56:171 Operations Research
Final Examination Solutions
December 15, 1999

- *Write your name on the first page, and initial the other pages.*
- *Answer both Parts A and B, and* **4 (out of 5)** *problems from Part C.*

		Possible
Part A:	Miscellaneous multiple choice	17
Part B:	Sensitivity analysis (LINDO)	14
Part C:	1. Discrete-time Markov chains I	15
	2. Discrete-time Markov chains II	15
	3. Continuous-time Markov chains	15
	4. Integer Programming Models	15
	5. Dynamic programming	15
	total possible:	88

PART A

Multiple Choice: Write the appropriate letter (a, b, c, d, or e) : (*NOTA* =None of the above).

__e_ 1. If, in the optimal *primal* solution of an LP problem (min cx st Ax≥b, x≥0), there is zero slack in constraint #1, then in the optimal dual solution,
a. dual variable #1 must be zero c. slack variable for dual constraint #1 must be zero
b. dual variable #1 must be positive d. dual constraint #1 must be slack e. *NOTA*

Note: Dual variable #1 may or may not be zero (if zero, solution is "degenerate").

__c_ 2. If, in the optimal solution of the *dual* of an LP problem (min cx subject to: Ax≥b, x≥0), dual variable #2 is positive, then in the optimal *primal* solution,
a. variable #2 must be zero c. slack variable for constraint #2 must be zero
b. variable #2 must be positive d. constraint #2 must be slack e. *NOTA*

__b_ 3. If $X_{ij}>0$ in the transportation problem, then dual variables U and V *must* satisfy
a. $C_{ij} > U_i + V_j$ c. $C_{ij} < U_i + V_j$ e. $C_{ij} = U_i - V_j$
b. $C_{ij} = U_i + V_j$ d. $C_{ij} + U_i + V_j = 0$ f. *NOTA*

__c_ 4. For a discrete-time Markov chain, let P be the matrix of transition probabilities. The sum of each...
a. column is 1 c. row is 1
b. column is 0 d. row is 0 e. *NOTA*

__d_ 5. In a birth/death process model of a queue, the time between arrivals is assumed to
a. have the Beta distribution c. be constant
b. have the Normal distribution d. have the exponential distribution e. *NOTA*

__c_ 6. In an M/M/1 queue, if the arrival rate = $\lambda < \mu$= service rate, then
a. $\pi_0 = 1$ in steady state c. $\pi_i > 0$ for all i e. the queue is not a birth-death process
b. no steady state exists d. $\pi_0 = 0$ in steady state f. *NOTA*

__d_ 7. If there is a tie in the "minimum-ratio test" of the simplex method, the solution in the next tableau
a. will be nonbasic c. will have a worse objective value
a. will be nonfeasible d. will be degenerate e. *NOTA*

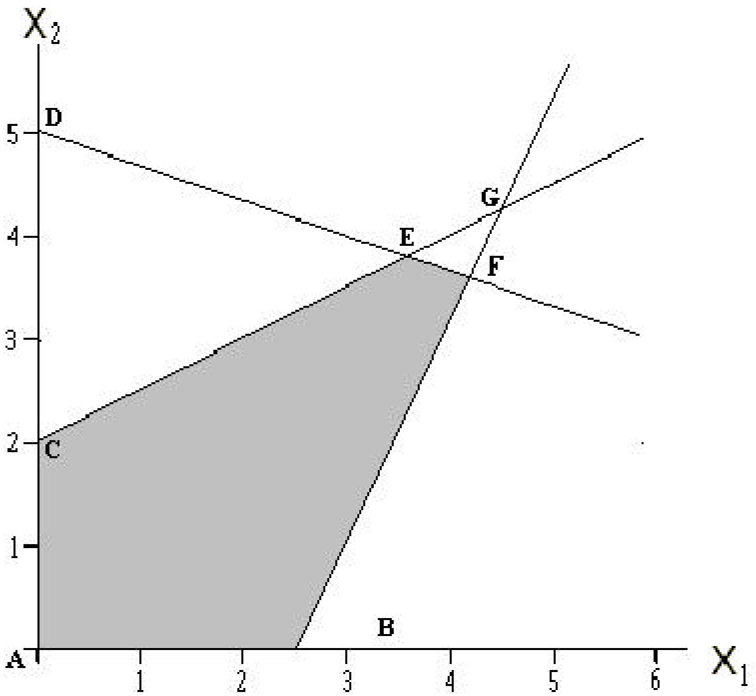
__a_ 8. An absorbing state of a Markov chain is one in which the probability of
a. moving out of that state is zero c. moving out of that state is one.
b. moving into that state is one. d. moving into that state is zero e. *NOTA*

