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		Possible	Score
A.	True/False & Multiple Choice	30	_____
B.	Sensitivity analysis (LINDO)	20	_____
C.1.	Transportation	15	_____
C.2.	Decision Tree	15	_____
C.3.	Simplex LP Method	15	_____
	total possible:	80	_____

▮❏▮❏▮❏▮❏ **Part A** ❏▮❏▮❏▮❏▮❏

True/False:

True 1. A "pivot" in a nonbasic column of an LP simplex tableau will make it a basic column.

True 2. If you increase the right-hand-side of a "greater-than-or-equal" constraint in a minimization LP, the optimal objective value will either increase or stay the same.

False 3. The "reduced cost" in LP provides an estimate of the change in the objective value when a right-hand-side of a constraint changes.

False 4. In the two-phase simplex method, Phase One computes the optimal dual variables, followed by Phase Two in which the optimal primal variables are computed.

True 5. At the completion of the revised simplex method applied to an LP, the simplex multipliers give the optimal solution to the dual of the LP.

True 6. In the LP formulation of the project scheduling problem, the constraints include $Y_B - Y_A$ d $_A$ if activity A must precede activity B, where d_A = duration of activity A.

False 7. In CPM, the "forward pass" is used to determine the latest time (LT) for each event (node).

True 8. The "minimum ratio test" is used to determine the pivot row in the simplex method.

True 9. The A-O-N project network does not require any "dummy" activities, except for the "begin" and "end" activities.

True 10. At the end of the first phase of the two-phase simplex method, the phase-one objective function must be zero if the LP is feasible.

False 11. In a transportation problem, if the current dual variables $U_2=3$ and $V_4=1$, and $C_{24}=5$, then the current basic solution *cannot* be optimal.

True 12. In a transportation problem, if the current dual variables $U_2=3$ and $V_4=1$, and $C_{24}=5$, then X_{24} *cannot* be basic.

True 13. All tasks on the critical path of a project schedule have their latest finish time equal to their earliest finish time.

True 14. A "dummy" activity in an A-O-A project network always has duration zero, but can be on the critical path.

False 15. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next simplex iteration you *cannot* pivot in row i.

False 16. When maximizing in the simplex method, the value of the objective function will not improve at the next pivot if the current tableau is degenerate.

False 17. When minimizing in the simplex method, you must select the column which has the smallest (i.e., the most negative) reduced cost as the next pivot column.

True 18. If the "float" ("slack") of an activity of a project is positive, then the activity cannot be "critical" in the schedule.

False 19. If a primal minimization LP problem has a cost which is unbounded below, then the dual maximization problem has an objective which is unbounded above.

<u>False</u> 20. If $X_{ij}=0$ in the transportation problem, then dual variables U and V *must* satisfy
$C_{ij}=U_i+V_j$.

Multiple Choice: Write the appropriate letter (a, b, c, d, or e) : (*NOTA* =<u>N</u>one <u>of</u> the <u>a</u>bove).
  __e__ 21.   If, in the optimal *primal* solution of an LP problem (min cx st Ax≥b, x≥0), there is
zero slack in constraint #1, then in the optimal dual solution,
   (a) dual variable #1 must be zero   (c) slack variable for dual constraint #1 must be zero
      (b) dual variable #1 must be positive (d) dual constraint #1 must be slack (e) *NOTA*
  __c__ 22.   If, in the optimal *dual* solution of an LP problem (min cx st Ax b, x 0), variable #2
is positive, then in the optimal primal solution,
      (a) variable #2 must be zero   (c) slack variable for constraint #2 must be zero
      (b) variable #2 must be positive   (d) constraint #2 must be slack   (e)*NOTA*
  __b__ 23. If you make a mistake in choosing the pivot row in the simplex method, the solution in
the next tableau
      (a) will be nonbasic   (c) will have a worse objective value
      (b) will be nonfeasible   (d) will be degenerate   (e) *NOTA*
  __c__ 24. If you make a mistake in choosing the pivot column in the simplex method, the solution
   in the next tableau will
      (a) be nonbasic   (c) have a worse objective value
      (b) be nonfeasible   (d) be degenerate   (e) *NOTA*
  __d__ 25. If there is a tie in the "minimum-ratio test" of the simplex method, the solution in the
next tableau
      (a) will be nonbasic   (c) will have a worse objective value
       (b) will be nonfeasible   (d) will be degenerate   (e) *NOTA*

The problems (26)-(30) below refer to the following LP: *(with inequalities converted to equations:)*

```
Maximize  3X1 + 2X2   Maximize  3X1 + 2X2
subject to   2X1 +  X2 ≤ 100    subject to   2X1 +  X2 + X3          = 100
             X1 +  X2 ≤ 80                   X1 +   X2     + X4      =  80
             X1         ≤   40               X1                  +X5 =  40
             X1 ≥ 0, X2 ≥ 0         Xj ≥ 0, j=1,2, 3,4,5
```
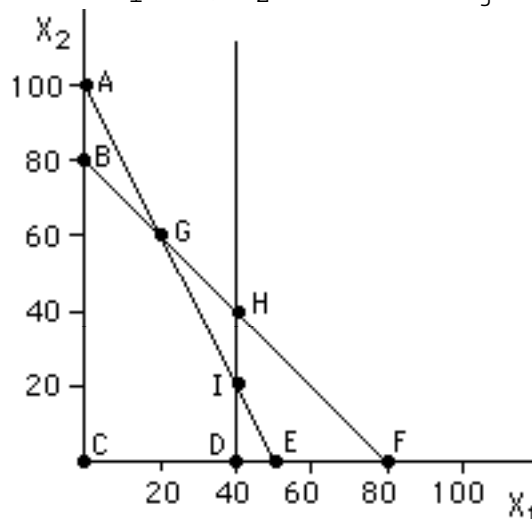

