56:171 Operations Research
Final Examination Solutions
Fall 2002

• *Answer **both** Parts **A** and **B**, and select any **4 (out of 5)** problems from Part **C**.*

		Possible
Part A:	Miscellaneous multiple choice	40
Part B:	Sensitivity analysis (LINDO)	14
Part C:	I. Discrete-time Markov chains	14
	II. Continuous-time markov chains	14
	III. Decision Analysis	14
	IV. Deterministic dynamic programming	14
	V. Integer Programming Models	14
	total possible:	110

▼▲▼▲▼▲▼ PART A ▼▲▼▲▼▲▼

_+__ 1. The minimum ratio test is used to select the pivot row in the simplex method for LP.
_+__ 2. The "Northwest Corner" method applied to an assignment problem will produce a feasible solution for the assignment problem.
_o__ 3. When minimizing an LP, selecting the column with the smallest (i.e., "most negative") reduced cost will produce the greatest improvement at the next pivot.
_+__ 4. The reduced cost of a nonbasic variable in the simplex method indicates the rate of change of the cost function as the variable increases.
_o__ 5. When you enter an LP formulation into LINDO, you must first convert all inequalities to equations.
_o__ 6. When you enter an LP formulation into LINDO, you must include any nonnegativity constraints.
_+__ 7. The pivot operation in the simplex method for LP never changes the total number of variables in the basis.
_+__ 8. A tie in the minimum ratio test can be broken by arbitrarily selecting either minimum ratio.
_o__ 9. The minimum ratio in the minimum ratio test is always positive.
_o__ 10. Either the dual variable or the slack variable of a constraint must be zero, but not both.
_+__ 11. The dual variable corresponding to a primal constraint is the rate at which the optimal value is changed as the right-hand-side is increased.
_o__ 12. Using the "revised" simplex method usually requires fewer pivots than the "ordinary" simplex method in order to find the optimal solution of an LP.
_o__ 13. If a primal problem has 3 rows and 5 columns, and the dual has 5 rows and 3 columns, then the revised simplex method would require less computation per pivot if it were applied to the dual problem.
_o__ 14. If the revised simplex method is used to solve the primal problem, each simplex multiplier vector computed at each iteration is a feasible solution to the dual problem.
_+__ 15. The diagonal of the transition rate matrix Λ of a continuous-time Markov chain cannot contain a positive number.
_o__ 16. Little's Law applies only to queues which have a continous-time Markov chain model (including birth-death models).
_+__ 17. The M/M/2/4 queue can be modeled as a birth-death process.
_+__ 18. If a random variable T has an exponential distribution, then $P\{T>2 \mid T\geq 1\} = P\{T > 1\}$.
_+__ 19. A random variable T with the Erlang-k distribution is the sum of k random variables, all with the same exponential distribution.
_+__ 20. An $M/E_k/1$ queueing system can be modeled as a continuous-time Markov chain.
_+__ 21. If an exponential and an Erlang-k (with k>1) distribution have the same mean, the Erlang-k distribution has a smaller variance.
_o__ 22. An $M/E_k/1$ queueing system can be modeled as a birth-death process.
_o__ 23. In a birth-death process, it is possible for a "catastrophe" to occur, causing the "death" (or departure) of the entire population.
_o__ 24. A Poisson process is a birth-death process where death is the result of poissoning.
_+__ 25. A Poisson process is "memoryless".

_o__ 26. In a continous-time Markov chain, the transition rate λ_{ii} of state i to itself is assumed to be zero.
_o__ 27. PERT and the Critical Path Method (CPM) are both names for the same procedure.

Multiple Choice: Write the appropriate letter (a, b, c, d, etc.) : (*NOTA* =None of the above).

_d__ 28. If X_1 and X_2 are binary variables, to require that "if $X_1 = 1$ then X_2 must also be 1", we add the constraint
a. $X_1 + X_2 = 1$ b. $X_1 + X_2 \leq 1$ c. $X_1 \geq X_2$ d. $X_1 \leq X_2$ e. *NOTA*

_c__ 29. If X_1 is the quantity of product 1 to be produced, up to a maximum of K_1, and Y_1 is the binary variable indicating that a setup cost is to be included for this product, then we add the constraint….
a. $X_1 = K_1Y_1$ b. $X_1 \geq K_1Y_1$ c. $X_1 \leq K_1Y_1$ d. $X_1\ Y_1 \leq K_1$ e. *NOTA*

_b__ 30. In an M/M/1 queue, if the arrival rate (λ) = service rate (μ) , then
a. $\pi_0 = 1$ in steady state c. $\pi_i > 0$ for all i e. the queue is not a birth-death process
b. no steady state exists d. $\pi_0 = 0$ in steady state f. *NOTA*

_e__ 31. A state in a closed set of states of a Markov chain has the property
a. the system cannot enter that state c. probability of moving out of that state is one.
b. the system must leave that state, once it is entered. d. moving into that state is zero e. *NOTA*

_b__ 32. A minimal closed set of states of a Markov chain has the property
a. it contains only transient states c. it has fewer states than any other closed set of states.
b. it contains only recurrent states d. the steadystate probabilities of states in that set are zero.
e. *NOTA*

_d__ 33. The number of basic variables in a solution of a transportation problem with m sources and n dest'ns is
a. $m \times n$ c. $m+n+1$ e. $n-m$ g. *NOTA*
b. $m \times n-1$ d. $m+n-1$ f. $m+n$