The problems (9)-(12) below refer to the following LP:

$$\begin{array}{ll} \text{Maximize} & 3X_1 + 5X_2 \\ \text{subject to} & 2X_1 - X_2 \le 5 \\ & X_1 + 3X_2 \le 15 \\ & X_1 - 2X_2 \ge -4 \\ & X_1 \ge 0,\ X_2 \ge 0 \end{array}$$

(with inequalities converted to equations:)

$$\begin{array}{ll} \text{Maximize} & 3X_1 + 5X_2 \\ \text{subject to} & 2X_1 - X_2 + X_3 = 5 \\ & X_1 + 3X_2 + X_4 = 15 \\ & X_1 - 2X_2 - X_5 = -4 \\ & X_j \ge 0,\ j=1,2,3,4,5 \end{array}$$



_a or b_ 9 .The feasible region includes (possibly with others) points:
a. A, B, & C c. D, C, & E e. D, E, & G
b. B & F d. E, F, &G f. *NOTA*

__e_ 10. At point **F**, the basic variables include the variables
a. X_1 , X_2 & X_3 c. X_2, X_4 & X_5 e. X_1 , X_2 & X_5
b. X_1 , X_3 & X_4 d. X_1, X_2 & X_4 f. *NOTA*

__f_ 11. Which point is degenerate in the primal problem?
a. point A c. point C e. point E
b. point B d. point D f. *NOTA*

__a_ 12. The *dual* of this LP has the following constraints (not including nonnegativity or nonpositivity):
a. 2 constraints of type (≥) d. one each of type ≥ & =
b. one each of type ≤ & ≥ e. 2 of type ≤ and 1 of type ≥
c. 2 constraints of type (≤) f. *NOTA*

__d_ 13. The dual of the LP has the following types of variables:
a. three non-negative variables e. three non-positive variables
b. one non-negative and two non-positive variables f. *NOTA*
c. two non-negative variables and one unrestricted in sign
d. two non-negative variables and one non-positive variable

__d_ 14. If point F is optimal, then which dual variables must be zero, according to the *Complementary Slackness Theorem* ?
a. Y_1 and Y_2 d. Y_1 only
b. Y_1 and Y_3 e. Y_2 only
c. Y_2 and Y_3 f. Y_3 only

15. The number of basic variables in a solution of a transportation problem with 5 sources and 7 destinations is _11_

__d_ 16. A balanced transportation problem is one in which
a. # sources = # destinations c. supplies & demands all 1 e. *NOTA*
b. cost coefficients are all 1 d. sum of supplies = sum of demand

__c_ 17. An assignment problem is a transportation problem for which
a. # sources = # destinations c. supplies & demands all 1 e. *NOTA*
b. cost coefficients are all 1 d. sum of supplies = sum of demand

▼▲▼▲▼▲▼ PART B ▼▲▼▲▼▲▼

Sensitivity Analysis in LP.

Zales Jewlers uses rubies and sapphires to produce two types of rings. A type 1 ring requires 2 rubies, 3 sapphires, and 1 hour of jeweler's labor. A type 2 ring requires 3 rubies, 2 sapphires, and 2 hours of jeweler's labor. Each type 1 ring sells for $400, and each type 2 ring sells for $500. All rings produced by Zales can be sold. At present, Zales has 100 rubies, 120 sapphires, and 70 hours of jeweler's labor available. Extra rubies can be purchased at a cost of $100 each. Market demand requires that the company produce at least 20 type 1 rings and at least 25 type 2 rings. To maximize profit, Zales should solve the following LP:

X1= type 1 rings produced.
X2= type 2 rings produced
R = number of rubies purchased.

$$
\begin{array}{ll}
\text{MAX} & z=400X1 + 500X2 - 100R \\
\text{s.t.} & 2X1 + 3X2 \;\; R \le 100 \\
& 3X1 + 2X2 \le 120 \\
& X1 + 2X2 \le 70 \\
& X1 \ge 20 \\
& X2 \ge 25 \\
& X1 0, \;\; X2 0, \;\; R 0
\end{array}
$$

