Name or Initials ____________

tststst 56:171 Operations Research tststst
stststs Final Examination ststst
tststst December 16, 1997 tststst

• ***Write your name on the first page, and initial the other pages.***
• ***Answer both questions of Part One, and 4 (out of 5) problems from Part Two.***

		Possible	Score
Part One:	1. True/False & multiple choice	25	_____
	2. Sensitivity analysis (LINDO)	15	_____
Part Two:	3. Integer LP Models	15	_____
	4. Discrete-time Markov chain (steadystate)	15	_____
	5. Discrete-time Markov chain (absorption)	15	_____
	6. Continuous-time Markov chain	15	_____
	7. Dynamic programming	15	_____
	Total possible:	100	_____

tststst PART ONE tststst

(1.) ***True/False:*** Indicate by "+" or "o" whether each statement is "true" or "false", respectively:

____ a. If there is a tie in choosing the minimum reduced cost in the simplex method, the next basic solution will be degenerate.

____ b. The transportation problem is a special case of an assignment problem.

____ c. A "pivot" in a nonbasic column of a tableau will make it a basic column.

____ d. If the primal LP objective function, to be minimized, is unbounded below, then the dual LP (a maximization) has no feasible solution.

____ e. The optimal basic solution to an LP with m constraints (excluding non-negativity constraints) can have at most m positive decision variables.

____ f. The Revised Simplex Method, for most LP problems, requires fewer pivots than the ordinary simplex method.

____ g. In the simplex method, every variable of the LP is either basic or nonbasic.

____ h. When maximizing in the simplex method, the value of the objective function increases at every iteration unless a degenerate tableau is encountered.

____ i. The critical path in a project network is the shortest path from a specified source node (beginning of project) to a specified destination node (end of project).

____ j. The assignment problem is a special case of a transportation problem.

____ k. If you make a mistake in choosing the pivot row in the simplex method, the next basic solution will be infeasible.

____ l. The optimal solution found by the simplex method is always a basic solution.

____ m. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *must* pivot in row i.

____ n. In setting up the tableau for a transportation problem to be solved by the transportation simplex method, the total supply must be exactly equal to the total demand.

____ o. If the optimal value of a *slack* variable of a primal LP constraint is zero, then the optimal value of the *dual* variable for that same constraint must also be zero.

____ p. If an LP problem has 3 constraints (not including non-negativity) and 5 variables, then its dual problem has 5 constraints (not including non-negativity) and 3 variables.

____ q. If the increase in the cost of a nonbasic variable remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.

____ r. If the increase in the right-hand-side of a "tight" constraint remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.

____ s. Vogel's method will always yield an optimal solution to a transportation problem, if the problem is nondegenerate.

____ t. The optimal dual variables for a balanced transportation problem must be nonnegative.

____ u. The Hungarian method might be used to solve the above transportation problem.

____ v. An assignment problem may be considered to be a special case of a transportation problem with all "transportation" costs equal to 1.

____ w. Considered as a special case of the TP, the AP *always* has a degenerate basic solution.

____ x. After row reduction in the Hungarian method, each row contains at least one zero.

____ y. Any state of a Markov chain which is not recurrent must be transient.

2. *Sensitivity Analysis in LP.* During each 6-hour period of the day, the Bloomington Police Department needs at least the number of police officers shown below:

#	Time Period	# of Officers Required
1	12 midnight- 6 a.m.	12
2	6 a.m. - 12 noon	8
3	12 noon - 6 p.m.	6
4	6 p.m. - 12 midnight	15

Officers can be hired to work either 12 consecutive hours or 18 consecutive hours. Officers are paid \$12 per hour for each of the first 12 hours a day they work, and are paid \$18 per hour for each of the next 6 hours they work in a day. The city wishes to minimize the cost of meeting Bloomington's daily police requirements.

