56:171 Operations Research
Midterm Exam
October 22, 1993

- ***Write your name on the first page, and initial the other pages.***
- ***Answer all questions.***

	Possible	Score
A. True/False	45	_____
B. Multiple Choice	30	_____
C. Sensitivity analysis (LINDO)	20	_____
total possible:	95	_____

(A.) ***True/False:*** Indicate by "+" ="true" or "o" ="false" :

____ 1. A "dummy" activity in CPM has duration zero and cannot be on the critical path.

____ 2. In PERT, the total completion time of the project is assumed to be a random variable with a normal distribution.

____ 3. A "pivot" in a nonbasic column of a tableau will make it a basic column.

____ 4. The earliest completion time of a project could be computed by formulating an LP problem and solving it with the simplex method.

____ 5. In the two-phase simplex method, Phase One computes the optimal dual variables, followed by Phase Two in which the optimal primal variables are computed.

____ 6. "Crashing" a project is a procedure for allowing additional time for critical activities.

____ 7. In CPM, the "backward pass" is used to determine the latest time (LT) for each event (node).

____ 8. Considered as an LP problem, every basic feasible solution of an assignment problem is degenerate.

____ 9. The "minimum ratio test" is used to determine the pivot row in the simplex method.

____ 10. During any iteration of the simplex method, if x_j is the variable entering the basis, the improvement in the cost function resulting from the pivot is the value of the reduced cost.

____ 11. In applying the Hungarian method to an assignment problem, the number of iterations required may depend upon the degree of degeneracy of the problem.

____ 12. Before you enter an LP formulation into LINDO, you must first convert all inequalities to equations.

____ 13. The A-O-N project network does not require any "dummy" activities, except for the "begin" and "end" activities.

____ 14. The revised simplex method, unlike the ordinary simplex method which pivots in the original tableau, leaves the original tableau unchanged.

____ 15. If an artificial variable is nonzero in the optimal solution of an LP problem, then the problem has no feasible solution.

____ 16. At the end of the first phase of the two-phase simplex method, the phase-one objective function must be zero if the LP is feasible.

____ 17. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *must* pivot in row i.

____ 18. In the Hungarian method, no further reduction of the cost matrix is necessary if the number of lines required to cover the zeroes is less than the number of rows in the cost matrix.

____ 19. In a transportation problem, if the total demand exceeds total supply, a "dummy" destination should be defined.

____ 20. In a transportation problem, the number of basic (primal) variables is less than the number of dual variables.

____ 21. In a transportation problem, if the current dual variables $U_2=3$ and $V_4=1$, and $C_{24}=2$, then the current basic solution cannot be optimal.

____ 22. In a transportation problem, if the current dual variables $U_2=3$ and $V_4=1$, and $C_{24}=5$, then X_{24} cannot be basic.

____ 23. If the "float" of an activity of a project is positive, then the activity cannot be "critical" in the schedule.

____ 24. All tasks on the critical path of a project schedule have their latest finish time equal to their earliest finish time.

____ 25. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *cannot* pivot in row i.

____ 26. When you enter an LP formulation into LINDO, you must manipulate your equality constraints so that all variables appear on the left, and all constants on the right of the "=".

____ 27. A transportation problem is called "balanced" if the number of supply points equals the number of demand points.

____ 28. When maximizing in the simplex method, the value of the objective function cannot improve at the next pivot if the current tableau is degenerate.

____ 29. When minimizing in the simplex method, the cost may be improved by selecting any column having a negative reduced cost as the pivot column.

____ 30. A basic solution of an LP is always feasible, but not all feasible solutions are basic.

____ 31. The optimal value of a primal minimization LP problem is less than or equal to the objective value of every dual feasible solution.

____ 32. The optimal values of the primal and dual LP problems, if they exist, must be equal.

____ 33. If a primal minimization LP problem has a cost which is unbounded below, then the dual maximization problem has an objective which is unbounded above.

____ 34. If $X_{ij}=0$ in the transportation problem, then dual variables U and V *must* satisfy $C_{ij}=U_i+V_j$.

____ 35. In project scheduling, the problem of finding the earliest completion time for the project can be stated as an LP, with a dual LP which will find the length of the longest path from beginning to ending of the project.

____ 36. The reduced cost of a slack variable in row i is the simplex multiplier π_i for that row (if $-z$ is used as the basic variable in the objective row).

____ 37. At the completion of the revised simplex method applied to an LP, the simplex multipliers give the optimal solution to the dual of the LP.

____ 38. In the revised simplex method, before entering variable X_j into the basis, the substitution rates (necessary for the minimum ratio test) are computed by multiplying the basis inverse matrix times the original column of constraint coefficients for X_j.

____ 39. One advantage of the revised simplex method is that it does not require the use of artificial variables.

____ 40. If you change the objective coefficients of an LP which you solved yesterday, you can use yesterday's optimal solution as the starting basic feasible solution to solve the new problem today.

