56:171 Operations Research
Final Exam
December 12, 1994

- **Write your name on the first page, and initial the other pages.**
- **The response *"NOTA "* = "None of the above"**
- **Answer both parts A & B, and five sections of part C.**

	Possible	Score
A. Multiple Choice	10	_____
B. Sensitivity analysis (LINDO)	15	_____
C. **Choose 5 of 6:**		
C.1. Project Scheduling	12	_____
C.2. Integer LP Models	12	_____
C.3. Discrete-time Markov chains I	12	_____
C.4. Discrete-time Markov chains II	12	_____
C.5. Birth-Death Processes	12	_____
C.6. Dynamic Programming	12	_____
total possible:	85	_____

Part A

Multiple Choice: Write the appropriate letter (a, b, c, d, or e) : (*NOTA* =None of the above).

____ 1. If $X_{ij}>0$ in the transportation problem, then dual variables U and V *must* satisfy
- a. $C_{ij}>U_i+V_j$
- b. $C_{ij}=U_i+V_j$
- c. $C_{ij}<U_i+V_j$
- d. $C_{ij}+U_i+V_j = 0$
- e. $C_{ij}=U_i - V_j$
- f. *NOTA*

____ 2. If, in the optimal *dual* solution of an LP problem (min cx st $Ax \leq b$, $x \geq 0$), variable #2 is zero, then in the optimal primal solution,
- a. variable #2 must be zero
- b. variable #2 must be positive
- c. slack variable for constraint #2 must be zero
- d. constraint #2 must be slack
- e. *NOTA*

____ 3. If, in the optimal *primal* solution of an LP problem (min cx st $Ax \leq b$, $x \geq 0$), there is positive slack in constraint #3, then in the optimal dual solution,
- a. dual variable #3 must be zero
- b. dual variable #3 must be positive
- c. slack variable for dual constraint #3 must be zero
- d. dual constraint #3 must be slack
- e. *NOTA*

____ 4. If there is a tie in the "minimum-ratio test" of the simplex method, the solution in the next tableau
- a. will be nonbasic
- b. will be nonfeasible
- c. will have a worse objective value
- d. will be degenerate
- e. *NOTA*

____ 5. For a continuous-time Markov chain, let Λ be the matrix of transition rates. The sum of each...
- a. column is 1
- b. column is 0
- c. row is 1
- d. row is 0
- e. *NOTA*

____ 6. For a discrete-time Markov chain, let P be the matrix of transition probabilities. The sum of each...
- a. column is 1
- b. column is 0
- c. row is 1
- d. row is 0
- e. *NOTA*

____ 7. In PERT, the completion time for the project is assumed to
- a. have the Beta distribution
- b. have the Normal distribution
- c. be constant
- d. have the exponential distribution
- e. *NOTA*

____ 8. In an M/M/1 queue, if the arrival rate = $\lambda > \mu$ = service rate, then
- a. $\pi_0 = 1$ in steady state
- b. no steady state exists
- c. $\pi_i > 0$ for all i
- d. $\pi_0 = 0$ in steady state
- e. the queue is not a birth-death process
- f. *NOTA*

____ 9. The Poisson process is a special case of the birth-death process with
- a. no births
- b. no deaths
- c. birth rate = death rate
- d. death is by Poissoning
- e. time between births &/or deaths has Poisson distribution
- f. *NOTA*

____ 10. An absorbing state of a Markov chain is one in which the probability of
a. moving into that state is zero
b. moving out of that state is one.
c. moving out of that state is zero.
d. *NOTA*

Part B

LINDO analysis

Problem Statement: McNaughton Inc. produces two steak sauces, spicy Diablo and mild Red Baron. These sauces are both made by blending two ingredients A and B. A certain level of flexibility is permitted in the formulas for these products. Indeed, the restrictions are that:

i) Red Baron must contain no more than 75% of A.
ii) Diablo must contain no less than 25% of A and no less than 50% of B

Up to 40 quarts of A and 30 quarts of B could be purchased. McNaughton can sell as much of these sauces as it produces at a price per quart of $3.35 for Diablo and $2.85 for Red Baron. A and B cost $1.60 and $2.05 per quart, respectively. McNaughton wishes to maximize its net revenue from the sale of these sauces.

Define *D = quarts of Diablo to be produced*
R = quarts of Red Baron to be produced
AD= quarts of A used to make Diablo
AR = quarts of A used to make Red Baron
BD = quarts of B used to make Diablo
BR = quarts of B used to make Red Baron

The LINDO output for solving this problem follows:

```
MAX     3.35 D + 2.85 R - 1.6 AD - 1.6 AR - 2.05 BD - 2.05 BR
SUBJECT TO
       2) - D + AD + BD =    0
       3) - R + AR + BR =    0
       4)   AD + AR <=   40
       5)   BD + BR <=   30
       6) - 0.25 D + AD >=   0
       7) - 0.5 D + BD >=   0
       8) - 0.75 R + AR <=   0
END
        OBJECTIVE FUNCTION VALUE
  1)        99.0000000

VARIABLE          VALUE                 REDUCED COST
      D           50.000000                0.000000
      R           20.000000                0.000000
      AD          25.000000                0.000000
      AR          15.000000                0.000000
      BD          25.000000                0.000000
      BR           5.000000                0.000000

    ROW           SLACK OR SURPLUS      DUAL PRICES
      2)             0.000000             -2.350000
      3)             0.000000             -4.350000
      4)             0.000000              0.750000
      5)             0.000000              2.300001
      6)            12.500000              0.000000
      7)             0.000000             -1.999999
      8)             0.000000              2.000000
```