Number the four 6-hour time intervals by integers 1,2,3, & 4. Define decision variables

X_t = number of officers assigned to 12-hour shift beginning in time interval t (t=1,2,3,4)

Y_t = number of officers assigned to 18-hour shift beginning in time interval t (t=1,2,3,4)

Thus, for example, Y_3 is the number of officers who begin their shift at noon and complete their shift at 6 a.m. the following morning.

Ignoring the restriction that the number of officers for each shift must be integer-valued and solving the problem as an ordinary LP, LINDO gives us the following solution, which happens to satisfy the integer restrictions:

```
MIN     144 X1 + 144 X2 + 144 X3 + 144 X4 + 252 Y1 + 252 Y2 + 252 Y3
       + 252 Y4
  SUBJECT TO
         2)   X1 + X4 + Y1 + Y3 + Y4 >=   12
         3)   X1 + X2 + Y1 + Y2 + Y4 >=   8
         4)   X2 + X3 + Y1 + Y2 + Y3 >=   6
         5)   X3 + X4 + Y2 + Y3 + Y4 >=   15
  END

        OBJECTIVE FUNCTION VALUE
        1)      3132.000

  VARIABLE        VALUE          REDUCED COST
        X1         3.000000          .000000
        X2          .000000          .000000
        X3         1.000000          .000000
        X4         9.000000          .000000
        Y1          .000000        72.000000
        Y2         5.000000          .000000
        Y3          .000000        72.000000
        Y4          .000000          .000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         -.000000       -36.000000
        3)         -.000000      -108.000000
        4)         -.000000       -36.000000
        5)         -.000000      -108.000000

 RANGES IN WHICH THE BASIS IS UNCHANGED:
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OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	144.000000	.000000	36.000000
X2	144.000000	INFINITY	.000000
X3	144.000000	.000000	.000000
X4	144.000000	.000000	.000000
Y1	252.000000	INFINITY	72.000000
Y2	252.000000	.000000	72.000000
Y3	252.000000	INFINITY	72.000000
Y4	252.000000	INFINITY	.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	12.000000	5.000000	1.000000
3	8.000000	1.000000	5.000000
4	6.000000	5.000000	.500000
5	15.000000	1.000000	5.000000

X1	3.000000	.000000
X2	.000000	.000000
X3	1.000000	.000000
X4	9.000000	.000000
Y1	.000000	72.000000
Y2	5.000000	.000000
Y3	.000000	72.000000
Y4	.000000	.000000

THE TABLEAU

ROW	(BASIS)	X1	X2	X3	X4	Y1	Y2
1	ART	.000	.000	.000	.000	72.000	.000
2	X1	1.000	1.000	.000	.000	2.000	.000
3	X4	.000	-1.000	.000	1.000	-1.000	.000
4	Y2	.000	.000	.000	.000	-1.000	1.000
5	X3	.000	1.000	1.000	.000	2.000	.000

ROW	Y3	Y4	SLK 2	SLK 3	SLK 4	SLK 5	
1	72.000	.000	36.000	108.000	36.000	108.000	-3132.000
2	1.000	.000	-1.000	.000	-1.000	1.000	3.000
3	.000	1.000	.000	.000	1.000	-1.000	9.000
4	-1.000	1.000	1.000	-1.000	1.000	-1.000	5.000
5	2.000	-1.000	-1.000	1.000	-2.000	1.000	1.000

___1. If the officers who work a 12-hour shift from midnight to noon were to receive a $1/hour raise, the number assigned to this shift would

a. increase　　c. remain the same　　e. *NOTA*