The LINDO output for this problem follows:

```
  MAX     400 X1 + 500 X2 - 100 R
  SUBJECT TO
         2)   2 X1 + 3 X2 - R <=   100
         3)   3 X1 + 2 X2 <=   120
         4)   X1 + 2 X2 <=   70
         5)   X1 >=   20
         6)   X2 >=   25
  END

     OBJECTIVE FUNCTION VALUE
1)      19000.00

  VARIABLE        VALUE          REDUCED COST
        X1        20.000000           .000000
        X2        25.000000           .000000
         R        15.000000           .000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)          .000000        100.000000
        3)        10.000000           .000000
        4)          .000000        200.000000
        5)          .000000           .000000
        6)         -.000000       -200.000000

 RANGES IN WHICH THE BASIS IS UNCHANGED:

                          OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
       X1      400.000000         INFINITY       100.000000
       X2      500.000000       200.000000         INFINITY
        R     -100.000000       100.000000       100.000000

                          RIGHTHAND SIDE RANGES
      ROW         CURRENT        ALLOWABLE        ALLOWABLE
                    RHS          INCREASE         DECREASE
        2      100.000000        15.000000         INFINITY
        3      120.000000         INFINITY        10.000000
        4       70.000000         3.333333          .000000
        5       20.000000          .000000         INFINITY
        6       25.000000          .000000         2.500000
```

```
  THE TABLEAU
ROW          X1    X2     R   SLK 2 SLK 3   SLK 4     SLK 5      SLK 6      RHS
1 ART      .000  .000  .000 100.000  .000 200.000      .000   200.000   19000.000
2    X2    .000 1.000  .000    .000  .000    .000      .000    -1.000     25.0000
3 SLK 3    .000  .000  .000    .000 1.000  -3.000      .000    -4.000      10.000
4     R    .000  .000 1.000  -1.000  .000   2.000      .000     1.000      15.000
5    X1   1.000  .000  .000    .000  .000   1.000      .000     2.000      20.000
6 SLK 5    .000  .000  .000    .000  .000   1.000     1.000     2.000        .000
```

_a_ 1. Suppose that instead of $100, each ruby costs $150. Would Zales still purchase rubies?
a. Yes b. No c. *Cannot be determined*

_e_ 2. What is the most that Zales would be willing to pay for another hour of jeweler's time? *Choose nearest answer:*
a. nothing b. $50 c. $100
d. $150 e. $200 f. *Cannot be determined*

_d_ 3. Your answer in (2) is valid for up to how many additional hours? *Choose nearest answer:*
a. zero b. 1 hour c. 2 hours
d. 3 hours e. four hours f. *Cannot be determined*

_a_ 4. Consider the labor availability constraint, after it is transformed by LINDO into equation form:
X1 + 2X2 (+/- ?) SLK 4 = 70
What sign should SLK 4 have in this equation?
a. Plus b. Minus

_b_ 5. If we wished to determine the effect on the above solution if 1 additional hour of jeweler's time were available, we would _____ the variable SLK 4 by 1 unit.
a. increase b. decrease

_b_ 6. If the variable SLK 4 were to increase by 1 hour, then according to the substitution rates, the number of rubies purchased would
a. increase b. decrease c. remain the same

_c_ 7. If the variable SLK 4 were to increase by 1 hour, then according to the substitution rates, the number of type 2 rings made would
a. increase b. decrease c. remain the same

_a_ 8. If the variable SLK 4 were to **de**crease by 1 hour, then according to the substitution rates, the number of type 1 rings made would
a. increase b. decrease c. remain the same

_b_ 9. Suppose that instead of $500, each type 2 ring were to have a profit of $400 each. Would Zales reduce the number of such rings produced?
a. Yes b. No c. *Cannot be determined*

_b_ 10. Suppose that instead of $500, each type 2 ring were to have a profit of $600 each. Would Zales increase the number of such rings produced?
a. Yes b. No c. *Cannot be determined*

▼▲▼▲▼▲▼ PART C ▼▲▼▲▼▲▼

1. **Discrete-Time Markov Chains I:** The Minnesota State University admissions office has modeled the path of a student through the university as a Markov Chain:

	Freshman	Sophomore	Junior	Senior	Quits	Graduates
Freshman	0.10	0.80	0	0	0.10	0
Sophomore	0	0.10	0.85	0	0.05	0
Junior	0	0	0.12	0.80	0.08	0
Senior	0	0	0	0.10	0.05	0.85
Quits	0	0	0	0	1.00	0
Graduates	0	0	0	0	0	1.00