RANGES IN WHICH THE BASIS IS UNCHANGED

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
D	3.350000	0.750000	0.500000
R	2.850000	0.500000	0.375000
AD	-1.600000	1.500001	0.666666
AR	-1.600000	0.666666	0.500000
BD	-2.050000	1.500001	1.000000
BR	-2.050000	1.000000	1.500001

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	0.000000	10.000000	10.000000
3	0.000000	16.666668	3.333333
4	40.000000	50.000000	10.000000
5	30.000000	10.000000	16.666664
6	0.000000	12.500000	INFINITY
7	0.000000	6.250000	5.000000
8	0.000000	2.500000	12.500000

THE TABLEAU:

ROW	(BASIS)	D	R	AD	AR	BD	BR	SLK 4	SLK 5	SLK 6
1	ART	0.000	0.000	0.000	0.000	0.000	0.000	0.750	2.300	0.000
2	AD	0.000	0.000	1.000	0.000	0.000	0.000	-0.500	1.500	0.000
3	R	0.000	1.000	0.000	0.000	0.000	0.000	2.000	-2.000	0.000
4	AR	0.000	0.000	0.000	1.000	0.000	0.000	1.500	-1.500	0.000
5	BR	0.000	0.000	0.000	0.000	0.000	1.000	0.500	-0.500	0.000
6	SLK 6	0.000	0.000	0.000	0.000	0.000	0.000	-0.250	0.750	1.000
7	D	1.000	0.000	0.000	0.000	0.000	0.000	-1.000	3.000	0.000
8	BD	0.000	0.000	0.000	0.000	1.000	0.000	-0.500	1.500	0.000

ROW	SLK 7	SLK 8	RHS
1	2.000	2.000	99.000
2	3.000	2.000	25.000
3	-4.000	-4.000	20.000
4	-3.000	-2.000	15.000
5	-1.000	-2.000	5.000
6	2.000	1.000	12.500
7	4.000	4.000	50.000
8	1.000	2.000	25.000

___1. If the profit on DIABLO sauce were to decrease from $3.35 /quart to $3.00/quart, the number of quarts of DIABLO to be produced would

a. increase
b. decrease
c. remain the same
d. insufficient info. given
e. *NOTA*

___2. The LP problem above has

a. exactly one optimal sol'n
b. exactly two optimal sol'ns
c. infinitely many sol'ns
d. no optimal solution
e. insufficient info. given
f. *NOTA*

___3. If an additional 8 quarts of ingredient A were available, McNaughton's profits would be

a. $0.75
b. $8.00
c. $105.00
d. $107.00
e. insufficient info. given
f. *NOTA*

___4. If the variable "SLK 5" were increased, this would be equivalent to

a. increasing A availability
b. decreasing A availability
c. increasing B availability
d. decreasing B availability
e. *NOTA*

___5. If the variable "SLK 5" were increased by 5, the quantity of DIABLO produced would be:

a. 35 quarts c. 53 quarts e. 65 quarts
b. 50 quarts d. 55 quarts f. *NOTA*

___6. If a pivot were to be performed to enter the variable SLK5 into the basis, then according to the "minimum ratio test", the value of SLK5 in the resulting basic solution would be

a. 16.66667 c. 10.0 e. 5.0
b. 0.06 d. 0.10 f. *NOTA*

___7. If the variable SLK5 were to enter the basis, then the variable leaving the basis is

a. A c. AD e. D g. SLK6 i. more than one answer is possible
b. B d. BD f. R h. any of the above j. *NOTA*

___8. If the variable SLK5 were to enter the basis, then the next tableau

a. indicates multiple optimal sol'ns c. both of the above
b. is degenerate d. *NOTA*

___9. The dual of the LP above has an objective function which is to be

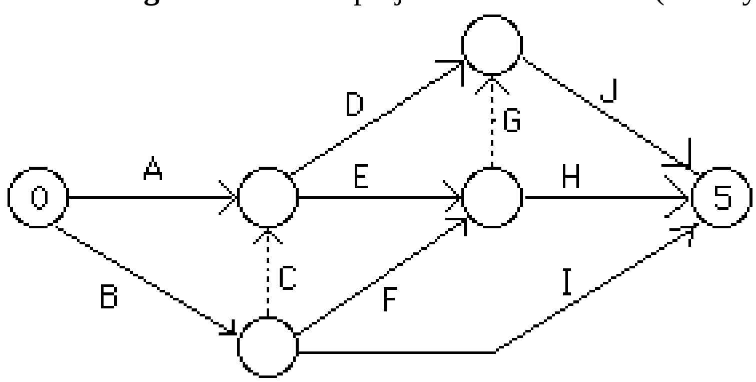
a. minimized c. both of the above
b. maximized d. *NOTA*