_d__ 34. A balanced transportation problem is one in which
a. # sources = # destinations c. supplies & demands all 1 e. *NOTA*
b. cost coefficients are all one's d. sum of supplies = sum of demand

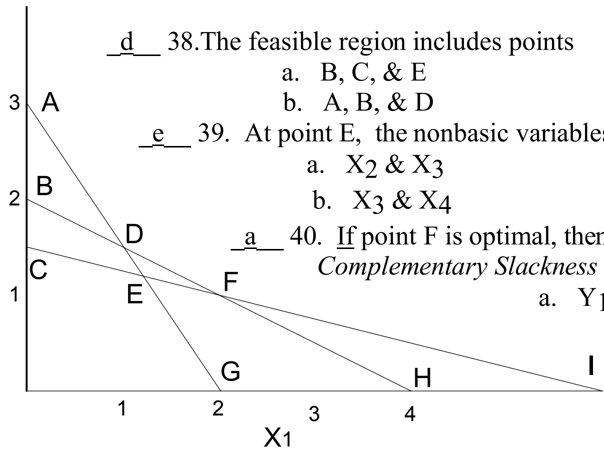
_a__ 35. If the assignment problem is treated as a linear programming problem and solved using the simplex method,
a. it has only degenerate basic solutions. b. it has a square constraint coefficient matrix.
c. the simplex method can give fractional optimal values of the variables. d. *NOTA*

_c__ 36. A critical path of a project network…
a. can have several activities in progress simultaneously
b. path from the "begin" node to the "end" node having shortest duration.
c. path from the "begin" node to the "end" node having longest duration.
d. *NOTA*

_b__ 37. Bayes' Rule is used to compute…
a. the joint probability of a "state of nature" and the outcome of an experiment.
b. the conditional probability of a "state of nature" given the outcome of an experiment.
c. the conditional probability of an experiment, given a state of nature.
d. *NOTA*

The problems below refer to the following LP:

	(with inequalities converted to equations:)
Minimize $4X_1 + 6X_2$	Minimize $4X_1 + 6X_2$
subject to $3X_1 + 2X_2 \geq 6$	subject to $3X_1 + 2X_2 - X_3 = 6$
$X_1 + 2X_2 \leq 4$	$X_1 + 2X_2 + X_4 = 4$
$X_1 + 4X_2 \leq 6$	$X_1 + 4X_2 + X_5 = 6$
$X_1 \geq 0,\ X_2 \geq 0$	$X_j \geq 0$, j=1,2, 3,4,5



_d__ 38.The feasible region includes points

a. B, C, & E
b. A, B, & D
c. C, E, & G
d. E, F, &G
e. F, H, & I
f. *NOTA*

_e__ 39. At point E, the nonbasic variables include

a. X_2 & X_3
b. X_3 & X_4
c. X_4 & X_5
d. X_1 & X_4
e. X_3 & X_5
f. *NOTA*

_a__ 40. If point F is optimal, then which dual variable must be zero, according to the *Complementary Slackness Theorem* ?

a. Y_1
b. Y_2
c. Y_3

▼▲▼▲▼▲▼ PART B ▼▲▼▲▼▲▼

Sensitivity Analysis in LP.

Ken and Larry, Inc., supplies its ice cream parlors with four flavors of ice cream: chocolate, vanilla, banana, and strawberry. Because of extremely hot weather and a high demand for its products, the company has run short of its supply of ingredients: milk, sugar, & cream. Hence, they will not be able to fill all the orders received from their retail outlets, the ice cream parlors. Owing to these circumstances, the company has decided to choose the amount of each product to produce that will maximize total profit, given the constraints on supply of the basic ingredients. The chocolate, vanilla, banana and strawberry flavors generate, respectively, \$1.00, \$0.90, \$0.95, and \$0.85 per profit per gallon sold. The company has only 185 gallons of milk, 165 pounds of sugar, and 65 gallons of cream left in its inventory. The LP formulation for this problem has variables C, V, B, and S representing gallons of chocolate, vanilla, banana, and strawberry ice cream produced, respectively.

```
!    Ken & Larry Ice Cream

MAXIMIZE  C+0.9V+0.95B + .85S
ST
0.45C + 0.50V + 0.40B + 0.43S <= 185  ! milk resource
0.50C + 0.40V + 0.40B + 0.35S <= 165  ! sugar resource
0.10C + 0.15V + 0.20B + 0.18S <= 65   ! cream resource
END
```

```
OBJECTIVE FUNCTION VALUE
   1)      373.8435

VAR  VALUE   REDUCED COST
 C  110.204    0.000000
 V   45.578    0.000000
 B    0.000    0.007823
 S  261.904    0.000000

ROW   SLACK   DUAL PRICES
2)    0.000    0.068027
3)    0.000    1.680272
4)    0.000    1.292517
```

```
RANGES IN WHICH THE BASIS IS UNCHANGED:

       OBJ COEFFICIENT RANGES
  CURRENT ALLOWABLE  ALLOWABLE
VAR COEF   INCREASE  DECREASE
C  1.000   0.012821  0.015972
V  0.900   0.006117  0.004545
B  0.950   0.007823  INFINITY
S  0.850   0.007143  0.004107

         RIGHTHAND SIDE RANGES
ROW  CURRENT  ALLOWABLE  ALLOWABLE
       RHS     INCREASE   DECREASE
2    185.000  20.769230   3.045455
3    165.000   4.407895  15.882353
4     65.000   2.913043  13.750000
```