Each student's state is observed at the *beginning of each fall semester*. For example, if a student who is a junior at the beginning of the current fall semester has an 80% chance of becoming a senior at the beginning of the next fall semester, a 15% chance of remaining a junior, and a 5% chance of quitting. (We will assume that a student who quits never re-enrolls.)

```
Powers of P:
                     \  1    2    3      4     5      6
                     m|
                     1|0.01 0.16 0.68   0     0.15   0
                     2|0    0.01 0.187  0.68  0.123  0
 P^2 =               3|0    0    0.0144 0.176 0.1296 0.68
                     4|0    0    0      0.01  0.055  0.935
                     5|0    0    0      0     1      0
                     6|0    0    0      0     0      1

                \  1     2     3        4       5        6
                m|
                1|0.001 0.024 0.2176   0.544   0.2134   0
                2|0     0.001 0.03094  0.2176  0.17246  0.578
P^3 =           3|0     0     0.001728 0.02912 0.139552 0.8296
                4|0     0     0        0.001   0.0555   0.9435
                5|0     0     0        0       1        0
                6|0     0     0        0       0        1

             \   1      2      3          4         5        6
         from|
            1|0.0001 0.0032 0.046512   0.22848   0.259308 0.4624
            2|0      0.0001 0.0045628  0.046512  0.185865 0.76296
 P^4 =      3|0      0      0.00020736 0.0042944 0.141146 0.854352
            4|0      0      0          0.0001    0.05555  0.94435
            5|0      0      0          0         1        0
            6|0      0      0          0         0        1

                          \     5         6
                      from|
 Absorption              1| 0.279212  0.720788
 Probabilities A:        2| 0.189113  0.810887
                         3| 0.141414  0.858586
                         4| 0.0555556 0.944444

                       \    1       2        3        4
                   from|
  Expected # of        1| 1.11111 0.987654 0.953984 0.847986
  Visits   E=          2| 0       1.11111  1.07323  0.953984
                       3| 0       0        1.13636  1.0101
                       4| 0       0        0        1.11111
```

Select the **nearest** available numerical choice in answering the questions below.

_e_ 1. The number of *transient* states in this Markov chain model is

a. 0	c. 2	e. 4	g. *NOTA*
b. 1	d. 3	f. 5	

_c_ 2. The number of *absorbing* states in this Markov chain model is

a. 0	c. 2	e. 4	g. *NOTA*
b. 1	d. 3	f. 5	

_c_ 3. The number of *recurrent* states in this Markov chain model is

a. 0	c. 2	e. 4	g. *NOTA*
b. 1	d. 3	f. 5	

4. The closed sets of states in this Markov chain model are (circle all that apply!)

a. {1 }	d. {4 }	g. {1,2,3,4 }	j. {2,3,4 }
b. {2 }	[✓ e. {5 }]	h. {1,2,3,4 }	k. {3,4 }
c. {3 }	[✓ f. {6 }]	[✓ i. {5,6}]	l. [✓ {1,2,3,4,5,6}]

5. The *minimal* closed sets of states in this Markov chain model are (circle <u>all</u> that apply!)

a. {1 } d. {4 } g. {1,2,3,4 } j. {2,3,4 }
b. {2 } [✓ e. {5 }] h. {1,2,3,4 } k. {3,4 }
c. {3 } [✓ f. {6 }] i. {5,6} l. {1,2,3,4,5,6}

Suppose that at the beginning of the Fall '99 semester, Joe Cool was a *Freshman.*

_g_ 6. What is the probability that Joe is a junior in Fall 2001? (choose nearest answer)
a. 10% c. 30% e. 50% g. 70% i. 90%
b. 20% d. 40% f. 60% h. 80% j. 100%

_e_ 7. What is the probability that Joe is a senior in Fall 2002? (choose nearest answer)
a. 10% c. 30% e. 50% g. 70% i. 90%
b. 20% d. 40% f. 60% h. 80% j. 100%

__*_ 8. What is the probability that Joe *first* becomes a senior in 2003? (choose nearest answer)
a. 5% c. 15% e. 25% g. 35% i. 45%
b. 10% d. 20% f. 30% h. 40% j. 50%

** This probability cannot be readily determined from the information given-- requires recursive computation of the first-passage probabilities.*

_g_ 9. What is the probability that Joe *eventually* graduates? (choose nearest answer)
a. 10% c. 30% e. 50% g. 70% i. 90%
b. 20% d. 40% f. 60% h. 80% j. 100%

_c_ 10. What is the expected length of his academic career, in years? (choose nearest answer)
a. 3 year c. 4 years e. 4.5 years g. 5 years i. 5.5 years
b. 3.75 years d. 4.25 years f. 4.75 years h. 5.25 years j. ≥5.75 years

_e_ 11. What fraction of students graduate in *exactly* four years? (choose nearest answer)
a. ≤25% c. 35% e. 45% g. 55% i. 65% k. 75% m. 85%
b. 30% d. 40% f. 50% h. 60% j. 70% l. 80% n. ≥90%

2. Discrete-time Markov Chains II: On New Year's Eve (December 31) of each year I determine whether my car is in <u>good</u>, <u>fair</u>, or <u>broken-down</u> condition. If my car is broken-down, I replace it on January 1st with a good used car.