___10. The dual of the LP above has an optimal value which is

a. 0 c. 99 e. insufficient infomation given
b. 1 d. 100 f. *NOTA*

Part C

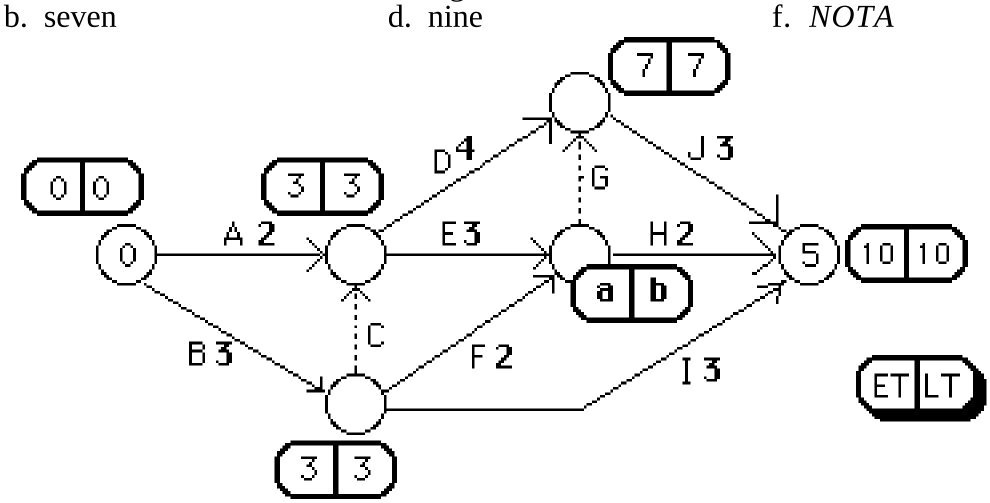
C.1. Project Scheduling. Consider the project with the A-O-A (activity-on-arrow) network:



1. Complete the labeling of the nodes on the network above.

___ 2. The number of activities (i.e., tasks), not including "dummies", which are required to complete this project is

a. six c. eight e. ten
b. seven d. nine f. *NOTA*



The activity durations are given above on the arrows. The Early Times (ET) and Late Times (LT) for each node are written in the box (with rounded corners) beside each node.

___ 3. The early time (ET) indicated by **a** in the network above is:
a. four c. six e. eight
b. five d. seven f. *NOTA*

___ 4. The late time (LT) ndicated by **b** in the network above is:
a. four c. six e. eight
b. five d. seven f. *NOTA*

___ 5. The slack ("total float") for activity A is
a. zero c. two e. four
b. one d. three f. *NOTA*

6. Which activities are critical? (*circle:* A B C D E F G H I J)

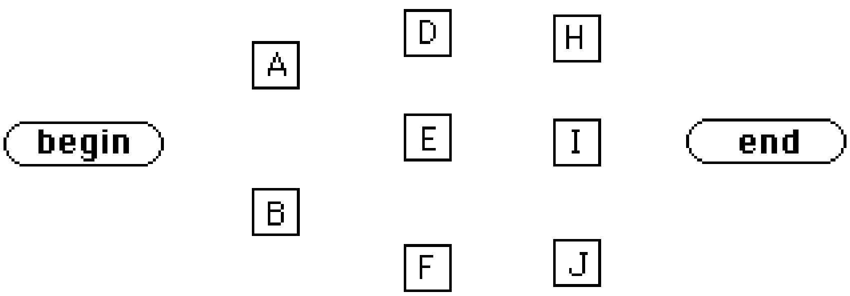
___ 7. The earliest completion time for the project is
a. six c. eight e. ten
b. seven d. nine f. *NOTA*

Suppose that the non-zero durations are *random*, with each value in the above network being the *expected* values and each *standard deviation* equal to 1.00. Then...

___ 8. The expected earliest completion time for the project is
a. six c. eight e. ten
b. seven d. nine f. *NOTA*

___ 9. The *standard deviation* σ of the earliest completion time for the project is
a. $\sqrt{3}$ c. 2 e. $\sqrt{7}$ g. $\sqrt{8}$
b. 3 d. 4 f. 7 h. *NOTA*

10. Add the arrows to complete the A-O-N (activity-on-node) network below for this same project.



C.2. Integer LP Models. To model a production planning problem, define
X_j = amount of item type j which is produced (a continuous variable), and
Y_j = 1 if the machine is set up to produce item type j,
otherwise 0 (a binary integer)
for j=1,2,3,....
Select a constraint (or set of constraints) to model each situation below:

____ 1. If the machine is set up for neither item #2 nor item #3, then item #1 *cannot* be produced.
a. $2Y_1 \geq Y_2+Y_3$ c. $2Y_1 = Y_2+Y_3$ e. $2Y_1 \leq Y_2+Y_3$
b. $Y_1 \leq Y_2 + Y_3$ d. $X_1 \leq Y_2 + Y_3$ f. *NOTA*

____ 2. If item #1 is produced, then *at least* 100 units of item #1 must be produced.
a. $X_1 \leq 100Y_1$ c. $100X_1 \leq Y_1$ e. $X_1 + Y_1 \leq 100$
b. $100X_1 \geq Y_1$ d. $X_1 \geq 100Y_1$ f. *NOTA*

____ 3. If both items #2 and #3 are produced, then item #1 must also be produced.
a. $2Y_1 \geq Y_2+Y_3$ c. $2Y_1 = Y_2+Y_3$ e. $2Y_1 \leq Y_2+Y_3$
b. $Y_1 \leq Y_2 + Y_3$ d. $X_1 \leq Y_2 + Y_3$ f. *NOTA*

____ 4. If it is decided to produce item #1, then *at most* 100 units of item #1 may be produced.

a. $X_1 \leq 100Y_1$ c. $100X_1 \leq Y_1$ e. $X_1 + Y_1 \leq 100$
b. $100X_1 \geq Y_1$ d. $X_1 \geq 100Y_1$ f. *NOTA*

