probabilities.



2. Which states are transient? *Circle:* 1 2 3 4 5 6

3. Which states are recurrent? *Circle:* 1 2 3 4 5 6

4. Which states are absorbing? *Circle:* 1 2 3 4 5 6

____ 5. Does this system have a steady-state probability distribution?
a. Yes b. No c. Maybe

____ 6. If a student enters the college as a freshman, how many years can he or she expect to spend as a student in the college before either dropping out or graduating? *(Choose nearest value)*
a. 3.0 years b. 3.5 years c. 4 years d. 4.5 years e. 5 years

____ 7. The probability that, at the beginning of the fourth year in the college, he or she is classified as a senior is *(Choose nearest value)*
a. 50% b. 60% c. 70% d. 80% e. 90%

____ 8. The probability that he or she eventually will graduate is *(Choose nearest value)*
a. 30% b. 50% c. 70% d. 90% e. >90%

____ 9. If a student has survived to the point that he or she has been classified as a junior, the probability that he or she eventually graduates is *(Choose nearest value)*
a. 30% b. 50% c. 70% d. 90% e. >90%

Transition probabilities (P)

from \ to	1	2	3	4	5	6
1	0.1	0.8	0	0	0.1	0
2	0	0.1	0.85	0	0.05	0
3	0	0	0.15	0.8	0.05	0
4	0	0	0	0.1	0.05	0.85
5	0	0	0	0	1	0
6	0	0	0	0	0	1

E = Expected No. Visits to Transient States

from \ to	1	2	3	4
1	1.1111111	0.98765432	0.98765432	0.87791495
2	0	1.1111111	1.1111111	0.98765432
3	0	0	1.1764706	1.0457516
4	0	0	0	1.1111111

P^2

from \ to	1	2	3	4	5	6
1	0.01	0.16	0.68	0	0.15	0
2	0	0.01	0.2125	0.68	0.0975	0
3	0	0	0.0225	0.2	0.0975	0.68
4	0	0	0	0.01	0.055	0.935
5	0	0	0	0	1	0
6	0	0	0	0	0	1

P^3

from \ to	1	2	3	4	5	6
1	0.001	0.024	0.238	0.544	0.193	0
2	0	0.001	0.040375	0.238	0.142625	0.578
3	0	0	0.003375	0.038	0.108625	0.85
4	0	0	0	0.001	0.0555	0.9435
5	0	0	0	0	1	0
6	0	0	0	0	0	1

A = Absorption Probabilities

from \ to	5	6
1	0.25377229	0.74622771
2	0.16049383	0.83950617
3	0.11111111	0.88888889
4	0.055555556	0.94444444

P^4

from \ to	1	2	3	4	5	6
1	0.0001	0.0032	0.0561	0.2448	0.2334	0.4624
2	0	0.0001	0.00690625	0.0561	0.15659375	0.7803
3	0	0	0.00050625	0.0065	0.11069375	0.8823
4	0	0	0	0.0001	0.05555	0.94435
5	0	0	0	0	1	0
6	0	0	0	0	0	1

VI. Markov Chain Model--(s,S) Inventory System: Consider the following inventory system for a certain spare part for a company's 2 production lines. A maximum of four parts may be kept on the shelf. At the end of each day, the parts in use are inspected and, if worn, replaced with one off the shelf. The probability distribution of the number replaced each day is:

n=	0	1	2
P{n}=	0.3	0.5	0.2

To avoid shortages, the current policy is to restock the shelf at the end of each day (after spare parts have been removed) so that the shelf is again filled to its limit (i.e., 4) **if** there are fewer than 2 parts on the shelf.

The inventory system has been modeled as a Markov chain, with the state of the system defined as the end-of-day inventory level (before restocking). Refer to the computer output which follows to answer the following questions: **Note that in the computer output, state #1 is inventory level 0, state #2 is inventory level 1, etc.**

$$P=\begin{bmatrix} 0 & 0 & 0.2 & 0.5 & 0.3 \\ 0 & 0 & 0.2 & 0.5 & 0.3 \\ 0.2 & 0.5 & \boxed{A} & 0 & 0 \\ 0 & 0.2 & 0.5 & 0.3 & 0 \\ 0 & 0 & 0.2 & 0.5 & 0.3 \end{bmatrix},\ P^2=\begin{bmatrix} 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \\ 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \\ 0.06 & 0.15 & 0.23 & 0.35 & 0.21 \\ 0.1 & 0.31 & 0.34 & 0.19 & 0.06 \\ 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \end{bmatrix}$$