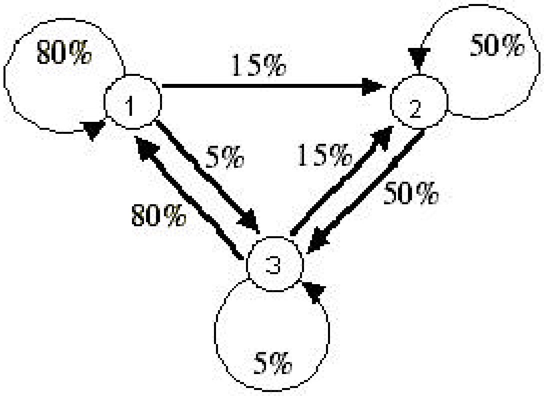
- A good car will be good at the end of next year with probability 80%, fair with probability 15%, or broken-down with probability 5%.
- A fair car will be fair at the end of the next year with probability 50%, or broken-down with probability 50%.
- It costs $10,000 to purchase a good used car; a fair car can be traded in for $3000; and a broken-down car can be sold as junk for $500.
- It costs $1000 per year to operate a good car and $1500 to operate a fair car. If a car breaks down during a year, the operating cost averages $2000.

Assume that the cost of operating a car during a year depends on the type of car on hand at the ***beginning*** of the year, and that any break-down occurs only at the ***end*** of a year.

My policy is to drive my car until it breaks down, at which time I replace it with a good used car. Define a Markov chain model representing the condition of the car which I own on Dec. 31, with three states:

1. Good condition
2. Fair condition
3. Broken-down

The diagram below indicates the transition probabilities:



Below are the 2nd, 3rd, & 4th powers of P:

```
P2= \   1       2      3   P3=\   1        2        3   P4 = \   1        2        3
from|                      from|                         from|
   1|0.6375 0.2025 0.16       1|0.598125 0.220875 0.181      1|0.584344 0.227306 0.18835
   2|0.375  0.325  0.3        2|0.50625  0.26375  0.23       2|0.552188 0.242313 0.2055
   3|0.6375 0.2025 0.16       3|0.598125 0.220875 0.181      3|0.584344 0.227306 0.18835
```

```
Mean 1st passage times
    \   1       2       3
from|
   1|1.73333 6.66667 5.2
   2|3.73333 4.33333 2
   3|1.73333 6.66667 5.2
```

_______Which ***one or more*** equations must be satisfied by the steady state probabilities π_1, π_2, & π_3?

a. $\pi_1 + \pi_2 + \pi_3 = 1$

b. $\pi_1 + \pi_2 + \pi_3 = 0$

c. $0.8\pi_1 + 0.8\pi_3 = \pi_1$

d. $0.8\pi_1 + 0.15\pi_2 + 0.05\pi_3 = \pi_1$

e. $0.8\pi_1 + 0.15\pi_2 + 0.05\pi_3 = \pi_3$

f. $0.05\pi_1 + 0.5\pi_2 + 0.05\pi_3 = \pi_3$

g. $0.8\pi_1 + 0.8\pi_3 = 0$

h. $0.8\pi_1 + 0.15\pi_2 + 0.05\pi_3 = 0$

Write the expression which represents my average cost per year:

__\$1000_ π_1 + _\$1500___ π_2 + _(\$2000 + 10000 - 500)_ π_3

I should expect to replace my car once every ___5.2_____ years.
If my current car is in *fair* condition, I should expect to replace it in ___2_______ years.

_a_ 7. The number of transient states in this Markov chain is

a. 0 c. 2 e. 4 g. *NOTA*
b. 1 d. 3 f. 5

_d_ 8. The number of recurrent states in this Markov chain is

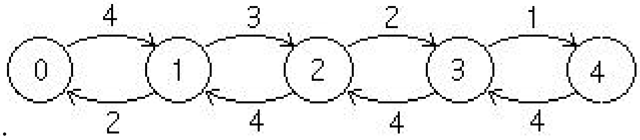
a. 0 c. 2 e. 4 g. *NOTA*
b. 1 d. 3 f. 5

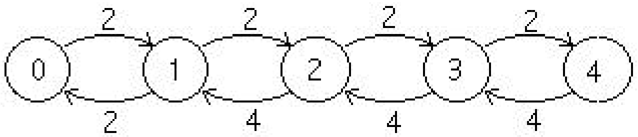
3. Birth/Death Model of a Queue:

For each birth/death process below, pick the classification of the queue and write it in the blank to the left:

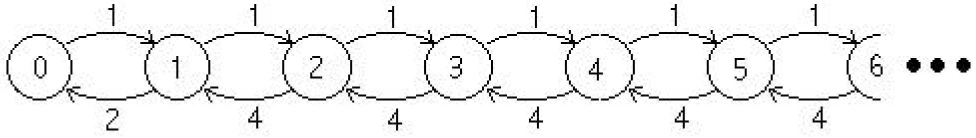
a. M/M/1
b. M/M/2
c. M/M/4
d. M/M/4/4
e. M/M/2/2
f. M/M/2/4/4
g. M/M/2
h. M/M/1/4
i. M/M/1/4/4
j. M/M/2/4
k. M/M/4/2
l. *NOTA*

M/M/2/4/4 1.