____ 5. *At most one* type of item may be produced.

a. $X_1 + X_2 + X_3 \leq 1$ c. $Y_1 + Y_2 + Y_3 \geq 1$ e. $Y_1 + Y_2 + Y_3 \leq 2$
b. $X_1 + X_2 + X_3 \geq 1$ d. $Y_1 + Y_2 + Y_3 \leq 1$ f. *NOTA*

____ 6. If a setup is done for *both* items #2 & #3, the machine should *not* be set up for item #1.

a. $2Y_1 \geq Y_2+Y_3$ c. $2Y_1 = Y_2+Y_3$ e. $2Y_1 \leq Y_2+Y_3$
b. $Y_1 \leq Y_2 + Y_3$ d. $Y_1 \geq Y_2 + Y_3$ f. *NOTA*

____ 7. At least two different item types must be produced.

a. $2Y_1 \geq Y_2+Y_3$ c. $2Y_1 = Y_2+Y_3$ e. $2Y_1 \leq Y_2+Y_3$
b. $Y_1 \leq Y_2 + Y_3$ d. $X_1 \leq Y_2 + Y_3$ f. *NOTA*

____ 8. The machine must be set up for at least one type of item.

a. $X_1 + X_2 + X_3 \leq 1$ c. $Y_1 + Y_2 + Y_3 \geq 1$ e. $Y_1 + Y_2 + Y_3 \leq 2$
b. $X_1 + X_2 + X_3 \geq 1$ d. $Y_1 + Y_2 + Y_3 \leq 1$ f. *NOTA*

____ 9. If the machine is set up for item #1, then it should also be set up for item #2.

a. $Y_1 \leq Y_2$ c. $X_1 \leq X_2$ e. $Y_1 = Y_2$
b. $Y_1 \geq Y_2$ d. $X_1 \geq X_2$ f. *NOTA*

____ 10. If a setup is done for item #1, the machine should also be set up for *both* items #2 and #3.

a. $2Y_1 \geq Y_2+Y_3$ c. $2Y_1 = Y_2+Y_3$ e. $2Y_1 \leq Y_2+Y_3$
b. $Y_1 \leq Y_2 + Y_3$ d. $X_1 \leq Y_2 + Y_3$ f. *NOTA*

____ 11. If a setup is done for *both* items #2 & #3, the machine should also be set up for item #1.

a. $2Y_1 \geq Y_2+Y_3$ c. $2Y_1 = Y_2+Y_3$ e. $2Y_1 \leq Y_2+Y_3$
b. $Y_1 \leq Y_2 + Y_3$ d. $Y_1 \geq Y_2 + Y_3$ f. *NOTA*

____ 12. If item #1 is produced, then *either* item #2 *or* item #3 (or both) must be produced.

a. $2Y_1 \geq Y_2+Y_3$ c. $2Y_1 = Y_2+Y_3$ e. $2Y_1 \leq Y_2+Y_3$
b. $Y_1 \leq Y_2 + Y_3$ d. $X_1 \leq Y_2 + Y_3$ f. *NOTA*

C.3. Discrete-Time Markov Chains I Customers buy cars from three auto companies. Given the company from which a customer last bought a car, the probability that she will buy her next car from each company is as follows:

	Will Buy Next From		
Last Bought From	Co. A	Co. B	Co.C
Company A	75%	15%	10%
Company B	5%	85%	10%
Company C	15%	5%	80%

The first several powers of the matrix (P) above, the first-passage probabilities, the mean-first-passage times, and the steadystate distribution, are:

$$P = \begin{bmatrix} 0.75 & 0.15 & 0.1 \\ 0.05 & 0.85 & 0.1 \\ 0.15 & 0.05 & 0.8 \end{bmatrix} \qquad F = \begin{bmatrix} 0.75 & 0.15 & 0.1 \\ 0.05 & 0.85 & 0.1 \\ 0.15 & 0.05 & 0.8 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.585 & 0.245 & 0.17 \\ 0.095 & 0.735 & 0.17 \\ 0.235 & 0.105 & 0.66 \end{bmatrix} \qquad F^2 = \begin{bmatrix} 0.0225 & 0.1175 & 0.09 \\ 0.0575 & 0.0125 & 0.09 \\ 0.1225 & 0.0625 & 0.02 \end{bmatrix}$$

$$P^3=\begin{bmatrix} 0.4765 & 0.3045 & 0.219 \\ 0.1335 & 0.6475 & 0.219 \\ 0.2805 & 0.1575 & 0.562 \end{bmatrix} \quad F^3=\begin{bmatrix} 0.020875 & 0.094375 & 0.081 \\ 0.061125 & 0.012125 & 0.081 \\ 0.100875 & 0.067625 & 0.018 \end{bmatrix}$$

$$P^4=\begin{bmatrix} 0.40545 & 0.34125 & 0.2533 \\ 0.16535 & 0.58135 & 0.2533 \\ 0.30255 & 0.20405 & 0.4934 \end{bmatrix} \quad F^4=\begin{bmatrix} 0.01925625 & 0.07754375 & 0.0729 \\ 0.06204375 & 0.01148125 & 0.0729 \\ 0.08375625 & 0.06825625 & 0.0162 \end{bmatrix}$$

$\pi_A = 0.27777$
$\pi_B = 0.38888$
$\pi_C = 0.33333$

$$m = \begin{bmatrix} 3.6 & 8.5714286 & 10 \\ 12 & 2.5714286 & 10 \\ 8 & 11.428571 & 3 \end{bmatrix}$$

Suppose that "Jane Doe" buys a new car from company C in 1994, and replaces her car *every year!* **In the multiple choices below, choose the number nearest to the correct answer.**