TABLEAU:

ROW	(BASIS)	C	V	B	S	SLK 2	SLK 3	SLK 4	RHS
1	ART	0.000	0.000	0.008	0.000	0.068	1.680	1.293	373.844
2	C	1.000	0.000	0.490	0.000	-5.306	6.939	-0.816	110.204
3	V	0.000	1.000	-1.279	0.000	14.966	-10.340	-15.646	45.578
4	S	0.000	0.000	1.905	1.000	-9.524	4.762	19.048	261.905

1. The optimal solution above is *(check as many as apply):*
   _X_ basic ___ degenerate _X_ unique

_c__ 2. The number of basic variables in this optimal solution (not including z, the objective value) is
a. one b. two c. three
d. four e. five f. *NOTA*

_a__ 3. In *any* basic feasible solution of this problem:
a. not every product will be included b. exactly three products will be included
c. at least one slack variable will be >0 d. *NOTA*

_e__ 4. Suppose the company discovers that 3 gallons of cream have gone sour and so must be thrown out. The decrease in profit is *(choose nearest value)*
a. zero b. $1.00 c. $2.00 d. $3.00 e. $4.00
f. $5.00 g. $6.00 h. $7.00 i. $8.00 j. $9.00

_d__ 5. To adjust for the loss of 3 gallons of cream, the change in gallons of *vanilla* ice cream to be produced should
a. be unchanged b. increase by less than 10 c. decrease by less than 10
d. increase by more than 10 e. decrease by more than 10 f. cannot be determined g. *NOTA*

_b__ 6. If it were required to make ten gallons of *banana* ice cream, the profit will decrease by *(choose the nearest value)*
a. zero b. $0.10 c. $0.25 d. $0.50 e. $0.75
f. $1.00 g. $1.25 h. $1.50 i. $1.75 j. *NOTA*

_c__ 7. If it were required to make ten gallons of *banana* ice cream, the production of *chocolate* ice cream would
a. be unchanged b. increase by less than 10 c. decrease by less than 10
d. increase by more than 10 e. decrease by more than 10 f. cannot be determined g. *NOTA*

_c_ 8. How much must the profit of *chocolate* ice cream drop before its production would be decreased? *(choose the nearest value)*
a. zero b. $0.01 c. $0.02 d. $0.03 e. $0.04
f. $0.05 g. $0.06 h. $0.07 i. $0.08 j. $0.09

_b__ 9. If the profit of *strawberry* ice cream were to be $0.88 per gallon,
a. the production plan would be unchanged b. production of strawberry i.c. would increase
c. production of strawberry i.c. would decrease d. *cannot be determined*

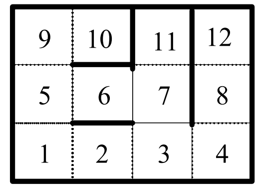
_c__ 10. The number of variables in the dual of this LP problem (not including variable z for objective row) is
a. one b. two c. three
d. four e. five f. *NOTA*

_a__ 11. The sign restrictions on the dual variables are
a. all nonnegative b. all nonpositive c. some nonpositive, some nonnegative
d. no sign restrictions e. *NOTA*

▼▲▼▲▼▲▼ PART C ▼▲▼▲▼▲▼

I. **Discrete-Time Markov Chains**

A rat is placed in location #1 of a maze shown below on the right. (Walls are indicated by solid lines.) A Markov chain model has been built where the state of the "system" is the location of the rat after he leaves his current location.



In assigning transition probabilities, it is assumed that the rat is equally likely to leave a location by any of the available paths:

P=

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0.5	0	0	0.5	0	0	0	0	0	0	0
2	0.5	0	0.5	0	0	0	0	0	0	0	0	0
3	0	0.333	0	0.333	0	0	0.333	0	0	0	0	0
4	0	0	0.5	0	0	0	0	0.5	0	0	0	0
5	0.333	0	0	0	0	0.333	0	0	0.333	0	0	0
6	0	0	0	0	0.5	0	0.5	0	0	0	0	0
7	0	0	0.333	0	0	0.333	0	0	0	0	0.333	0
8	0	0	0	0.5	0	0	0	0	0	0	0	0.5
9	0	0	0	0	0.5	0	0	0	0	0.5	0	0
10	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	1	0	0	0	0	0
12	0	0	0	0	0	0	0	1	0	0	0	0

(If he arrives at a "dead end", he will retrace his last move with probability 1.)