____ 41. If the simplex method is applied to the transportation problem, all of the "substitution rates" which are computed for the optimal solution will be either +1, -1, or zero.

____ 42. In the LP formulation of the project scheduling problem, the constraints include $Y_A - Y_B = d_A$ if activity A must precede activity B, where d_A is the given duration of activity A.

____ 43. Bayes' Rule can be used for revising one's estimates of the defective rate of a manufacturing process after one has inspected a sample of items obtained from the process.

____ 44. If you increase the right-hand-side of a "less-than-or-equal" constraint in a minimization LP, the optimal objective value will either increase or stay the same.

____ 45. The "reduced cost" in LP provides an estimate of the change in the objective value when a right-hand-side of a constraint changes.

____ 46. The transportation problem is a special case of an assignment problem.

(B.) ***Multiple Choice:*** Write the appropriate letter (a, b, c, d, or e) : (*NOTA* =None of the above).

____ 1. If, in the optimal *primal* solution of an LP problem (min cx st Ax ≤ b, x 0), there is zero slack in constraint #1, then in the optimal dual solution,
- (a) dual variable #1 must be zero
- (b) dual variable #1 must be positive
- (c) slack variable for dual constraint #1 must be zero
- (d) dual constraint #1 must be slack
- (e) *NOTA*

____ 2. If, in the optimal *dual* solution of an LP problem (min cx st Ax ≤ b, x 0), variable #2 is positive, then in the optimal primal solution,
- (a) variable #2 must be zero
- (b) variable #2 must be positive
- (c) slack variable for constraint #2 must be zero
- (d) constraint #2 must be slack
- (e) *NOTA*

____ 3. If you make a mistake in choosing the pivot row in the simplex method, the solution in the next tableau
- (a) will be nonbasic
- (b) will be nonfeasible
- (c) will have a worse objective value
- (d) will be degenerate
- (e) *NOTA*

____ 4. If you make a mistake in choosing the pivot column in the simplex method, the solution in the next tableau
- (a) will be nonbasic
- (b) will be nonfeasible
- (c) will have a worse objective value
- (d) will be degenerate
- (e) *NOTA*

____ 5. If there is a tie in the "minimum-ratio test" of the simplex method, the solution in the next tableau
- (a) will be nonbasic
- (b) will be nonfeasible
- (c) will have a worse objective value
- (d) will be degenerate
- (e) *NOTA*

The problems (6)-(10) below refer to the following LP:

$$\begin{array}{ll} \text{Minimize} & 8X_1 + 4X_2 \\ \text{subject to} & 3X_1 + 4X_2 \quad 6 \\ & 5X_1 + 2X_2 \le 10 \\ & X_1 + 4X_2 \le 4 \\ & X_1 \quad 0,\ X_2 \quad 0 \end{array}$$

(with inequalities converted to equations:)

$$\begin{array}{ll} \text{Minimize} & 8X_1 + 4X_2 \\ \text{subject to} & 3X_1 + 4X_2 - X_3 = 6 \\ & 5X_1 + 2X_2 + X_4 = 10 \\ & X_1 + 4X_2 + X_5 = 4 \\ & X_j \quad 0,\ j=1,2,3,4,5 \end{array}$$

____ 6 .The feasible region includes points
(a) A, B, &C (c) C, E, &F
(b) B, F, &G (d) B, D, &G (e) *NOTA*

____ 7. At point F, the basic variables include the variables
(a) X_2 & X_3 (c) X_4 & X_5
(b) X_3 & X_4 (d) X_1 & X_4 (e) *NOTA*

____ 8. Which point is degenerate in the primal problem?
(a) point A (c) point C
(b) point B (d) point D (e) *NOTA*

____ 9. If point F is optimal, then which dual variables must be zero, according to the Complementary Slackness Theorem?
(a) Y_1 and Y_2 (d) Y_1 only
(b) Y_1 and Y_3 (e) Y_2 only
(c) Y_2 and Y_3 (f) Y_3 only

10. For each alternative pair in parentheses, check the appropriate choice to obtain the dual LP of the above primal problem (with the inequality constraints):

(___Max/ ___Min) $6Y_1 + 10Y_2 + 4Y_3$
subject to $3Y_1 + 5Y_2 + Y_3$ (___ / ___$\leq$) 8
$4Y_1 + 2Y_2 + 4Y_3$ (___ / ___$\leq$) 4
Y_1 (___ / ___$\leq$) 0, Y_2 (___ / ___$\leq$) 0, Y_3 (___ / ___$\leq$) 0

(C.) ***Sensitivity Analysis in LP.***

"A manufacturer produces two types of plastic cladding. These have the trade names Ankalor and Beslite. One yard of Ankalor requires 8 lb of polyamine, 2.5 lb of diurethane and 2 lb of monomer. A yard of Beslite needs 10 lb of polyamine, 1 lb of diurethane, and 4 lb of monomer. The company has in stock 80,000 lb of polyamine, 20,000 lb of diurethane, and 30,000 lb of monomer. Both plastics can be produced by alternate parameter settings of the production plant, which is able to produce sheeting at the rate of 12 yards per hour. A total of 750 production plant hours are available for the next planning period. The contribution to profit on Ankalor is \$10/yard and \$20/yard on Beslite.
The company has a contract to deliver at least 3,000 yards of Ankalor. What production plan should be implemented in order to maximize the contribution to the firm's profit from this product division."