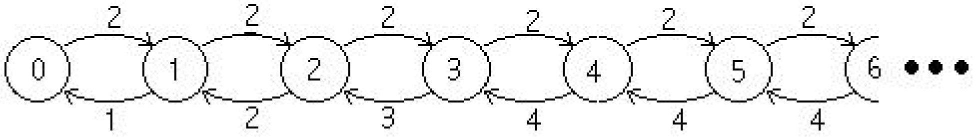




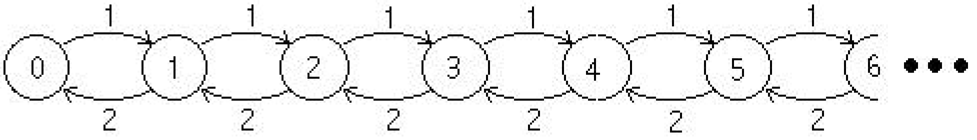
M/M/2/4 2.



M/M/2 3.



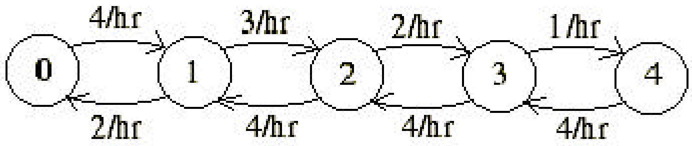
M/M/4 4.



M/M/1 5.

A job shop has four numerically controlled machines that are capable of operating on their own (i.e., without a human operator) once they have been set up with the proper cutting tools and all adjustments are made. Each setup requires the skills of an experienced machine operator, and the time need to complete a setup is exponentially distributed with a mean of 30 minutes. When the setup is complete, the machine operator pushes a button, and the machine requires no further attention until it has finished its job, when it is ready for another setup. The job times are exponentially distributed with a mean of one hour. Two operators have been assigned to this group of machines. (If only one machine requires attention, only one operator will tend it, rather than both working together.)

6. Indicate the transition rates on the diagram:



7. Write the expression which is used to evaluate π_0:

$$\frac{1}{\pi_0} = 1 + \frac{4}{2} + \frac{4}{2}\times\frac{3}{4} + \frac{4}{2}\times\frac{3}{4}\times\frac{2}{4} + \frac{4}{2}\times\frac{3}{4}\times\frac{2}{4}\times\frac{1}{4}$$

The steady-state probability distribution is:

i	πi	CDF
0	0.1839	0.1839
1	0.3678	0.5517
2	0.2759	0.8276
3	0.1379	0.9655
4	0.0345	1

_c_ 8. What is the percent of the time that both machinists are idle? *(Choose nearest value.)*

a. 10% c. 20% e. 30% g. 40%
b. 15% d. 25% f. 35% h. 45% or more

_e_ 9. What is the average number of machines in operation? *(Choose nearest value.)*

a. 0.5 c. 1.5 e. 2.5 (2.52) g. 3.5
b. 1.0 d. 2.0 f. 3.0 h. 4.0

_d_10. What is the utilization of each machine, i.e., the percent of the time that each machine is busy? *(Choose nearest value.)*

a. ≤50% c. 60% e. 70% g. 80% i. 90%
b. 55% d. 65% (63.2%) f. 75% h. 85% j. 95% or more

Suppose that the average rate at which machines complete jobs is 2.5/hour

_c_11. What is the average length of the time interval between a machine's completion of a job and the starting of another job (in hours)? *(Choose nearest value.)*

a. 0.5 c. 0.6 e. 0.7 g. 0.8 i. 0.9
b. 0.55 d. 0.65 f. 0.75 h. 0.85 j. 0.95 or more

Little's Law implies that $L = \lambda W$ where $L = 0\pi_0 + 1\pi_1 + 2\pi_2 + 3\pi_3 + 4\pi_4$ is the average number in the queueing system. (Or L could be calculated as 4 minus your answer in (9) above. Hence L = 4–2.52 = 1.48 and $W = \frac{L}{\lambda} = \frac{1.48}{2.52\ /\ \text{hr}} = 0.592$ hr.

4. Integer Programming Models

The board of directors of a large manufacturing firm is considering the set of investments shown below: Let R_i be the annual revenue (in \$millions) from investment *i* and C_i the cost (in \$millions) to make investment *i*. The board wishes to maximize total annual revenues and invest no more than a total of 50 million dollars.

Investment	Revenue	Cost	
i	R_i	C_i	Condition
1	1	5	*None*
2	2	8	Only if #1
3	3	12	*None*
4	4	18	Must if #1 and #2
5	5	24	Not if both #3 and #4
6	6	27	*None*
7	7	30	Only if both #3 and #6

Define variables:

X_i = 1 if investment i is selected, else 0.