Mean-first-passage matrix:

	1	2	3	4	5	6	7	8	9	10	11	12
1	12	9.33	12.67	31.66	8.66	15.33	16	52.65	29.66	52.66	39	75.64
2	10.67	12	7.333	26.33	15.33	18	14.67	47.32	36.33	59.32	37.6	70.31
3	19.33	12.67	8	19	20	18.6	11.33	39.99	40.99	63.99	34.34	62.98
4	24.33	17.67	5	12	25	23.67	16.33	20.99	45.99	68.99	39.34	43.99
5	11.33	16.67	16	34.99	8	10.67	15.33	55.98	21	44	38.34	78.97
6	16.67	18	13.33	32.33	9.333	12	8.66	53.32	30.33	53.33	31.67	76.3
7	20	17.33	8.66	27.66	16.67	11.33	8.00	48.65	37.66	60.66	23	71.64
8	27.33	20.67	8	3	28	26.67	19.33	12	48.99	71.99	42.34	22.99
9	14.33	19.67	19	37.99	3	13.67	18.33	58.98	12	23	41.34	81.97
10	15.33	20.67	20	38.99	4	14.67	19.33	59.98	0.99	24	42.34	82.97
11	21	18.33	9.66	28.66	17.67	12.33	1	49.65	38.66	61.66	24	72.64
12	28.33	21.67	9	4	29	27.67	20.33	0.99	49.99	72.99	43.34	23.99

=M

n	$f_{1,12}^{(n)}$
1	0
2	0
3	0
4	0
5	0.02083
6	0
7	0.02546
8	0
9	0.02592
10	0
11	0.02545
12	0

First-visit distribution

state	$\Pi\{i\}$
1	0.08332
2	0.08332
3	0.12499
4	0.08334
5	0.12499
6	0.08332
7	0.12498
8	0.08335
9	0.08333
10	0.04167
11	0.04166
12	0.04168

Steadystate distribution

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c or d 1. If we record the rat's location over a period of several days, which location do you expect to be visited most frequently by the rat?
- a. all equally often
- b. location 7 more often than others
- c. locations 3 & 5 equally often
- d. locations 3, 5, &7 equally often
- e. locations 1, 3, 5, &7 equally often

_a_ 2. The number of transient states in this Markov chain model is
- a. 0
- b. 6
- c. 9
- d. 10
- e. 12
- f. none of the above

_e_ 3. If the rat begins in location #1, what is the expected number of moves required to reach location #12?
- a. five
- b. between 5 and 20
- c. between 20 and 50
- d. between 50 and 75
- e. between 75 and 100
- f. over 100

_ b_ 4. If the rat begins in location #1, how many locations will the rat visit before returning to his starting point?
- a. five
- b. between 5 and 20
- c. between 20 and 50
- d. between 50 and 75
- e. between 75 and 100
- f. over 100

PART TWO: Suppose that locations 11 and 12 both contain food, so that the rat does not leave when he finds it. States 11 & 12 then become *absorbing states*.

The matrices A (absorption probabilities) and E (expected # visits) for this Markov chain are:

	11	12
1	0.6897	0.3103
2	0.6552	0.3448
3	0.6207	0.3793
4	0.4138	0.5862
5	0.7241	0.2759
6	0.7586	0.2414
7	0.7931	0.2069
8	0.2069	0.7931
9	0.7241	0.2759
10	0.7241	0.2759

=A

	1	2	3	4	5	6	7	8	9	10	row sum
1	4.069	2.966	2.793	1.241	4.759	2.276	2.069	0.620	3.172	1.586	25.552
2	2.966	3.517	3.103	1.379	3.621	1.862	1.966	0.689	2.414	1.207	22.724
3	1.862	2.069	3.414	1.517	2.483	1.448	1.862	0.758	1.655	0.827	17.897
4	1.241	1.379	2.276	2.345	1.655	0.965	1.241	1.172	1.103	0.551	13.931
5	3.172	2.414	2.483	1.103	5.897	2.69	2.172	0.551	3.931	1.966	26.379
6	2.276	1.862	2.172	0.965	4.034	3.103	2.276	0.482	2.69	1.345	21.207
7	1.379	1.31	1.862	0.827	2.172	1.517	2.379	0.413	1.448	0.724	14.034
8	0.6207	0.689	1.138	1.172	0.827	0.482	0.620	1.586	0.551	0.275	7.966
9	3.172	2.414	2.483	1.103	5.897	2.69	2.172	0.551	5.931	2.966	29.379
10	3.172	2.414	2.483	1.103	5.897	2.69	2.172	0.551	5.931	3.966	30.379

=E

_c_ 6. If the rat begins at location #1, the probability that the rat finds the food at location #11 first (before the food at #12) is (nearest to)
- a. 50%
- b. 60%
- c. 70%
- d. 80%
- e. 90%
- f. 95%

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_a_ 7. The expected number of times that the rat returns to his initial location before finding food is

a. less than 5 b. between 5 and 10 c. between 10 and 25
d. between 25 and 40 e. between 40 and 80 f. more than 80

_d_ 8. If the rat manages to reach location #7 before finding food, the probability that he first finds the food at location #11 is

a. 50% b. 60% c. 70%
d. 80% e. 90% f. 95%

_d_ 9. The number of transient states in this Markov chain model is

a. 0 b. 6 c. 9
d. 10 e. 12 f. *NOTA*

A **black** rat is placed at location #**2** and a **white** rat at location #**6**. Assume they are otherwise identical and there is no interaction between the rats.