___1. What is the probability that Jane's *next* car (i.e., in 1995) is a Company A car?
a. 5% c. 15% e. 25% g. 70% i. 80% k. 90%
b. 10% d. 20% f. 30% h. 75% j. 85% l. 95%

___ 2. What is the probability that the car which Jane purchases in 1996 is a Company A car?
a. 5% c. 15% e. 25% g. 70% i. 80% k. 90%
b. 10% d. 20% f. 30% h. 75% j. 85% l. 95%

___ 3. What is the probability that *at least one* of the next two cars which Jane buyswill be a Company A car?
a. 5% c. 15% e. 25% g. 70% i. 80% k. 90%
b. 10% d. 20% f. 30% h. 75% j. 85% l. 95%

___ 4. What is the probability that the first Company A car which Jane buys will be in 1997?
a. 5% c. 15% e. 25% g. 70% i. 80% k. 90%
b. 10% d. 20% f. 30% h. 75% j. 85% l. 95%

___ 5. What is the expected number of years until Jane buys a Company A car?
a. 1 c. 3 e. 5 g. 7 i. 9 k. 11
b. 2 d. 4 f. 6 h. 8 j. 10 l. 12

___ 6. Over a "long" period of time, which company would you expect to have the *largest* market share?
a. Company A c. Company C e. All 3 share equally
b. Company B d. Both Co. A & C equal f. *NOTA*

___ 8. The number of *transient* states in this Markov chain model is
a. 0 c. 2 e. 9
b. 1 d. 3 f. *NOTA*

___ 9. The number of *recurrent* states in this Markov chain model is
a. 0 c. 2 e. 9
b. 1 d. 3 f. *NOTA*

___ 10. The probabilities in a Markov chain transition matrix are
a. simple probabilities. c. conditional probabilities.
b. joint probabilities. d. more than one of the above are correct. e. *NOTA*

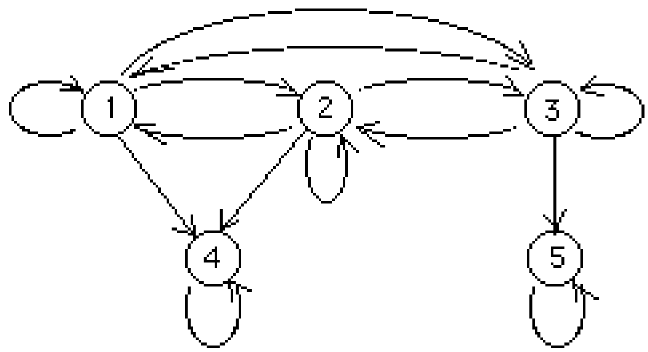
___ 11. The steady-state probability vector π of a discrete Markov chain with transition probability matrix P satisfies the matrix equation
a. $P^t \pi = 0$ c. $\pi (I+P) = 0$ e. $P \pi = \pi$ g. $\pi P = 0$
b. $P \pi = 0$ d. $(I+P) \pi = 0$ f. $\pi P = \pi$ h. *NOTA*

___ 12. The equations to be solved for the steadystate probabilities include:
a. $0.1\pi_A+0.1\pi_B+0.8\pi_C = \pi_A$ c. $0.15\pi_A+0.05\pi_B+0.8\pi_C=\pi_C$ e. $\pi_A+\pi_B+\pi_C = 0$
b. $0.1\pi_A+0.1\pi_B+0.8\pi_C = \pi_C$ d. $0.1\pi_A+0.1\pi_B+0.8\pi_C = 0$ f. *NOTA*

C.4. Discrete-Time Markov Chains II A baseball team consists of 2 stars, 13 starters, and 10 substitutes. For tax purposes, the team owner must place a value on the players. The value of each player is defined to be the total value of the salary he will earn until retirement. At the beginning of each season, the players are classified into one of four categories:

Category 1: Substitute (earns $100,000 per year).
Category 2: Starter (earns $400,000 per year).
Category 3: Star (earns $1 million per year).
Category 4: Retired while not a star (earns no more salary).
Category 5: Retired while Star (earns no salary, but is paid $100,000/year for product endorsements).

Given that a player is a star, starter, or substitute at the beginning of the current season, the probabiliites that he will be a star, starter, substitute, or retired at the beginning of the next season are shown in the transition probability matrix P below. Also shown are a diagram of the Markov chain model of a "typical" player, several powers of P, the first-passage probability matrices, the absorption probabilities, and the matrix of expected number of visits.