b. decrease　　d. insufficient info. given

___2. The LP problem above has

a. exactly one optimal sol'n　　c. multiple solutions　　e. insufficient info. given
b. a degenerate solution　　d. no optimal solution　　f. *NOTA*

___3. Increasing the requirement for officers working during the midnight-6 a.m. period (from 12 to 13) would increase the total cost by *(choose nearest value)* :

a. no change　　c. $36　　e. insufficient info. given
b. $18　　d. $108　　f. *NOTA*

___4. If the variable "SLK 5" were increased, this would be equivalent to
- a. increasing the requirement for officers during the 6pm-midnight period
- b. decreasing the requirement for officers during the 6pm-midnight period
- c. *NOTA*

___5. If the variable "SLK 5" were <u>de</u>creased by 1, the number of officers working a 12-hour shifts beginning at 6 pm would be *(choose nearest value)*

a. become zero	c. decrease by 1	e. decrease by 2
b. increase by 1	d. increase by 2	f. *NOTA*

___6. If a pivot were to be performed to enter the variable SLK4 into the basis, then according to the "minimum ratio test", the value of SLK4 in the resulting basic solution would be *(choose nearest value)*

a. 0	c. 1	e. 5
b. 0.5	d. 2	f. 10

___7. If the variable SLK4 were to enter the basis, then the variable leaving the basis is

a. X1	c. X3	e. SLK1	g. SLK3	i. more than one answer is possible
b. X2	d. X4	f. SLK2	h. SLK4	j. *NOTA*

___8. If the variable SLK4 were to enter the basis, then the next tableau

a. indicates multiple optimal sol'ns	c. both of the above
b. is degenerate	d. *NOTA*

___9. The dual of the LP above has an objective function which is to be

a. minimized	c. both of the above
b. maximized	d. *NOTA*

___10. If one officer were to be assigned an 18-hour shift from noon to 6 a.m., this will

a. increase Y2 by 1	c. increase X3 by 1
b. decrease Y2 by 1	d. decrease X3 by 1
e. *More than one answer above is correct*	f. *None of the above*

___11. If one officer were to be assigned an 18-hour shift from noon to 6 a.m., this will increase the total cost by *(choose nearest value):*

a. zero	c. $72	e. $144
b. $36	d. $108	f. $252

tstst **PART TWO** tstst

3. *Integer LP modeling* Comquat owns four production plants at which personal computers are produced. Comquat can sell up to 2000 computers per year at a price of $3500 per computer. For each plant, the production capacity, the production cost per computer, and the fixed cost of operating a plant for a year are given in the table below:

Plant #	Production Capacity	Plant Fixed Cost	Cost per Computer
1	100	$9 million	$1000
2	800	$5 million	$1700
3	900	$3 million	$2300
4	600	$1 million	$2900

The company wishes to determine how many computers it should produce at each plant in order to maximize its yearly revenue. (Note that if no computers are produced by a plant during the year, Comquat need not pay the fixed cost of operating the plant that year.)

We require two sets of **decision variables** :