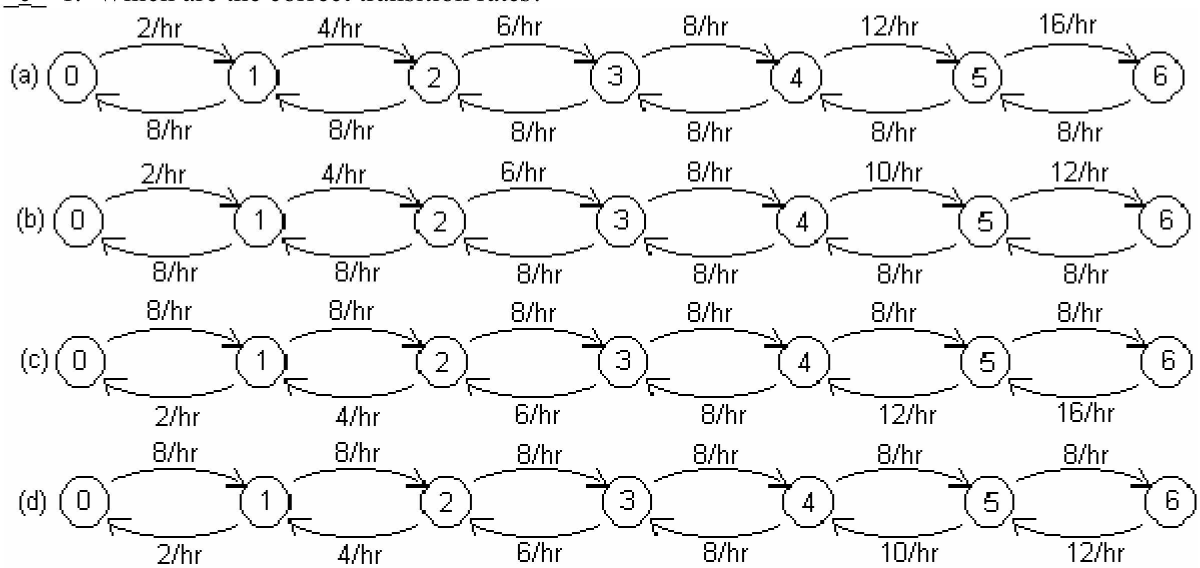
_a_ 10. Which rat do you expect to find food first?

a. White rat b. Black rat c. Tie!

(Note: compare row sums in expected # visits matrix!)

II. Continuous-time Markov chains: A parking lot consists of **four** spaces. Cars making use of these spaces arrive according to a Poisson process at the rate of ***eight cars per hour.*** Parking time is exponentially distributed with mean of ***30 minutes***. Cars who cannot find an empty space immediately on arrival may temporarily wait inside the lot until a parked car leaves, but may get impatient and leave before a parking space opens up. Assume that the time that a driver is willing to wait has exponential distribution with an average of ***15 minutes***. The temporary space can hold only **two** cars. All other cars that cannot park or find a temporary waiting space must go elsewhere. Model this system as a birth-death process, with states 0, 1, … 6.

_c_ 1. Which are the correct transition rates?



(e) *NOTA*

The steadystate probability distribution of the number of cars in the system is:

n	0	1	2	3	4	5	6
π_n	0.02	0.08	0.18	0.24	0.24	0.16	0.08

_e_ 2. What is the fraction of the time that there is at least one empty parking space? *(Choose nearest value!)*

a. 10% c. 30% e. 50% g. 70%
b. 20% d. 40% f. 60% h. 80%

_f_ 3. What is the average *total* number of cars in the lot? *(Choose nearest value!)*

a. 1 c. 2 e. 3 g. 4
b. 1.5 d. 2.5 f. 3.5 h. 4.5

_c_ 4. What is the average number of cars *waiting*? *(Choose nearest value!)*

a. 0.1 c. 0.3 *(0.32)* e. 0.5 g. 0.7
b. 0.2 d. 0.4 f. 0.6 h. 0.8

_c_ 5. What is the average arrival rate? *(Choose nearest value!)*

a. 5/hr c. 7/hr *(7.36/hr)* e. 9/hr g. 11/hr
b. 6/hr d. 8/hr f. 10/hr h. 12/hr

_b_ 6. According to Little's Law, what is the average time that a car waits for a parking space? *(Choose nearest value!)*

a. 0.025 hr c. 0.075 hr e. 0.25 hr g. 0.75 hr
b. 0.05hr *(0.32/7.36)* d. 0.1 hr f. 0.5 hr h. 1 hr.

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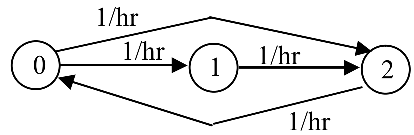
(2, continued)

Consider the queue with the continuous=time Markov chain model on the right. *(When the system is empty, customers can come singly or as a pair. Only when two customers have arrived does the server begin, processing both customers simultaneously.)*



7. Check all equations below that describe the steadystate probability distribution π:

_**X**_ $-2\pi_0 + \pi_2 = 0$	_**X**_ $\pi_0 - \pi_1 = 0$	___ $\pi_1 - \pi_2 = 0$
___ $\pi_0 + \pi_1 - 2\pi_2 = 0$	___ $2\pi_0 - 2\pi_2 = 0$	___ $2\pi_0 + \pi_1 - \pi_2 = 0$
___ $-2\pi_0 + \pi_1 + \pi_2 = 0$	_**X**_ $\pi_0 + \pi_1 - \pi_2 = 0$	___ $2\pi_0 - \pi_1 + \pi_2 = 0$

_c_ 8. The steadystate distribution is π =

a. $\left[\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right]$ b. $\left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]$ c. $\left[\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right]$

d. $\left[\frac{1}{6}, \frac{1}{3}, \frac{1}{2}\right]$ e. $\left[\frac{1}{3}, \frac{1}{6}, \frac{1}{2}\right]$ f. $\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right]$

g. *NOTA*

9. Is this a birth-death process? circle: (yes) [(no)]

10. Does Little's Law apply to this queue? circle: [(yes)] (no)

III. ***Decision analysis***

T. Bone Puckett, a corporate raider, has acquired a textile company and is contemplating the future of one of its major plants located in South Carolina. Three alternative decisions are being considered:

- Expand the plant and produce light-weight, durable materials for possible sales to the military, a market with little foreign competition.
- Maintain the status quo at the plant, continuing production of textile goods that are subject to heavy foreign competition.
- Sell the plant now.