Definition of variables:

A = Number of yards of Ankalor produced
B = Number of yards of Beslite produced

LP model:

1)	Maximize 10 A + 20 B subject to	
2)	8 A + 10 B = 80,000	(lbs. Polyamine available)
3)	2.5 A + 1 B = 20,000	(lbs. Diurethane available)
4)	2 A + 4 B = 30,000	(lbs. Monomer available)
5)	A + B = 9,000	(lbs. Plant capacity)
6)	A = 3,000	(Contract)
	A = 0, B = 0	

The LINDO solution is:

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OBJECTIVE FUNCTION VALUE
        1)    142000.000
    VARIABLE           VALUE        REDUCED COST
       A             3000.000         0.000
       B             5600.000         0.000

    ROW          SLACK OR SURPLUS   DUAL PRICES
     2)                  0.000           2.000
     3)               6900.000           0.000
     4)               1600.000           0.000
     5)                400.000           0.000
     6)                  0.000          -6.000

 RANGES IN WHCH THE BASIS IS UNCHANGED
                                    OBJ COEFFICIENT RANGES
    VARIABLE      CURRENT     ALLOWABLE    ALLOWABLE
                   COEF       INCREASE     DECREASE
       A          10.000        6.000      INFINITY
       B          20.000       INFINITY       7.500

                                    RIGHTHAND SIDE RANGES
    ROW           CURRENT        ALLOWABLE      ALLOWABLE
                    RHS          INCREASE       DECREASE
     2           80000.000        4000.000       56000.000
     3           20000.000        INFINITY        6900.000
     4           30000.000        INFINITY        1600.000
     5            9000.000        INFINITY         400.000
     6            3000.000        2000.000        1333.333

THE TABLEAU
    ROW    (BASIS)    A         B        SLK 2     SLK 3     SLK 4     SLK 5
     1      ART       .000      .000     2.000      .000      .000      .000
     2      B         .000     1.000      .100      .000      .000      .000
     3      SLK 3     .000      .000     -.100     1.000      .000      .000
     4      SLK 4     .000      .000     -.400      .000     1.000      .000
     5      SLK 5     .000      .000     -.100      .000      .000     1.000
     6      A        1.000      .000      .000      .000      .000      .000

    ROW         SLK 6
     1           6.0            0.14E+06
     2            .800       5600.000
     3           1.700       6900.000
     4          -1.200       1600.000
     5            .200        400.000
     6          -1.000       3000.000
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Consult the LINDO output above to answer the following questions. If there is not sufficient information in the LINDO output, answer "NSI".

___ 1. How many yards of Beslite should be manufactured?

a. 3000 yards c. 5600 yards e. *NSI*

b. 1600 yards d. 400 yards

___ 2. How much of the available diurethane will be used?

a. 6900 pounds c. 13100 pounds e. *NSI*
b. 1600 pounds d. 400 pounds

___ 3. How much of the available diurethane will be unused?
a. 6900 pounds c. 13100 pounds e. *NSI*
b. 1600 pounds d. 400 pounds

___ 4. Suppose that the company can purchase 2000 pounds of additional polyamine for $2.50 per pound. Should they make the purchase? a. yes b. no c. *NSI*

___ 5. If the profit contribution from Beslite were to decrease to $12/yard, will the optimal solution change? a. yes b. no c. *NSI*

___ 6. If the profit contribution from Ankelor were to increase to $15/yard, will the optimal solution change? a. yes b. no c. *NSI*

___ 7. Suppose that the company could deliver 1000 yards less than the contracted amount of Ankalor by paying a penalty of $5/yard shortage. Should they do so?
a. yes b. no c. *NSI*

___ 8. Regardless of your answer in (7), suppose that they <u>do</u> deliver 1000 yards less Ankalor. This is equivalent to
a. increasing the slack in row 6 by 1000
b. increasing the surplus in row 6 by 1000
c. decreasing the slack in row 6 by 1000
d. decreasing the surplus in row 6 by 1000
e. none of the above
f. *NSI*

___ 9. If the company delivers 1000 yards less of Ankalor, how much Beslite should they deliver?
a. 800 yards
b. 4800 yards
c. 5600 yards
d. 6400 yards
e. none of the above
f. *NSI*

___ 10. How will the decision to deliver 1000 yards less Ankalor change the quantity of diurethane used during the next planning period?
a. increase by 1700 pounds
b. decrease by 1700 pounds
c. increase by 2500 pounds
d. decrease by 2500 pounds
e. none of the above
f. *NSI*