Y_i = 1 if the computers are produced at plant i, 0 otherwise *(binary)*

and

X_i = quantity of computers produced at plant i *(continuous)*

Choose the appropriate constraint below for each of the following possible restrictions:

___ 1. Computers are to be produced at <u>no more than</u> 3 plants.

___ 2. If the production line at plant 2 is set up, then that plant can produce up to 800 computers; otherwise, none can be produced at that plant.
___ 3. The production lines at plants 2 and 3 cannot both be set up.
___ 4. The total production must be at least 2000 computers.
___ 5. If the production line at plant 2 is set up, that plant must produce at least 200 computers.
___ 6. If the production line at plant 3 is not set up, then the production line at plant 2 cannot be set up.

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The Tower Engineering Corporation is considering undertaking several proposed projects for the next fiscal year. The projects, together with the number of engineers, number of support personnel required for each project, and the expected project profit, are:

Project #:	1	2	3	4	5	6
Engineers req'd:	20	55	47	38	90	63
Support req'd:	15	45	50	40	70	70
Profit (x$\$10^6$)	1.0	1.8	2.0	1.5	3.6	2.2

Define the decision variables, for i=1,2,...6:

X_i = 1 if the company undertakes project i
0 otherwise

The corporation faces the following restrictions. Choose the appropriate constraint below for each.

___ 7. Only 200 engineers are available
___ 8. 80 support personnel are available
___ 9. Project #2 can be selected only if Project #1 is selected
___ 10. If project #5 is selected, project #3 cannot be selected, and vice versa
___ 11. No more than three projects may be selected in all
___ 12. If both projects 1 & 2 are selected, then project 3 cannot be selected

Constraints:

a. $\sum_j Y_j \leq 3$
b. $Y_2 \leq Y_3$
c. $Y_3 \leq Y_2$
d. $\sum_j Y_j \geq 2000$
e. $Y_3 Y_2 \geq 1$
f. $X_2 \geq 200Y_2$
g. $\sum_j X_j \geq 2000$
h. $X_2 \leq 800Y_2$
i. $X_2 \leq 200Y_2$
j. $\sum_j Y_j \geq 3$
k. $X_2 \geq 800Y_2$
l. $800X_2 \leq Y_2$
m. $\sum_j Y_j = 3$
n. $200X_2 \leq Y_2$
o. $Y_2 + Y_3 \leq 1$
p. $X_2 \leq Y_2$
q. $Y_2 + Y_3 \geq 1$
r. $Y_2 - Y_3 = 1$
s. $Y_3 \leq Y_2$
t. $Y_3 Y_2 = 1$
u. $Y_3 Y_2 \leq 1$
v. $Y_2 \leq Y_3$
w. $X_2 Y_2 \geq 200$
x. $X_2 Y_2 \leq 800$
y. $20Y_1+55Y_2+47Y_3+38Y_4+90Y_5+63Y_6 \leq 200$
z. $15Y_1+45Y_2+50Y_3+40Y_4+70Y_5+70Y_6 \leq 80$

NOTA: ***None of the above***

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4. ***Discrete-time Markov chains (steadystate analysis)***: A city's water supply comes from a reservoir. Careful study of this reservoir over the past twenty years has shown that, if the reservoir was full at the beginning of one summer, then the probability that it would be full at the beginning of the next summer is 80%; however, if the reservoir was not full at the beginning of one summer, the probability that it would be full at the beginning of the next summer is only 40%.

