Name ________________________

56:171 Operations Research
Midterm Exam--15 October 2002

	Possible	Score
1. True/False	25	_____
2. LP sensitivity analysis	25	_____
3. Transportation problem	15	_____
4. LP tableaux	15	_____
Total	80	_____

Part I: ***True***(+) or ***False***(o)?

#1-#10 refer to the "symmetic" primal/dual pair of LPs:

$$P:\begin{cases} \max\ cx \\ st\quad Ax \le b \\ \quad\quad x \ge 0 \end{cases} \qquad D:\begin{cases} \min\ by \\ st\quad A^T y \ge c \\ \quad\quad y \ge 0 \end{cases}$$

____ 1. If $\hat{x}$ is feasible in problem P above and $\hat{y}$ is feasible in problem D, then $c\hat{x} \le b\hat{y}$.

____ 2. If problem P is infeasible, then problem D must be infeasible also.

____ 3. If problem P has an unbounded feasible region, then problem D must be infeasible.

____ 4. If the nonnegativity restriction in problem P is removed, then its dual is unchanged except that the inequality $A^Ty \ge c$ is replaced with $A^Ty=c$.

____ 5. A point in the interior of problem P's feasible region must be nonbasic.

____ 6. Replacing $x\ge0$ with $x\le0$ in problem P will have the effect of replacing $y\ge0$ with $y\le0$ in its dual LP.

____ 7. If problem P has an unbounded objective function, then the dual problem D must have a degenerate optimal solution.

____ 8. If the revised simplex method is applied to problem P, and π is the final simplex multiplier vector, then π is the optimal solution of D.

____ 9.Increasing b_i in problem P above cannot improve the optimal value of the objective function cx.

____ 10. The dual variable for row i of problem P gives the rate of change of the optimal value of P as b_i increases.

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____ 11. If supplies and demands of a transportation problem are all integers, then there exists an optimal solution with all shipments equal to integers.

____ 12. If # rows of an assignment problem is less than # columns, then enough "dummy" rows must be appended to make the cost matrix square.

____ 13. If a transportation problem is not balanced, it may be made so by adding either a single dummy row or a single dummy column (but not both).

____ 14. If "Float" of an activity in a project schedule is positive, then its "Slack" must be zero.

____ 15. When a variable X_{ij} enters the basis of a transportation problem, then the variable which leaves the basis is in either row i or column j.

____ 16. Two activities on the critical path of a project may be in progress simultaneously.

____ 17. Substitution rates are computed in the RSM by multiplying the basis inverse matrix times a column in the original matrix A.

____ 18. If two or more activities of a project have no predecessor, then a dummy activity must be created in the AoA project network.

____ 19. The critical path in a project network is the *longest* path from a specified source node (beginning of project) to a specified destination node (end of project).

____ 20. A "dummy" activity in an A-O-A project network always has duration zero and cannot be a “critical” activity.

____ 21. If at some iteration of the Hungarian method, the zeroes of a n×n assignment cost matrix cannot be covered with fewer than n lines, this cost matrix must have more than one optimal solution.

____ 22.The number of basic variables in a n×n assignment problem is n.

____ 23. At each iteration of the Hungarian method, the number of zeroes in the cost matrix will increase.

Multiple choice:

____ 24. The “backward pass” of the critical path method computes

a. the latest time (LT) for events b. the earliest time (ET) for events
c. the “float” of the events d. *None of the above*

____ 25. If an artificial variable is positive in the optimal solution of the Phase I LP, then the LP must be

a. infeasible b. degenerate c. unbounded d. *None of the above*

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Part II. LP Sensitivity Analysis *Recall the following LP problem which appeared earlier in homework assignments:*

Marky Dee Sod operates three ranches in Texas. The acreage and irrigation water available for the three farms are shown below:

Farm	Acreage	Water available (acre-ft)
1	400	1500
2	600	2000
3	300	900

Three crops can be grown. However, the maximum acreage that can be grown of each crop is limited by the amount of appropriate harvesting equipment available. The three crops are described below. Any combination of crops may be grown on a farm.

Crop	Total harvesting capacity (in acres)	Water Reqmts (acre-ft per acre)	Expected profit ($/acre)
Milo	700	6	400
Cotton	800	4	300
Wheat	300	2	100

Decision variables: X_{ij} = # acres of crop j planted on farm i.