If one the first two alternatives is chosen, the plant will still be sold at the end of a year. The amount of profit that could be earned by selling the plant in a year depends upon foreign market conditions, including the status of a trade embargo bill in Congress. The following payoff table describes this decision situation.

Decision	Good foreign competitive conditions	Poor foreign competitive conditions
Expand	$800,000	$500,000
Maintain status quo	$1,300,000	– $150,000
Sell now	$320,000	$320,000

Determine the best decision using the following decision criteria: Enter the values and an X marking the best decision in the last 2 columns. In the case of the "Minimax Regret" criterion, you should also complete the missing entry in the table.

1. MAXIMAX Criterion

Decision	Good foreign competitive conditions	Poor foreign competitive conditions	Maximum payoff	Opt?
Expand	$800,000	$500,000	$800000	
Maintain status quo	$1,300,000	-$150,000	$1300000	X
_Sell now	$320,000	$320,000	$320000	

2. **MAXIMIN Criterion**

Decision	Good foreign competitive conditions	Poor foreign competitive conditions	Minimum payoff	Opt?
Expand	$800,000	$500,000	$500000	X
Maintain status quo	$1,300,000	–$150,000	–$150000	
Sell now	$320,000	$320,000	$320000	

3. **MINIMAX REGRET Criterion**

Decision	Good foreign competitive conditions	Poor foreign competitive conditions	Maximum Regret	Opt?
Expand	$500,000	$ 0	$500,000	X
Maintain status quo	$0	$650,000	$650,000	
Sell now	$980,000	$180,000	$980,000	

(continued on next page!)

(III. Decision Analysis, continued) The chief executive officer of a firm in a highly competitive industry believes that one of her key employees is providing confidential information to the competition.

She is **90%** certain that this informer is the **vice-president of finance**, whose contacts have been extremely valuable in obtaining financing for the company.

- If she decides to fire this VP and he *is* the informer, she estimates that the company will avoid any further losses, i.e., the cost is **zero**
- If she decides to fire this VP but he *is not* the informer, the company will lose his expertise and still have an informer within the staff—the CEO estimates that this outcome would cost her company about **$3 million**!
- If she decides <u>not</u> to fire this VP, she estimates that the firm will lose **$1 million** whether or not he is actually the informer (since in either case the informer is still with the company).

Before deciding whether to fire the VP for finance, the CEO could order ***lie detector tests***.

To avoid possible lawsuits, the lie detector tests would have to be administered to all company employees, at a total cost of **$100,000**.

Notation:

"States of nature":
- **Y**: VP is mole
- **N**. VP not mole

"Observations of experiment":
- **+**: positive test result (he is lying)
- **-**: negative test result (he is truthful)

Another problem she must consider is that the available lie detector tests are not perfectly reliable:
- the probability of a *false positive* is **10%**
- the probability of a *false negative* is **5%**.

That is, since here *"positive"* means *detecting a lie,*
- if a person is not lying, the test will incorrectly suggest that the person is lying 10% of the time, i.e., **P{+ | N } = 0.10**
- if a person is lying, the test will incorrectly suggest that the person is telling the truth 5% of the time, i.e., **P{ - | Y } = 0.05**

In order to minimize the expected total cost of managing this difficult situation, what strategy should the CEO adopt?

Complete the decision tree, with the probabilities and expected payoffs at the various nodes.