$$P^3=\begin{bmatrix} 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \\ 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \\ 0.046 & 0.185 & 0.328 & 0.315 & 0.126 \\ 0.068 & 0.208 & 0.291 & 0.292 & 0.141 \\ 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \end{bmatrix},\ P^4=\begin{bmatrix} 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \\ 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \\ 0.065 & 0.227 & 0.327 & 0.273 & 0.107 \\ 0.058 & 0.203 & 0.316 & 0.296 & 0.125 \\ 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \end{bmatrix}$$

$$\sum_{n=1}^{4} P^n=\begin{bmatrix} 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \\ 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \\ 0.3716 & 1.062 & 1.1853 & 0.938 & 0.4431 \\ 0.2262 & 0.9219 & 1.4477 & 1.0781 & 0.3261 \\ 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \end{bmatrix}$$

$$M=\begin{bmatrix} 15.769 & 4.641 & 3.076 & 2.571 & 8.367 \\ 15.769 & 4.641 & 3.076 & 2.571 & 8.367 \\ 12.692 & 2.754 & 3.153 & 4 & 9.795 \\ 15 & 3.396 & 2.307 & 3.514 & 10.816 \\ 15.769 & 4.641 & 3.076 & 2.571 & \boxed{B} \end{bmatrix}$$

i	π_i
1	0.063
2	0.215
3	0.317
4	0.284
5	0.119

____ 1. the value $P_{5,3}$ is
a. P{demand=0} b. P{demand=1}
c. P{demand=2} d. P{demand≤1}
e. P{demand≥1} f. none of the above

____ 2. the value $P_{1,4}$ is
a. P{demand=0} b. P{demand=1}
c. P{demand=2} d. P{demand≤1}
e. P{demand≥1} f. none of the above

____ 3. the value $P_{3,1}$ is
a. P{demand=0} b. P{demand=1}
c. P{demand=2} d. P{demand≤1}
e. P{demand≥1} f. none of the above

____ 4. the numerical value **A** in the matrix above is *(select nearest value)*
a. 0 b. 0.1 c. 0.2 d. 0.3
e. 0.4 f. 0.5

____ 5. the numerical value **B** in the mean-first-passage time matrix (M) above is *(select nearest value)*
a. 1 b. 2 c. 4 d. 6
e. 8 f. 10

____ 6. If the shelf is full Monday morning, the expected number of days until a stockout occurs is *(select nearest value):*
a. 2 b. 5 c. 10 d. 15
e. 20 f. more than 20

____ 7. If the shelf is full Monday morning, the probability that the shelf is full Thursday night is *(select nearest value):*
a. 7% b. 8% c. 9% d. 10%
e. 11% f. more than 12%

____ 8. If the shelf is full Monday morning, the probability that the shelf is restocked Thursday night is (select nearest value):
a. 10% b. 15% c. 20% d. 25%
e. 30% f. more than 30%

____ 9. If the shelf is full Monday morning, the expected number of nights that the shelf is restocked before Friday morning is (select nearest value):
a. 0.6 b. 0.7 c. 0.8 d. 0.9
e. more than once but less than twice f. more than 2

____ 10. The number of transient states in this Markov chain model is
a. zero b. 1
c. 2 d. 5 e. none of the above

____ 11. The number of recurrent states in this Markov chain model is
a. zero b. 1
c. 2 d. 5 e. none of the above

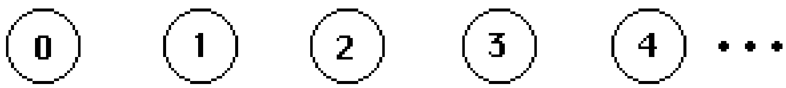
____ 12. The number of absorbing states in this Markov chain model is
a. zero b. 1
c. 2 d. 5 e. none of the above