1. Formulate this problem without the "side conditions" as an integer LP.

$$\text{Max} \sum_{i=1}^{7} C_i X_i$$

$$\text{s.t.} \sum_{i=1}^{7} R_i X_i \leq 50$$

$$X_i \in \{0,1\} \text{ for } i = 1,2,...7$$

This is an example of a special class of integer programming problems called *knapsack* problems.

2. Add a constraint or constraints to enforce the condition "Investment #2 can be selected only if #1 is selected".

$$X_2 \leq X_1$$

3. Add a constraint or constraints to enforce the condition "Investment #4 *must* be selected if both #1 & #2 are selected".

$$X_4 \geq X_1 + X_2 - 1$$

4. Add a constraint or constraints to enforce the condition "Investment #5 *cannot* be selected if both #3 & #4 are selected".

$$X_1 + X_2 + X_5 \leq 2$$

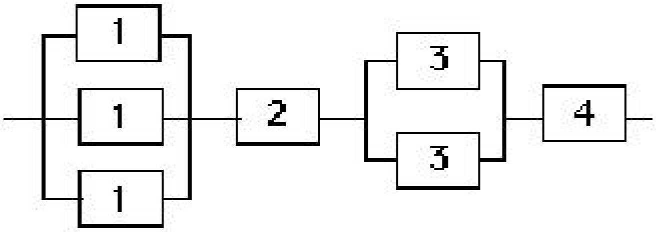
5. Add a constraint or constraints to enforce the condition "Investment #7 only if both #3 and #6 are selected".
Can use the pair of constraints

$$\begin{cases} X_7 \leq X_3 \\ X_7 \leq X_6 \end{cases}$$

or (summing the two constraints) $2X_7 \leq X_3 + X_6$

5. Optimization of System Reliability by DP: A system consists of 4 devices, each subject to possible failure, such that the system fails if any one or more of the devices fail:

Device	Reliability	Weight (kg)
1	75%	2
2	90%	3
3	80%	1
4	85%	2



Suppose that redundant units of devices 1 and 3 are included as shown above. (That is, system failure occurs if all 3 of device 1, or both of device 3, or device 2, or device 4 were to fail.)

__d_1. The reliability of **device 3** in the system shown is:

a. $.2^2 = 4\%$
b. $.8^2 = 64\%$
c. $1\text{-}e^{-2\times 0.2} = 32.97\%$
d. $1\text{-}.2^2 = 96\%$
e. $1\text{-}e^{-2\times 0.8} = 79.8\%$
f. $1\text{-}.8^2 = 36\%$
g. *NOTA*

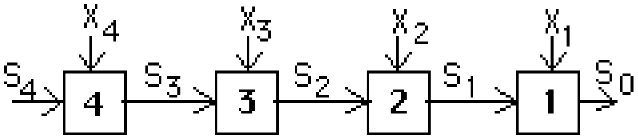
__c_2. The reliability of this entire **system** is:

a. $1 - (0.75^3)\,(0.9)\,(0.8^2)\,(0.85) = 79.35\%$
b. $(1 - e^{-3\times 0.75})\,(1 - e^{-0.9})\,(1 - e^{-2\times 0.8})\,(1 - e^{-0.85}) = 24.26\%$
c. $(1 - 0.25^3)\,(1 - 0.1)\,(1 - 0.2^2)\,(1 - 0.15) = 72.29\%$
d. $(1 - 0.75^3)\,(1 - 0.9)\,(1 - 0.8^2)\,(1 - 0.85) = 0.3\%$
e. $(1 - e^{-3\times 0.25})\,(1 - e^{-0.1})\,(1 - e^{-2\times 0.2})\,(1 - e^{-0.15}) = 0.23\%$
f. $1 - (1 - 0.25^3)\,(1 - 0.1)\,(1 - 0.2^2)\,(1 - 0.15) = 27.71\%$
g. *NOTA*

__b_3. The weight of this system is:

a. 10 kg.
b. 13 kg.
c. 11 kg.
d. 14 kg.
e. 12 kg.
f. none of the above

Suppose that we wish to find the system design having maximum reliability subject to a limit of 14 kg. weight.



__c_4. Define a dynamic programming model with optimal value function $f_n(S)$, where $f_n(S)$ is

a. the reliability of S redundant units of device #n.
b. the maximum reliability of the system if n redundant units are allowed.
c. the maximum reliability of devices 1 through n, if S kg. of weight is allocated to them.
d. the maximum reliability of devices n through 4, if S kg. of weight is allocated to them.

__c, f_5. The value of S_4 is (choose one or more!):

a. the safety factor for device 4
b. 15%
c. the state of the DP system at stage 4
d. 1 kg.
e. the reliability of device #4
f. 14 kg.