Node #8: Expected payoff: <u>0.3 $M</u>

Node #7: Expected payoff: <u>0.3 $M</u>

Node #6: Expected payoff: <u>2.000 $M</u>

Branches from node #6: probabilities = <u>0.333</u> & <u>0.667</u>

Node #5: Expected payoff: <u>1.0 $M</u>

Node #2: Expected payoff: <u>0.165 $M</u>

Branches from node #2: probabilities = <u>0.865</u> & <u>0.135</u>

Node #1: Expected payoff: <u>0.265 $M</u>

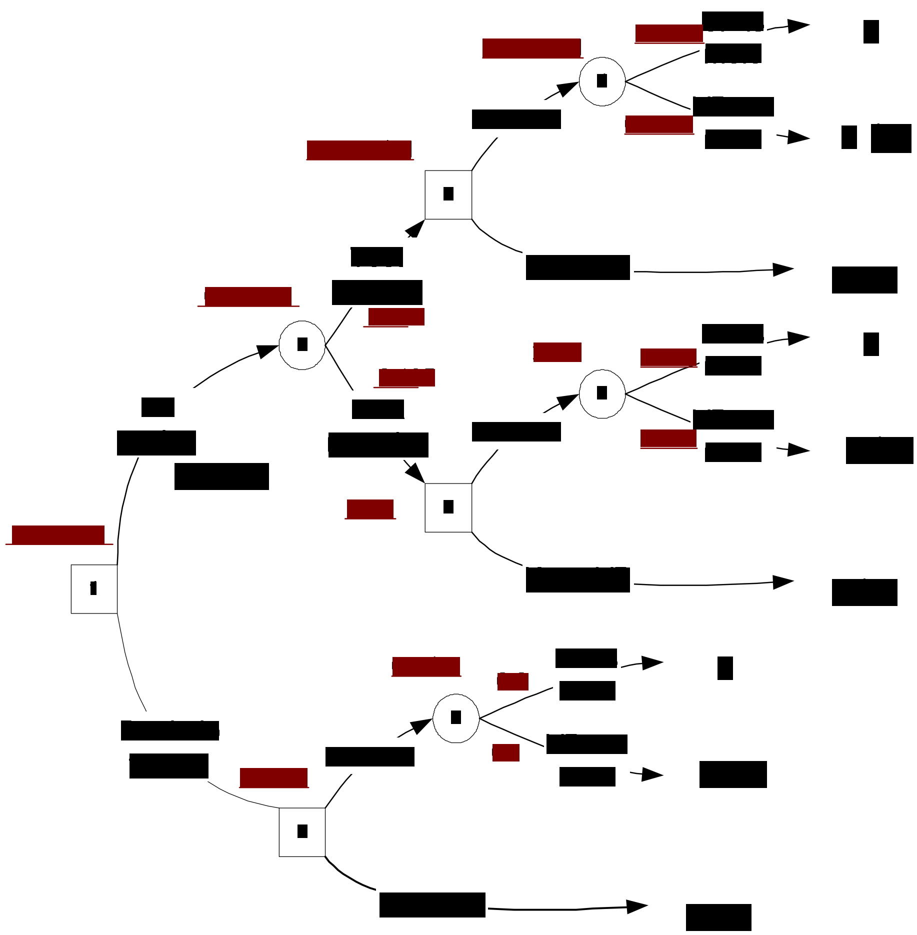
(Decision tree on next page)

(Decision tree on next page)

Note that the cost of the lie detector test has not been added to the terminal nodes on the far right, but is to be added as you fold back the tree!

Data:		P(Finding I State)	
State of	Prior	Finding	
Nature	Probability	POS	NEG
Y	0.9	0.95	0.05
N	0.1	0.1	0.9

Posterior		P(State I Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Y	N
POS	0.865	0.988439	0.011561
NEG	0.135	0.333333	0.666667



IV. *Production Planning* Production must be planned for the next eight days in order to meet scheduled shipments which have already been determined.
Other data:

- **Production cost** is $7 for setup, plus $3 per unit produced, up to a maximum of 4 units.
- **Storage cost**: $1 per unit stored (based upon **beginning**-of-day stock), up to a maximum of 6 units in storage. (For simplicity, assume any stock in excess of 6 units is scrapped.)
- **Shortages** are not allowed!
- **Salvage value**: $3 per unit in stock remaining in storage at the end of 8 days.
- **Initial inventory**: 1 unit is in stock at the beginning of the first day.
- **Orders to be delivered:**

Day	1	2	3	4	5	6	7	8
Demand	3	2	1	3	2	1	3	2

A dynamic programming model was used to compute the optimal production quantities for each day in order to minimize the cost. *Note that the recursion is forward, so that stage 1 is the first day, etc.*

1. What is the minimum total cost of the eight-day schedule? $_87___

2. Complete the computation of the missing element in the table for stage 1 (first day) below. $_87_

3. The *initial* inventory is **1 unit**. What is the optimal production schedule? *(If more than one solution, only one is required.) There is only one optimal solution:*

Day	1	2	3	4	5	6	7	8
Demand	3	2	1	3	2	1	3	2
Beginning stock	1	2	0	3	0	2	1	2
Production	4	0	4	0	4	0	4	0

_c_4. Suppose that at the beginning of day #2, a unit of the product in storage is discovered to be flawed and must be discarded. How will this change the production schedule for day #2?

a. unchanged b. increase 1 unit c. increase 2 units
d. increase 3 units e. increase 4 units f. *NOTA*

5. (***Stochastic DP***) Suppose now that on day #1 the demand is equally likely to be 1, 2, and 3 units. What is the total ***expected*** cost if you use the production decision that you have specified above? *(Assume that all other demands are known with certainty as before.)*

(Storage) $___1__ + *(Production)* $ __19__

+ *(expected future costs)* $_64.33__ = $ _84.33__

$$\frac{1}{3}f_2(4)+\frac{1}{3}f_2(3)+\frac{1}{3}f_2(2)=\frac{1}{3}\times 63+\frac{1}{3}\times 63+\frac{1}{3}\times 67=64.3333$$

Computer output on next page!