The below computer output may be consulted to help answer some of the following questions. Note that state #1 represents the condition "full" and state #2 represents the condition "not full"

Powers of P

$$P = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.72 & 0.28 \\ 0.56 & 0.44 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 0.688 & 0.312 \\ 0.624 & 0.376 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 0.6752 & 0.3248 \\ 0.6496 & 0.3504 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 0.67008 & 0.32992 \\ 0.65984 & 0.34016 \end{bmatrix}$$

Steady State Distribution

i	Pi
1	0.66666667
2	0.33333333

Mean First Passage Times

from \ to	1	2
1	1.5	5
2	2.5	3

Expected no. of visits during first 5 stages

from \ to	1	2
1	3.55328	1.44672
2	2.89344	2.10656

First Passage Probabilities

stage 1: $\begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$

stage 2: $\begin{bmatrix} 0.08 & 0.16 \\ 0.24 & 0.08 \end{bmatrix}$

stage 3: $\begin{bmatrix} 0.048 & 0.128 \\ 0.144 & 0.064 \end{bmatrix}$

stage 4: $\begin{bmatrix} 0.0288 & 0.1024 \\ 0.0864 & 0.0512 \end{bmatrix}$

stage 5: $\begin{bmatrix} 0.01728 & 0.08192 \\ 0.05184 & 0.04096 \end{bmatrix}$

1. Complete the diagram of a Markov Chain model of this reservoir:

___ 2. What property guarantees that this system has a steady-state probability distribution?

a. There are only transient states
b. There are no absorbing states
c. The Markov chain is normal
d. The Markov chain is regular
e. There are no periodic states
e. *None of the above*

___3. Indicate which of the below equations are among those solved to compute the steady-state distribution.

a. $0.8\ \pi_1 + 0.2\ \pi_2 = 1$ c. $0.8\ \pi_1 + 0.2\ \pi_2 = \pi_1$ e. $0.8\ \pi_1 + 0.2\ \pi_2 = 0$

b. $0.8\ \pi_1 + 0.4\ \pi_2 = 1$ d. $0.8\ \pi_1 + 0.4\ \pi_2 = \pi_1$ f. $0.8\ \pi_1 + 0.4\ \pi_2 = 0$

g. *None of the above*

___4. Over a 100-year period, how many summers can the reservoir be expected to be full? *(Choose nearest answer.)*

a. 10 c. 30 e. 50 g. 70 i. 90

b. 20 d. 40 f. 60 h. 80

___5. If the reservoir was full at the beginning of summer 1997, what is the probability that it will be full at the beginning of summer 1998? *(Choose nearest answer.)*

a. 10% c. 30% e. 50% g. 70% i. 90%

b. 20% d. 40% f. 60% h. 80%

___6. If the reservoir was full at the beginning of summer 1997, what is the probability that it will be full at the beginning of summer 1999? *(Choose nearest answer.)*

a. 10% c. 30% e. 50% g. 70% i. 90%

b. 20% d. 40% f. 60% h. 80%

___7. If the reservoir was full at the beginning of summer 1997, what is the expected number of summers during the next 5 years ('98 through 2002) that the reservoir will <u>not</u> be full? *(Choose nearest answer.)*

a. 0.5 c. 1.5 e. 2.5 g. 3.5 i. 4.5

b. 1 d. 2 f. 3 h. 4 j. 5

___8. If the reservoir was full at the beginning of summer 1997, what is the expected number of years before the reservoir will <u>not</u> be full at the beginning of the summer?*(Choose nearest answer.)*

a. 0.5 c. 1.5 e. 2.5 g. 3.5 i. 4.5