$$P = \begin{bmatrix} 0.5 & 0.15 & 0.05 & 0.3 & 0 \\ 0.2 & 0.5 & 0.2 & 0.1 & 0 \\ 0.15 & 0.3 & 0.45 & 0 & 0.1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad F = \begin{bmatrix} 0.5 & 0.15 & 0.05 & 0.3 & 0 \\ 0.2 & 0.5 & 0.2 & 0.1 & 0 \\ 0.15 & 0.3 & 0.45 & 0 & 0.1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.2875 & 0.165 & 0.077 & 0.465 & 0.005 \\ 0.23 & 0.34 & 0.2 & 0.21 & 0.02 \\ 0.2025 & 0.307 & 0.27 & 0.075 & 0.145 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad F^{(2)} = \begin{bmatrix} 0.0375 & 0.09 & 0.055 & 0.165 & 0.005 \\ 0.13 & 0.09 & 0.11 & 0.11 & 0.02 \\ 0.1275 & 0.157 & 0.067 & 0.075 & 0.045 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 0.188 & 0.148 & 0.082 & 0.567 & 0.012 \\ 0.213 & 0.264 & 0.169 & 0.313 & 0.04 \\ 0.203 & 0.265 & 0.193 & 0.166 & 0.172 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad F^{(3)} = \begin{bmatrix} 0.0258 & 0.0528 & 0.044 & 0.1027 & 0.007 \\ 0.0905 & 0.0495 & 0.066 & 0.103 & 0.02 \\ 0.0963 & 0.0843 & 0.041 & 0.0915 & 0.027 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 0.136 & 0.127 & 0.076 & 0.639 & 0.020 \\ 0.184 & 0.215 & 0.139 & 0.403 & 0.056 \\ 0.183 & 0.220 & 0.150 & 0.253 & 0.191 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad F^{(4)} = \begin{bmatrix} 0.018 & 0.030 & 0.031 & 0.071 & 0.008 \\ 0.064 & 0.027 & 0.041 & 0.090 & 0.016 \\ 0.070 & 0.045 & 0.026 & 0.087 & 0.019 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A =

	4	5
1	0.93103	0.068966
2	0.86207	0.13793
3	0.72414	0.27586

E =

	1	2	3
1	2.6959	1.2226	0.68966
2	1.7555	3.3542	1.3793
3	1.6928	2.163	2.7586

Suppose that at the beginning of the 1994 season, Joe Blough was a Starter (category #2).

Select the **nearest** available numerical choice in answering the questions below.

___ 1. The number of *transient* states in this Markov chain model is

a. 0 c. 2 e. 4 g. *NOTA*
b. 1 d. 3 f. 5

___ 2. The number of *recurrent* states in this Markov chain model is

a. 0 c. 2 e. 4 g. *NOTA*
b. 1 d. 3 f. 5

3. The closed sets of states in this Markov chain model are (circle all that apply!)

a. {1} d. {4} g. {1,2,3} j. {3,4,5}
b. {2} e. {5} h. {1,2,3,4} k. {2,3,4,5}
c. {3} f. {1,2} i. {4,5} l. *NOTA*

4. The *minimal* closed sets of states in this Markov chain model are (circle all that apply!)

a. {1} d. {4} g. {1,2,3} j. {3,4,5}
b. {2} e. {5} h. {1,2,3,4} k. {2,3,4,5}
c. {3} f. {1,2} i. {4,5} l. *NOTA*

___ 5. What is the probability that Joe is a star in 1995? (choose nearest answer)

a. 5% c. 15% e. 25% g. 35% i. 45%
b. 10% d. 20% f. 30% h. 40% j. 50%

___ 6. What is the probability that Joe is a star in 1996? (choose nearest answer)

a. 5% c. 15% e. 25% g. 35% i. 45%
b. 10% d. 20% f. 30% h. 40% j. 50%

___ 7. What is the probability that Joe *first* becomes a star in 1996? (choose nearest answer)

a. 5% c. 15% e. 25% g. 35% i. 45%
b. 10% d. 20% f. 30% h. 40% j. 50%

___ 8. What is the probability that Joe *eventually* becomes a star before he retires? (choose nearest answer)

a. 5% c. 15% e. 25% g. 35% i. 45%
b. 10% d. 20% f. 30% h. 40% j. 50%

___ 9. What is the expected length of his playing career, in years? (choose nearest answer)

a. 1 year c. 3 years e. 5 years g. 7 years i. 9 years
b. 2 years d. 4 years f. 6 years h. 8 years j. *NOTA*

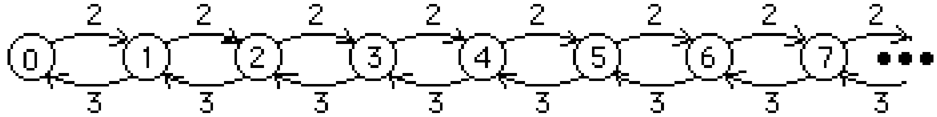
___ 10. What fraction of players who achieve "stardom" retire while still a star? (choose nearest answer)

a. 10% c. 30% e. 50% g. 70% i. 90%
b. 20% d. 40% f. 60% h. 80% j. 100%

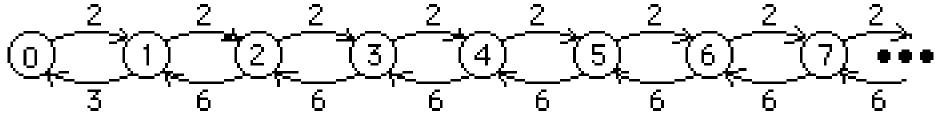
C.5. Birth-Death Processes For each birth-death model of a queue in diagrams (1) through (8) below, indicate the correct Kendall's classification from among the following choices. (Note that some classifications might not be matched with any birth-death diagram, while others might be matched to more than one!)

a. M/M/1 d. M/M/2 g. M/M/1/4 j. M/M/2/3
b. M/M/4 e. M/M/2/4 h. M/M/2/4/4 k. M/M/2/2/4
c. M/M/1/2/4 f. M/M/4/2 i. M/M/4/4 l. *NOTA*