The LINDO model is:

```
MAX     400 X1MILO + 300 X1COTTON + 100 X1WHEAT + 400 X2MILO
       + 300 X2COTTON + 100 X2WHEAT + 400 X3MILO + 300 X3COTTON
       + 100 X3WHEAT
SUBJECT TO
         2)   X1MILO + X1COTTON + X1WHEAT          <=   400
         3)   6 X1MILO + 4 X1COTTON + 2 X1WHEAT    <=   1500
         4)   X2MILO + X2COTTON + X2WHEAT          <=   600
         5)   6 X2MILO + 4 X2COTTON + 2 X2WHEAT    <=   2000
         6)   X3MILO + X3COTTON + X3WHEAT          <=   300
         7)   6 X3MILO + 4 X3COTTON + 2 X3WHEAT    <=   900
         8)   X1MILO + X2MILO + X3MILO             <=   700
         9)   X1COTTON + X2COTTON + X3COTTON       <=   800
        10)   X1WHEAT + X2WHEAT + X3WHEAT          <=   300
  END

OPTIMAL VALUE
        1)      320000.00
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VARIABLE	**VALUE**	**REDUCED COST**
X1MILO	0.00	0.00
X1COTTON	375.00	0.00
X1WHEAT	0.00	33.33
X2MILO	50.00	0.00
X2COTTON	425.00	0.00
X2WHEAT	0.00	33.33
X3MILO	150.00	0.00
X3COTTON	0.00	0.00
X3WHEAT	0.00	33.33

ROW	**SLACK/SURPLUS**	**DUAL PRICES**
2)	25.00	0.00
3)	0.00	66.66
4)	125.00	0.00
5)	0.00	66.66
6)	150.00	0.00
7)	0.00	66.66
8)	500.00	0.00
9)	0.00	33.33
10)	300.00	0.00

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	**CURRENT COEF**	**ALLOWABLE INCREASE**	**ALLOWABLE DECREASE**
X1MILO	400.00	0.00	INFINITY
X1COTTON	300.00	INFINITY	0.000
X1WHEAT	100.00	33.33	INFINITY
X2MILO	400.00	0.00	0.000
X2COTTON	300.00	0.00	0.000
X2WHEAT	100.00	33.33	INFINITY
X3MILO	400.00	INFINITY	0.000
X3COTTON	300.00	0.00	INFINITY
X3WHEAT	100.00	33.33	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	400.00	INFINITY	25.00
3	1500.00	100.00	300.00
4	600.00	INFINITY	125.00
5	2000.00	750.00	300.00
6	300.00	INFINITY	150.00
7	900.00	900.00	900.00
8	700.00	INFINITY	500.00
9	800.00	75.00	425.00
10	300.00	INFINITY	300.00

THE TABLEAU:

ROW	(BASIS)	X1MILO	X1COTTON	X1WHEAT	X2MILO	X2COTTON	X2WHEAT
1	ART	0.00	0.00	33.333	0.00	0.00	33.333
2	SLK 2	-0.500	0.00	0.500	0.00	0.00	0.000
3	X1COTTON	1.500	1.00	0.500	0.00	0.00	0.000
4	SLK 4	0.500	0.00	0.167	0.00	0.00	0.667
5	X2MILO	1.00	0.00	0.333	1.00	0.00	0.333
6	SLK 6	0.00	0.00	0.00	0.00	0.00	0.000
7	X3MILO	0.00	0.00	0.00	0.00	0.00	0.000
8	SLK 8	0.00	0.00	-0.333	0.00	0.00	-0.333
9	X2COTTON	-1.500	0.00	-0.500	0.00	1.00	0.000
10	SLK 10	0.00	0.00	1.00	0.00	0.00	1.000

ROW	X3MILO	X3COTTON	X3WHEAT	SLK 2	SLK 3	SLK 4	SLK 5
1	0.00	0.00	33.333	0.00	66.667	0.00	66.667
2	0.00	0.00	0.00	1.00	-0.250	0.00	0.000
3	0.00	0.00	0.00	0.00	0.250	0.00	0.000
4	0.00	-0.333	0.00	0.00	0.083	1.00	-0.167
5	0.00	-0.667	0.00	0.00	0.167	0.00	0.167
6	0.00	0.333	0.667	0.00	0.00	0.00	0.000
7	1.00	0.667	0.333	0.00	0.00	0.00	0.000
8	0.00	0.00	-0.333	0.00	-0.167	0.00	-0.167
9	0.00	1.00	0.00	0.00	-0.250	0.00	0.000
10	0.00	0.00	1.00	0.00	0.00	0.00	0.000

ROW	SLK 6	SLK 7	SLK 8	SLK 9	SLK 10	RHS
1	0.00	67.00	0.00	33.00	0.00	0.32E+06
2	0.00	0.00	0.00	0.00	0.00	25.00
3	0.00	0.00	0.00	0.00	0.00	375.00
4	0.00	0.00	0.00	-0.333	0.00	125.00
5	0.00	0.00	0.00	-0.667	0.00	50.00
6	1.00	-0.167	0.00	0.00	0.00	150.00
7	0.00	0.167	0.00	0.00	0.00	150.00
8	0.00	-0.167	1.00	0.667	0.00	500.00
9	0.00	0.00	0.00	1.00	0.00	425.00
10	0.00	0.00	0.00	0.00	1.00	300.00

___ 1. How much land is left idle on Farm #1 in the optimal solution? *(choose nearest value)*

a. 20 acres or less b. 40 acres c. 80 acres d. 160 acres or more

Suppose Mr. Sod decides to plant the number of idle acres (from question 1) in *milo*.

____ 2. What would be the decrease in the profit? *(choose nearest value)*
a. $0 b. $500 c. $1000 d. $1500

____ 3. How would this change the optimal # acres of cotton to be planted on Farm #1? *(choose nearest value)*
a. no change b. decrease 40 acres c. decrease 80 acres d. decrease 160 acres