b. 1 d. 2 f. 3 h. 4 j. 5

___9. The steady-state probability vector π of a discrete Markov chain with transition probability matrix P satisfies the matrix equation

a. $P^t \pi = 0$ c. $P\pi = \pi$ e. *none of the above*

b. $P\pi = 0$ d. $\pi P = \pi$

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5. *Discrete-time Markov chains (absorption analysis)*: The college admissions officer has modeled the path of a student through the Engineering College as a Markov chain:

Each student's state is observed at the beginning of each fall semester. For example, if a student is a junior at the beginning of the current fall semester, there is an 80% chance that he will be a senior at the beginning of the next fall semester, a 15% chance that he will still be a junior, and a 5% chance that he will have quit. (Assume that once a student quits, he never re-enrolls.)

State	Description
1	Freshman
2	Sophomore
3	Junior
4	Senior
5	Drop-out
6	Graduate

Consult the computer output below to answer the questions that follow.

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Transition probabilities (P)

from \ to	1	2	3	4	5	6
1	0.1	0.8	0	0	0.1	0
2	0	0.1	0.85	0	0.05	0
3	0	0	0.15	0.8	0.05	0
4	0	0	0	0.1	0.05	0.85
5	0	0	0	0	1	0
6	0	0	0	0	0	1

E = Expected No. Visits to Transient States

from \ to	1	2	3	4
1	1.11111	0.98765	0.98765	0.87791
2	0	1.11111	1.11111	0.98765
3	0	0	1.17647	1.04575
4	0	0	0	1.11111

P^2

from \ to	1	2	3	4	5	6
1	0.01	0.16	0.68	0	0.15	0
2	0	0.01	0.2125	0.68	0.0975	0
3	0	0	0.0225	0.2	0.0975	0.68
4	0	0	0	0.01	0.055	0.935
5	0	0	0	0	1	0
6	0	0	0	0	0	1

P^3

from \ to	1	2	3	4	5	6
1	0.001	0.024	0.238	0.544	0.193	0
2	0	0.001	0.040375	0.238	0.142625	0.578
3	0	0	0.003375	0.038	0.108625	0.85
4	0	0	0	0.001	0.0555	0.9435
5	0	0	0	0	1	0
6	0	0	0	0	0	1

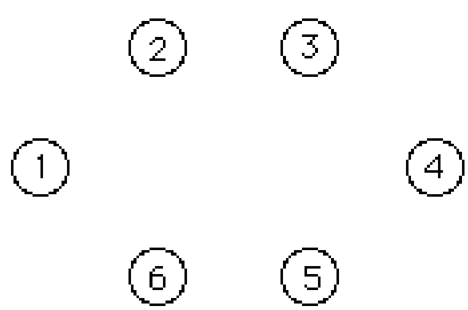
A = Absorption Probabilities

from \ to	5	6
1	0.253772	0.746227
2	0.160493	0.839506
3	0.111111	0.888888
4	0.055555	0.944444

P^4

from \ to	1	2	3	4	5	6
1	0.0001	0.0032	0.0561	0.2448	0.2334	0.4624
2	0	0.0001	0.00690625	0.0561	0.15659375	0.7803
3	0	0	0.00050625	0.0065	0.11069375	0.8823
4	0	0	0	0.0001	0.05555	0.94435
5	0	0	0	0	1	0
6	0	0	0	0	0	1

1. Indicate the possible transitions in the diagram for this Markov chain. *(You need not include probabilities!)*



2. Which states are transient? _______

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3. Which states are recurrent? _______

4. Which states are absorbing? _______

5. Does this system have a steady-state probability distribution? *Justify your answer.*

___6. If a student transfers into the college as a junior, how many years can he or she expect to spend as a student in the college? *(Choose nearest answer)*

a. 0.5	c. 1.5	e. 2.5	g. 3.5	i. 4.5
b. 1	d. 2	f. 3	h. 4	j. 5

___7. What is the probability that, at the beginning of his/her fourth year in the college, an entering freshman is classified as a senior? *(Choose nearest answer)*

a. 10%	c. 30%	e. 50%	g. 70%	i. 90%