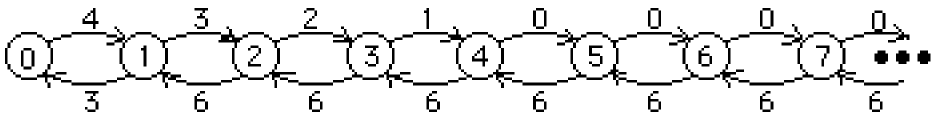
___ 1

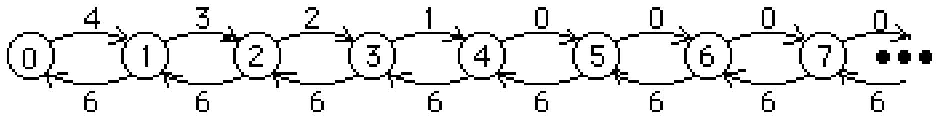


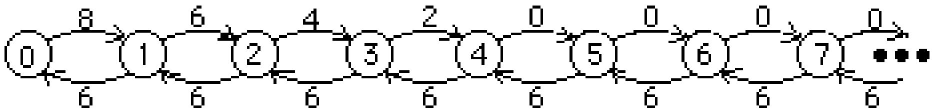
___ 2

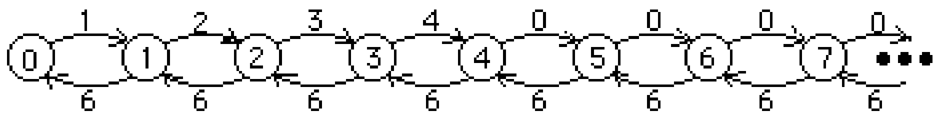


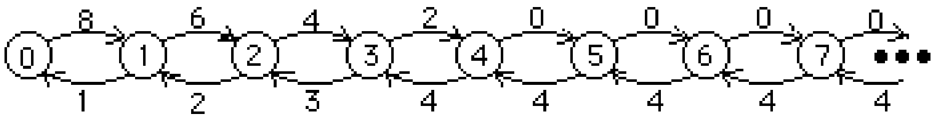
___ 3

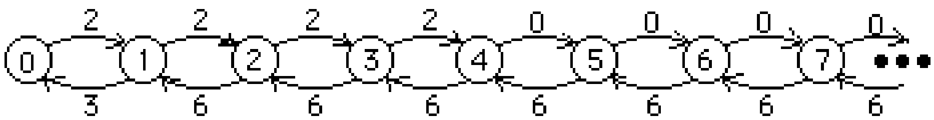


___4 

___5 

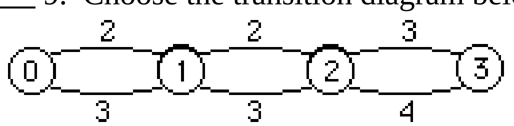
___6 

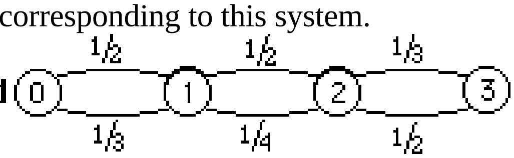
___7 

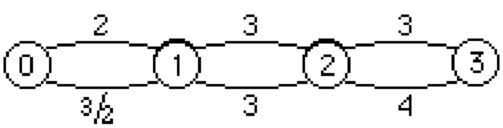
___8 

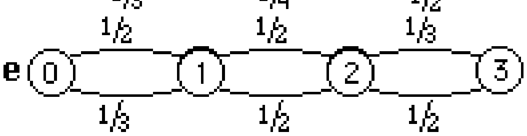
Two mechanics work in an auto repair shop, with a capacity of 3 cars. If there are 2 or more cars in the shop, each mechanic works individually, each completing the repair of a car in an average of **4** hours (the actual time being random with exponential distribution). If there is only one car in the shop, both mechanics work together on it, completing the repair in an average time of **3** hours (also exponentially distributed). Cars arrive randomly, according to a Poisson process, at the rate of one every two hours when there is at least one idle mechanic, but one every **3** hours when both mechanics are busy. If 3 cars are in the shop, no cars arrive.

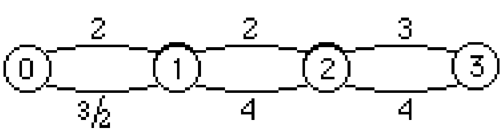
____ 9. Choose the transition diagram below corresponding to this system.

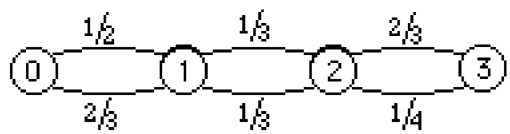
a 

d 

b 

e 

c 

f 

g. *NOTA*

The steady-state probabilities for this system are: π_0=20%, π_1=30%, π_2=30%, & π_3=20%.

____ 9. What fraction of the day will both mechanics be idle?

a. 10%	c. 30%	e. 50%	g. 70%
b. 20%	d. 40%	f. 60%	h. *NOTA*

____ 10. What fraction of the day will both mechanics be working on the same car?

a. 10%	c. 30%	e. 50%	g. 70%
b. 20%	d. 40%	f. 60%	h. *NOTA*

____ 11. What is the average number of cars in the shop? (Choose nearest answer.)

a. 0.5	c. 1.0	e. 1.5	g. 2.0
b. 0.75	d. 1.25	f. 1.75	h. 2.5

____ 12. If an average of 2.8 cars arrive during an 8-hour day, then according to Little's Law, the average time spent by a car in the repair shop is (choose nearest answer)

a. 3.0 hours	c. 3.5 hours	e. 4.0 hours	g. 4.5 hours
b. 3.25 hours	d. 3.75 hours	f. 4.25 hours	h. 4.75 hours

C.6. Dynamic Programming. *Match Problem.* Suppose that there are 15 matches originally on the table, and you are challenged by your friend to play this game. Each player must pick up either 1, 2, or 3 matches, with the player who picks up the last match paying $1.