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---Stage 8---

s \ x:	0	1	2	3	4	Min
0	999	999	13	13	13	13
1	999	11	11	11	11	11
2	2	9	9	9	9	2
3	0	7	7	7	7	0
4	¯2	5	5	5	5	¯2
5	¯4	3	3	3	999	¯4
6	¯6	1	1	999	999	¯6

---Stage 7---

s \ x:	0	1	2	3	4	Min
0	999	999	999	29	30	29
1	999	999	27	28	22	22
2	999	25	26	20	21	20
3	16	24	18	19	20	16
4	15	16	17	18	19	15
5	7	15	16	17	18	7
6	6	14	15	16	999	6

---Stage 6---

s \ x:	0	1	2	3	4	Min
0	999	39	35	36	35	35
1	30	33	34	33	35	30
2	24	32	31	33	28	24
3	23	29	31	26	28	23
4	20	29	24	26	999	20
5	20	22	24	999	999	20
6	13	22	999	999	999	13

---Stage 5---

s \ x:	0	1	2	3	4	Min
0	999	999	48	46	43	43
1	999	46	44	41	43	41
2	37	42	39	41	41	37
3	33	37	39	39	42	33
4	28	37	37	40	36	28
5	28	35	38	34	999	28
6	26	36	32	999	999	26

---Stage 4---

s \ x:	0	1	2	3	4	Min
0	999	999	999	59	60	59
1	999	999	57	58	57	57
2	999	55	56	55	54	54
3	46	54	53	52	50	46
4	45	51	50	48	51	45
5	42	48	46	49	50	42
6	39	44	47	48	999	39

---Stage 3---

s \ x:	0	1	2	3	4	Min
0	999	69	70	70	65	65
1	60	68	68	63	65	60
2	59	66	61	63	63	59
3	57	59	61	61	61	57
4	50	59	59	59	999	50
5	50	57	57	999	999	50
6	48	55	999	999	999	48

---Stage 2---

s \ x:	0	1	2	3	4	Min
0	999	999	78	76	78	76
1	999	76	74	76	77	74
2	67	72	74	75	71	67
3	63	72	73	69	72	63
4	63	71	67	70	71	63
5	62	65	68	69	999	62
6	56	66	67	999	999	56

---Stage 1---

s \ x:	0	1	2	3	4	Min
0	999	999	999	92	93	92
1	999	999	90	91	87	87
2	999	88	89	85	84	84
3	79	**87**	83	82	85	79
4	78	81	80	83	85	78
5	72	78	81	83	80	72
6	69	79	81	78	999	69

V. ***Integer LP modeling*** *Comquat* owns four production plants at which personal computers are produced. *Comquat* can sell up to 20000 computers per year at a price of $750 per computer. For each plant, the production capacity, the production cost per computer, and the fixed cost of operating a plant for a year are given in the table below:

Plant #	Annual Production Capacity	Plant Annual Fixed Cost	Production Cost per Computer
1	4000	$9 million	$180
2	8000	$5 million	$310
3	9000	$3 million	$340
4	6000	$1 million	$350

The company wishes to determine how many computers it should produce at each plant in order to maximize its yearly revenue. (Note that if no computers are produced by a plant during the year, *Comquat* need not pay the fixed cost of operating the plant that year.)

We require two sets of **decision variables** :

Y_i = 1 if the computers are produced at plant i, 0 otherwise *(binary)*

and

X_i = quantity of computers produced at plant i *(continuous)*

Complete the mixed-integer programming model to impose the constraints specified. (Assume that other similar constraints will also be imposed.)

Minimize *(Annual fixed costs of plants)* $9\times10^3 Y_1 + 5\times10^3 Y_2 + 3\times10^3 Y_3 + 10^3 Y_4$ *(in $K)*

\+ *(Annual production costs)* $0.18X_1 + 0.31X_2 + 0.34X_3 + 0.35X_4$ *(in $K)*

subject to:

1. *Computers are to be produced at* <u>*no more than*</u> *3 plants*. $Y_1 + Y_2 + Y_3 + Y_4 \le 3$
2. *If the production line at plant 2 is set up, then that plant can produce up to 8000 computers; otherwise, none can be produced at that plant.* $X_2 \le 8000Y_2$
3. *The total production must be* <u>*at least*</u> *20000 computers*. $X_1 + X_2 + X_3 + X_4 \ge 20000$
4. If the production line at plant 2 is set up, that plant must produce <u>at least</u> 2000 computers.

$$2000Y_2 \le X_2$$

(Continued next page!)

(V, continued) The *Tower Engineering Corporation* is considering undertaking several proposed projects for the next fiscal year. The projects, together with the number of engineers required for each project, and the expected project profit, are:

Project #	1	2	3	4	5	6
Engineers req'd	20	55	47	38	90	63
Profit ($\times\$10^6$)	1.0	1.8	2.0	1.5	3.6	2.2

Define the decision variables, for i=1,2,...6:

Y_i = 1 if the company undertakes project i
0 otherwise

Complete the integer programming model to impose the constraints specified. (Assume that other similar constraints will also be imposed.)

Maximize $Y_1 + 1.8Y_2 + 2Y_3 + 1.5Y_4 + 3.6Y_5 + 2.2Y_6$
subject to

5. *Only 200 engineers are available* $20Y_1 + 55Y_2 + 47Y_3 + 38Y_4 + 90Y_5 + 63Y_6 \leq 200$

6. *Project #1 can be selected* *<u>only if</u>* *Project #2 is selected* $Y_1 \leq Y_2$

7. *Projects 4 and 5 cannot both be selected.* $Y_4 + Y_5 \leq 1$

8. *No more than three projects may be selected in all* $Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6 \leq 3$

9. *If* *<u>both</u>* *projects 2 & 3 are selected, then project 1 cannot be selected* $Y_1 \leq 2 - (Y_2 + Y_3)$

($\Rightarrow Y_1 + Y_2 + Y_3 \leq 2$)