b. 20%	d. 40%	f. 60%	h. 80%	

___8. What is the probability that an entering freshman eventually will graduate? *(Choose nearest answer)*

a. 10%	c. 30%	e. 50%	g. 70%	i. 90%
b. 20%	d. 40%	f. 60%	h. 80%	

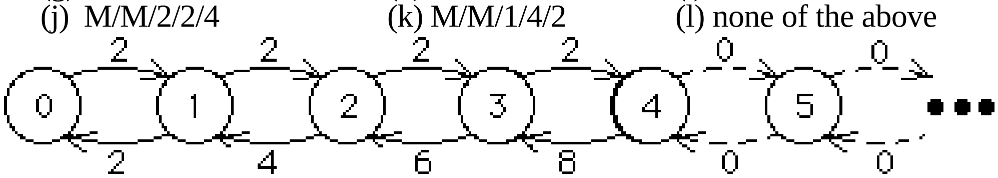
___9. If a student has survived to the point that he or she has been classified as a junior, what is the probability that he or she eventually graduates? *(Choose nearest answer)*

a. 10%	c. 30%	e. 50%	g. 70%	i. 90%
b. 20%	d. 40%	f. 60%	h. 80%	

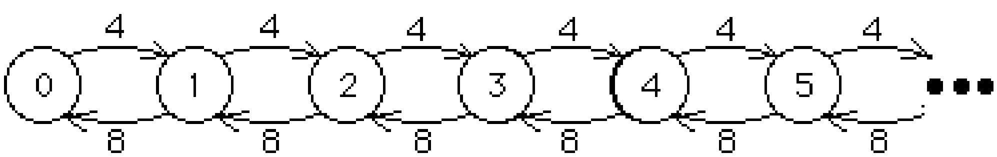
6. *Continuous-time Markov chains.* For each diagram of a Markov model of a queue in (1) through (5) below, indicate the correct Kendall's classification from among the following choices :

(a) M/M/1	(b) M/M/2	(c) M/M/1/4
(d) M/M/4	(e) M/M/2/4	(f) M/M/2/4/4
(g) M/M/1/2/4	(h) M/M/4/2	(i) M/M/4/4
(j) M/M/2/2/4	(k) M/M/1/4/2	(l) none of the above

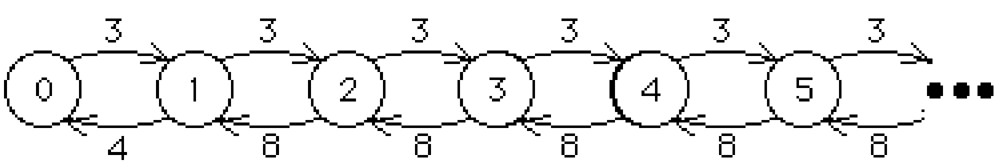
___ 1.



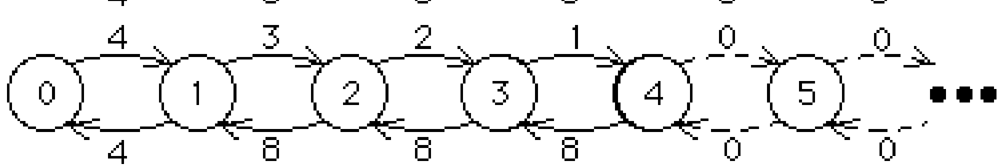
___ 2.



___ 3.



___ 4.



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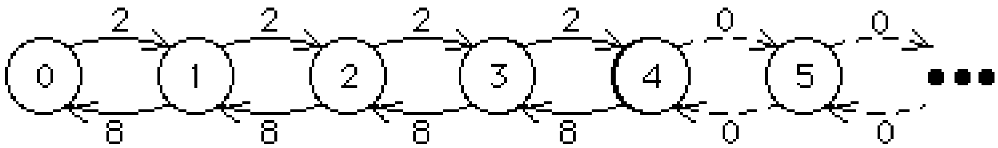
___ 5. 

____6. For a continuous-time Markov chain, let Λ be the matrix of transition rates. The sum of each...

a. column is 1
b. column is 0
c. row is 1
d. row is 0
e. none of the above

____7. . To compute the steady state distribution π of a continuous-time Markov chain, one must solve (in addition to sum of components 0f π equal to 1) the matrix equation (where Λ^t is the transpose of Λ):

a. $\pi\Lambda = 1$
b. $\Lambda^t\pi = 1$
c. $\Lambda^t\pi = \pi$
d. $\pi\Lambda = \pi$
e. $\pi\Lambda = 0$
f. none of the above

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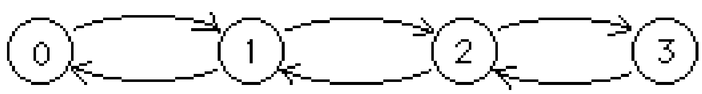
Consider a barber shop with two chairs, two barbers, and room for 1 customer to wait. The state of the system is the number of customers in the shop: 0, 1, 2, or 3. Potential customers arrive in a random fashion, an average of one every ten minutes. If there is an empty chair when a customer arrives, he enters the shop and his haircut begins. If both barber chairs are occupied when he arrives, there is a 50% probability that he will "balk", i.e., leave without entering the shop. If the chair in the waiting area is in use, he does not enter the shop. The time required to cut a customer's hair averages 15 minutes, having the exponential distribution. As soon as a customer's haircut is completed, he leaves the shop immediately.

8. Draw the diagram for this birth/death process, indicating the birth & death rates.



___9. What is the probability that a potential customer arrives at the door during the first 15 minutes that the shop is open (and empty)? *(Choose nearest value.)*

a. 10%
b. 20%
c. 30%
d. 40%
e. 50%
f. 60%
g. 70%
h. 80%
i. 90%

Steady-State Distribution

i	Lambda	Mu	ρ	Pi	CDF
0	0.1000	0.0000	1.000000	0.206853	0.206853
1	0.1000	0.0667	1.499993	0.310278	0.517132
2	0.0500	0.0667	1.499993	0.465415	0.982547