Define **F(i)** to be the **minimal cost** to you (either $1 or $0) if

- it is your turn to pick up matches, and
- **i** matches remain on the table.

Thus, F(1) = 1, F(2) = 0 (since you can pick up one match, forcing your opponent to pick up the last match), etc.

_____ 1. What is the value of F(3)?

_____ 2. What is the value of F(4)?

_____ 3. What is the value of F(6)?

_____ 4. What is the value of F(15)?

_____ 5. If you are allowed to decide whether you or your friend should take the first turn, what is your optimal decision?

a. You take first turn	c. You are indifferent about this choice
b. Friend takes first turn	d. You refuse to play the game

Auto Replacement Problem. Suppose that a new car costs $15,000 and that the annual operating cost and resale value of the car are as shown in the table below:

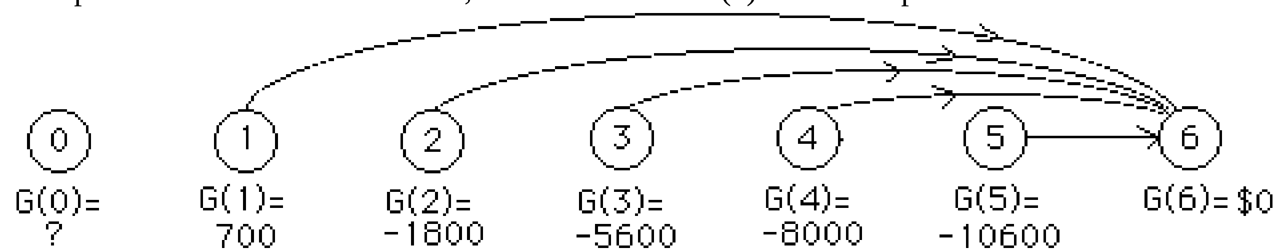
Age of Car (years)	Resale Value	Operating Cost in year ending
1	$11000	$400 (year 1)
2	$9000	$600 (year 2)
3	$7500	$900 (year 3)
4	$5000	$1200 (year 4)
5	$4000	$1600 (year 5)
6	$3000	$2200 (year 6)

(The operating cost specified above is for the year which is ending; thus, the cost of operating a car its first year is $400, for its second year the cost is $600, etc.) If I have a new car now (time 0, and this initial car is assumed to be "free", i.e. a "sunk" cost), I wish to determine a replacement policy that minimizes my net cost of owning and operating a car for the next six years.

Define G(t) = minimum cost of owning and operating car(s) through the end of the sixth year, **given that I have a new car at the end of year t.**

(As in the example solved in class, this includes the cost of the replacement car if I trade in my current car before the end of the sixth year, but does not include the cost of the car which is new at the beginning of this period.)

The optimal solution is shown below, with the value of G(0) & initial replacement time omitted:



___ 6. The value of G(5), i.e., the cost for the final year if I have a new car at the end of year 5, is
a. 0 c. $2200 e. $11000-2200 = $8800 g. *NOTA*
b. $400 d. $400-11000 = - $10600 f. $11000 - 400 = $10600

___ 7. If I have a new car at the end of year 4 and replace it after one year, my cost for the remainder of the six-year period is
a. $400 d. $400+4000-10600 = -$6200 g. $400-10600 = -$10200
b. $1000 e. $2200-10600 = - $8400 h. $ 1600 + 4000 = $5600
c. - $8000 f. $1600-10600 = - $9000 i. *NOTA*

___ 8. If I have a new car at the end of year 4 and keep it until the end of the sixth year, my cost for that period is
a. $600 d. $1600-3000 = - $1400 g. $400-3000 = -$2600
b. $1000 e. $1000+6000-9000 = -$2000 h. $ 1000 - 9000 = -$8000
c. $1600 f. $600-3000 = - $2000 i. *NOTA*

___ 9. If I have a new car at the end of year 2, how old will it be when I should replace it?
a. 1 year old c. 3 years old e. 5 years old
b. 2 years old d. 4 years old f. 6 years old

___ 10. If I have a new car at the end of year 0 (beginning of year 1) and replace it at the end of the first year, my total cost for the six-year period is
a. $400 d. $400+4000=$4400 g. $400+15000=$15400
b. $700 e. $400+4000+700=$5100 h. $700+15000-11000=$4700
c. $400+700=$1100 f. $600+700=$1300 i. *NOTA*

___ 11. If I have a new car at the end of year 0 (beginning of year 1) and replace it at the end of the second year, my total cost for the six-year period is
a. $600 d. $1000-1800= -$800 g. $600+15000-1800=$14800
b. $400+600=$1000 e. $600-1800= -$1200 h. $700+15000-9000=$6700
c. $400+700=$1100 f. $1000+6000-1800=$5200 i. *NOTA*

___ 12. If I have a new car at the end of year 0 (beginning of year 1) and keep my original car until the end of the sixth year, my total cost for the six-year period is
a. $400+600+900+1200+1600+2200 - 3000 = $3900
b. $400+600+900+1200+1600+2200 = $6900
c. $15000 400+600+900+1200+1600+2200 - 3000 = $18900
d. $700 + 1800 + 5600 + 8000 + 10600 + 0 = $26700
e. *NOTA*