Mr. Sod notices that his cotton acreage is limited by his harvesting capacity (800 acres). He investigates and discovers that he has the opportunity to contract with an outside firm to harvest **40** acres of his cotton crop, so that he can increase his cotton acreage by 40 acres.

____ 4. What is the largest amount per acre that he can afford to pay for this service? *(choose nearest value)*
a. $25 or less b. $50 c. $75 d. $100 or more

____ 5. What is the effect of this increased cotton acreage on the variable SLK_9 in the solution above?
a. no change b. decrease 40 acres c. increase 40 acres

____ 6. On which farm should the additional 40 acres be planted?
a. Farm #1 b. Farm #2 c. Farm #3

____ 7. How does this change the acreage of *milo* on this farm? *(choose nearest value)*
a. no change b. decrease 40 acres c. decrease 80 acres d. decrease 160 acres

____ 8. How does this change the acreage of *wheat* on this farm? *(choose nearest value)*
a. no change b. decrease 40 acres c. decrease 80 acres d. decrease 160 acres

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Part III. Transportation Problem

	1	2	3	Supply
A	|_5_ **3**	|_7_ **2**	|_2_	**5**
B	|_4_	|_8_ **2**	|_4_ **1**	**3**
C	|_3_	|_9_	|_3_ **3**	**3**
Demand	**3**	**4**	**4**	

1. Is the above basic solution of the transportation problem degenerate? |__| Yes |__| No

2. Suppose that the dual variable $U_A = 0$. Then the value of dual variable V_1 = _____

3. Based upon the values of the dual variables, the reduced cost of the nonbasic variable X_{C1} is _______.

4. If X_{C1} were to enter the basis (regardless of whether it would improve the solution), then its value would become _____ and the basic variable X___ would leave the basis.

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Name ______________________

Part IV. Below are two **simplex tableaus**. Note that the objective in each case is to be **MIN**imized *(and, unlike the H&L textbook, –z rather than z is basic in the objective row!)*

1)

	-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS
MIN	1	-3	0	1	3	0	0	2	2	-36
	0	3	0	4	0	0	1	3	0	9
	0	-1	1	-2	-5	0	0	-2	1	4
	0	6	0	3	-2	1	0	-4	3	5

Is solution in tableau **1** above a *feasible* solution? |__| Yes |__| No
If so…
Is it *degenerate*? |__| Yes |__| No
Is it *optimal*? |__| Yes |__| No
If optimal….
Is it |__| unique, or |__| one of multiple optima?
If multiple optima exist…
variable _____ could replace variable ____ in the basis without increasing the cost.
If *not* optimal…
Does it indicate an unbounded solution? |__| Yes |__| No
If cost is unbounded …
entering variable ___ $\rightarrow +\infty$ would decrease cost z to $-\infty$.
If not unbounded…
entering variable ___ into the basis would remove variable ___ from the basis.
with improvement in objective? |__| Yes |__| No

2)

	-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS
MIN	1	3	0	1	3	0	0	2	0	-36
	0	3	0	4	1	0	1	3	0	9
	0	-1	1	2	5	0	0	-2	1	2
	0	6	0	3	-2	1	0	-4	-3	0

____

Is solution in tableau **2** above a *feasible* solution? |__| Yes |__| No
If so…
Is it *degenerate*? |__| Yes |__| No
Is it *optimal*? |__| Yes |__| No
If optimal….
Is it |__| unique, or |__| one of multiple optima?
If multiple optima exist…
variable _____ could replace variable ____ in the basis without increasing the cost.
If *not* optimal…
Does it indicate an unbounded solution? |__| Yes |__| No
If cost is unbounded …
entering variable ___ $\rightarrow +\infty$ would decrease cost z to $-\infty$.
If not unbounded…
entering variable ___ into the basis would remove variable ___ from the basis.
with improvement in objective? |__| Yes |__| No