____ 13. Which (one or more) of the following equations are among those solved to compute the steady state probability distribution?
a. $\pi_1 = 0.2\pi_3$
b. $\pi_1 = 0.2\pi_3 + 0.5\pi_4 + 0.3\pi_5$

c. $\pi_3 = 0.2\pi_1 + 0.2\pi_2 + 0.3\pi_3 + 0.5\pi_4 + 0.2\pi_5$
d. $\pi_4 = 0.2\pi_2 + 0.5\pi_3 + 0.3\pi_4$
e. $\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1$

VII. Continuous-Time Markov Chain Two mechanics work in an auto repair shop, with a **maximum capacity** of **3** cars, so that any cars arriving when there are already 3 in the shop are turned away. Each mechanic works individually, completing the repair of a car in an average of **6** hours (the actual time being random with exponential distribution). (If there is only one car in the shop, only one mechanic works on it, while the other takes a break.) Cars arrive randomly, according to a Poisson process, at the rate of one every **two** hours when there are no cars in the shop, but one every **three** hours when one or more cars are in the shop. (If 3 cars are in the shop, of course, no cars will enter the shop.)

1. Draw a transition diagram below, with transition rates included, for this system.



____ 2. What is the name of the distribution of the time between arrivals when the shop is empty?

a. Markov	b. Poisson	c. Uniform	d. Exponential
e. Normal	f. Weibull	g. None of the above	

____ 3. The steady-state probability that the shop is empty is *(choose nearest value):*

a. 10%	b. 20%	c. 30%	d. 40%
e. 50%	f. 60%	g. 70%	h. >80%

____ 4. In steady state, the fraction of the day that *exactly one* car will be in the shop is *(choose nearest value):*

a. 10%	b. 20%	c. 30%	d. 40%
e. 50%	f. 60%	g. 70%	h. >80%

____ 5. In steady state, the average number of cars in the shop is *(choose nearest value):*

a. .5	b. 1	c. 1.5	d. 2
	e. 2.5	f. 3	

____ 6. The average arrival rate in steady state is approximately one every 4 hours, i.e., 0.25/hour. According to Little's Formula, the average total time spent by a car in the shop (including both waiting and repair time) is *(choose nearest value):*

a. 6 hours	b. 7 hours	c. 8 hours	d. 9 hours
	e. 10 hours	f. 11 hours	

VIII. Dynamic Programming We wish to plan production of an expensive, low-demand item for the next three months (January, February, & March).

- the cost of production is \$15 for setup, plus \$5 per unit produced, up to a maximum of 4 units.
- the storage cost for inventory is \$2 per unit, based upon the level at the beginning of the month.
- a maximum of 3 units may be kept in inventory at the end of each month; any excess inventory is simply discarded.
- the demand each month is random, with the same probability distribution each month:

d	0	1	2
P{D=d}	0.3	0.4	0.3

- there is a penalty of \$25 per unit for any demand which cannot be satisfied. Backorders are not allowed.
- there is zero inventory at the end of December.
- a salvage value of \$4 per unit is received for any inventory remaining at the end of the last month (March)

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 3 = January, stage 2 = February, etc.** (i.e., n = # months remaining in planning period.)

1. What is the optimal production quantity for January (starting with zero inventory)? _______
2. What is the total expected cost for the three months? __________
3. If, during January, the demand is 2 units, what should be produced in February? ______
4. Three values have been blanked out in the computer output, What are they?
   i. the optimal value $f_2(1)$ ___________
   ii. the optimal decision $x_2^*(1)$ ___________
   iii. the cost associated with the decision to produce 1 unit in February when the inventory is 0 at the end of January. ____________

The table of costs for each combination of state & decision at stage 2 is:

STAGE 2

s \ x:	0	1	2	3	4
0	46.00		34.62	31.89	33.60
1	26.69	31.62	28.89	30.60	35.00
2	13.62	25.89	27.60	32.00	37.00
3	7.89	24.60	29.00	34.00	39.00

The tables of the optimal value function $f_n(S_n)$ at each stage are:

Stage 3

State	Optimal Values	Optimal Decisions
0	44.61	4
1	39.83	0
2	28.33	0
3	21.82	0

Stage 2

State	Optimal Values	Optimal Decisions
0	31.89	3
1		
2	13.62	0
3	7.89	0

Stage 1

State	Optimal Values	Optimal Decisions
0	21.00	2
1	8.30	0
2	0.00	0
3	¯2.00	0