The following table shows the reliability of a device for various numbers of redundant units:

# units			
Device	1	2	3
1	.75	.9375	.984375
2	.9	.99	.999
3	.8	.96	.992
4	.85	.9775	.996625

The following output is obtained during the solution of the DP model, where several values have been omitted. *Note that the value -99.99999 is used to indicate -¥, i.e., an infeasible combination of s & x.*

6. Enter the missing value for each. of the entries in the tables:

α: 1–0.252 = 0.9375 (see table above).
β: 0.92813 (maximum value in row of table)
γ: 2 (value of X in which β appears)
δ: 10 – 2×3 = 4 (remaining capacity after 2 units of weight 3 have been added to system)
ε: 0.992 × f_2(12–3×1) = 0.992 × f_2(9) = 0992× 0.88594 = **0.87885**

7. The optimal design, weighing 14 kg., has reliability: f_4(14) = 0.81817

8. The optimal design has _2_ units of #1, _1_ unit of #2, _3_ units of #3, and _2_ units of #4.

```
---Stage 1---
                                                 f1(s)    optimal next
      s  \x:       1      2         3                           x    state
------------------------------------------------------------------------------
      2  |    0.75000 ¯99.99999 ¯99.99999 |     0.75000         1      0
      3  |    0.75000 ¯99.99999 ¯99.99999 |     0.75000         1      1
      4  |    0.75000  [__α__]  ¯99.99999 |     0.93750         2      0
      5  |    0.75000   0.93750 ¯99.99999 |     0.93750         2      1
      7  |    0.75000   0.93750   0.98438 |     0.98438         3      1
      8  |    0.75000   0.93750   0.98438 |     0.98438         3      2
      9  |    0.75000   0.93750   0.98438 |     0.98438         3      3
     10  |    0.75000   0.93750   0.98438 |     0.98438         3      4
     11  |    0.75000   0.93750   0.98438 |     0.98438         3      5
     12  |    0.75000   0.93750   0.98438 |     0.98438         3      6
     13  |    0.75000   0.93750   0.98438 |     0.98438         3      7
     14  |    0.75000   0.93750   0.98438 |     0.98438         3      8
```

```
                  ---Stage 2---
                                                 f1(s)    optimal next
      s  \x:       1      2         3                           x    state
------------------------------------------------------------------------------
      5  |    0.67500 ¯99.99999 ¯99.99999 |     0.67500         1      2
      6  |    0.67500 ¯99.99999 ¯99.99999 |     0.67500         1      3
      7  |    0.84375 ¯99.99999 ¯99.99999 |     0.84375         1      4
      8  |    0.84375   0.74250 ¯99.99999 |     0.84375         1      5
      9  |    0.8859    0.74250 ¯99.99999 |     0.88594         1      6
     10  |    0.88594   0.92813 ¯99.99999 |     [__β__]        [γ ]   [_δ_]
     11  |    0.88594   0.92813   0.74925 |     0.92813         2      5
     12  |    0.88594   0.97453   0.74925 |     0.97453         2      6
     13  |    0.88594   0.97453   0.93656 |     0.97453         2      7
     14  |    0.88594   0.97453   0.93656 |     0.97453         2      8
```

```
                  ---Stage 3---
                                                 f₁(s)    optimal next
      s  \x:          1       2         3                       x    state
       6   |     0.54000 ¯99.99999 ¯99.99999 |     0.54000        1      5
       7   |     0.54000   0.64800 ¯99.99999 |     0.64800        2      5
       8   |     0.67500   0.64800   0.66960 |     0.67500        1      7
       9   |     0.67500   0.81000   0.66960 |     0.81000        2      7
      10   |     0.70875   0.81000   0.83700 |     0.83700        3      7
      11   |     0.74250   0.85050   0.83700 |     0.85050        2      9
      12   |     0.74250   0.89100      ε    |     0.89100        2     10
      13   |     0.77963   0.89100   0.92070 |     0.92070        3     10
      14   |     0.77963   0.93555   0.92070 |     0.93555        2     12
```

```
                  ---Stage 4---
                                                 f₁(s)    optimal next
      s  \x:          1       2         3                       x    state
       8   |     0.45900 ¯99.99999 ¯99.99999 |     0.45900        1      6
       9   |     0.55080 ¯99.99999 ¯99.99999 |     0.55080        1      7
      10   |     0.57375   0.52785 ¯99.99999 |     0.57375        1      8
      11   |     0.68850   0.63342 ¯99.99999 |     0.68850        1      9
      12   |     0.71145   0.65981   0.53818 |     0.71145        1     10
      13   |     0.72293   0.79178   0.64581 |     0.79178        2      9
      14   |     0.75735   0.81817   0.67272 |     0.81817        2     10
```