3	0.0000	1.3333	0.037500	0.017453	1.000000

___10. What is the average number of customers in the shop? *(Choose nearest value.)*

a. 0
b. 0.5
c. 1
d. 1.5
e. 2
f. 2.5
g. 3

Name or Initials ___________

___11. What is average arrival rate of customers in the shop? *(Choose nearest value.)*

a. 0.02/min.	c. 0.06/min.	e. 0.1/min
b. 0.04/min	d. 0.08/min.	

___12. What is the average amount of time (in minutes) that a customer spends in the shop (both waiting and receiving his haircut)? *(Choose nearest value.)*

a. ≤15	c. 17	e. 19	g. 21	i. 23
b. 16	d. 18	f. 20	h. 22	j. ≥24

7. *Dynamic programming.* Production of a certain high-valued, low-demand item is to be scheduled for the next three months (January, February, & March. The relevant data are (where costs are measured in multiples of $100):

- maximum lot size per month is 3
- maximum inventory level at the end of each month is 4
- demand is random, with the following probability distributions each month:

D	0	1	2
P{D}	0.4	0.4	0.2

- Production cost includes a setup cost $5 for each month in which production is scheduled, plus $3 per unit produced.
- A storage cost of $2 is incurred for each unit in storage at the beginning of each month
- A salvage value is received for 1, 2, or 3 units remaining in storage at the end of March (month 3):

Inventory:	1	2	3	4
Value:	$3	$5	$7	$8

- If demand cannot be satisfied, a penalty of $10 per unit short is incurred, but the demand is not backordered.

A dynamic programming model was formulated to find the production schedule having minimum expected cost, with the stage **n** defined as the number of months into the planning period. (That is, stage 1 is January, 2 is February, 3 is March.) Consult the **APL** output to answer the following questions:

1. Four values in the output have been removed (represented by boxes). Compute these four values and insert in the boxes.
2. Suppose that we have one unit in storage at the end of this month (December). What should be our production quantity in January? What will be our total *expected* cost for the 3-month period?
3. If the demand happens to be two during January, what should be the production quantity during February?

Stage 3

s \ x:	0	1	2	3
0	8.00	8.80	7.80	8.60
1	2.80	6.80	7.60	9.00
2	0.80	6.60	8.00	9999.99
3	0.60	7.00	9999.99	9999.99
4	1.00	9999.99	9999.99	9999.99

Stage 3:

State	Optimal Values	Optimal Decisions
0	7.80	2
1	2.80	0
2	0.80	0
3	0.60	0
4	1.00	0

Stage 2

s \ x:	0	1	2	3
0	15.80	15.80	14.00	15.12
1	9.80	$\boxed{}$	14.12	16.80
2	7.00	13.12	15.80	9999.99
3	7.12	14.80	9999.99	9999.99
4	8.80	9999.99	9999.99	9999.99

Stage 2:

State	Optimal Values	Optimal Decisions
0	$\boxed{}$	$\boxed{}$
1	9.80	0
2	7.00	0
3	7.12	0
4	8.80	0

Stage 1: s \ x:	0	1	2	3
0	$\boxed{}$	22.32	20.52	21.61
1	16.32	19.52	20.61	23.77
2	13.52	19.61	22.77	9999.99
3	13.61	21.77	9999.99	9999.99
4	15.77	9999.99	9999.99	9999.99

Stage 1:

State	Optimal Values	Optimal Decisions
0	20.52	2
1	16.32	0
2	13.52	0
3	13.61	0
4	15.77	0