  __d__   26 .The feasible region includes points

     (a) I, D, & E   (c) E, F, H, & I

     (b) G, H, & I   (d) B, D, G, I, & C (e) *NOTA*

__c__   27. At point G, the basic variables <u>include</u> the variables

     (a) X_2 & X_3 (c) X_1 & X_5

     (b) X_3 & X_4 (d) X_1 & X_4 (e) *NOTA*

__e__   28. Which point is degenerate in this problem?

     (a) point B   (c) point H

     (b) point G   (d) point I   (e) *NOTA*

__f__   29. <u>If</u> point G is optimal, then which dual variables must be zero, according to the Complementary Slackness Theorem?

     (a) both Y_1 and Y_2 (d) Y_1 only

     (b) both Y_1 and Y_3 (e) Y_2 only

     (c) both Y_2 and Y_3 (f) Y_3 only

30. For each alternative pair in parentheses, check the appropriate choice to obtain the dual LP of the above primal problem (with the inequality constraints):

```
   (___Max/ _X_Min)                    100Y1 + 80Y2 + 40Y3

      subject to     2Y1 +  Y2  +  Y3  (___≤ / _X_≥ )   3

                      Y1 +   Y2         (___≤ / _X_≥ )   2

   Y1 (___≤ / _X_≥ ) 0, Y2 (___≤ / _X_≥ ) 0, Y3 (___≤ / _X_≥ ) 0
```

◼◻◼◻◼◻◼◻ Part B ◻◼◻◼◻◼◻◼

LINDO analysis

Problem Statement: McNaughton Inc. produces two steak sauces, spicy Diablo and mild Red Baron. These sauces are both made by blending two ingredients A and B. A certain level of flexibility is permitted in the formulas for these products. Indeed, the restrictions are that:

    i) Red Baron must contain no more than 75% of A.

    ii) Diablo must contain no less than 25% of A and no less than 50% of B

Up to 40 quarts of A and 30 quarts of B could be purchased. McNaughton can sell as much of these sauces as it produces at a price per quart of $3.35 for Diablo and $2.85 for Red Baron. A and B cost $1.60 and $2.05 per quart, respectively. McNaughton wishes to maximize its net revenue from the sale of these sauces.

Define   *D = quarts of Diablo to be produced*

               R = quarts of Red Baron to be produced

               AD= quarts of A used to make Diablo

               AR = quarts of A used to make Red Baron

               BD = quarts of B used to make Diablo

               BR = quarts of B used to make Red Baron

The LINDO output for solving this problem follows:

```
  MAX    3.35 D + 2.85 R - 1.6 AD - 1.6 AR - 2.05 BD - 2.05 BR
  SUBJECT TO
        2) - D + AD + BD =    0
        3) - R + AR + BR =    0
        4)   AD + AR <=   40
        5)   BD + BR <=   30
        6) - 0.25 D + AD >=   0
        7) - 0.5 D + BD >=   0
        8) - 0.75 R + AR <=   0
  END
         OBJECTIVE FUNCTION VALUE
   1)       99.0000000

  VARIABLE           VALUE                      REDUCED COST
```

```
        D                  50.000000                  0.000000
        R                  20.000000                  0.000000
        AD                 25.000000                  0.000000
        AR                 15.000000                  0.000000
        BD                 25.000000                  0.000000
        BR                  5.000000                  0.000000

        ROW            SLACK OR SURPLUS            DUAL PRICES
         2)                0.000000                 -2.350000
         3)                0.000000                 -4.350000
         4)                0.000000                  0.750000
         5)                0.000000                  2.300001
         6)               12.500000                  0.000000
         7)                0.000000                 -1.999999
         8)                0.000000                  2.000000
```

RANGES IN WHICH THE BASIS IS UNCHANGED

```
                         OBJ COEFFICIENT RANGES
      VARIABLE        CURRENT        ALLOWABLE        ALLOWABLE
                       COEF          INCREASE         DECREASE
        D            3.350000        0.750000         0.500000
        R            2.850000        0.500000         0.375000
        AD          -1.600000        1.500001         0.666666
        AR          -1.600000        0.666666         0.500000
        BD          -2.050000        1.500001         1.000000
        BR          -2.050000        1.000000         1.500001

                         RIGHTHAND SIDE RANGES
      ROW            CURRENT        ALLOWABLE        ALLOWABLE
                      RHS           INCREASE         DECREASE
        2            0.000000       10.000000        10.000000
        3            0.000000       16.666668         3.333333
        4           40.000000       50.000000        10.000000
        5           30.000000       10.000000        16.666664
        6            0.000000       12.500000         INFINITY
        7            0.000000        6.250000         5.000000
        8            0.000000        2.500000        12.500000
```

THE TABLEAU:

```
ROW    (BASIS) D      R      AD     AR     BD     BR    SLK 4  SLK 5  SLK 6
  1    ART    0.000  0.000  0.000  0.000  0.000  0.000  0.750  2.300  0.000
  2    AD     0.000  0.000  1.000  0.000  0.000  0.000 -0.500  1.500  0.000
  3    R      0.000  1.000  0.000  0.000  0.000  0.000  2.000 -2.000  0.000
  4    AR     0.000  0.000  0.000  1.000  0.000  0.000  1.500 -1.500  0.000
  5    BR     0.000  0.000  0.000  0.000  0.000  1.000  0.500 -0.500  0.000
  6    SLK 6  0.000  0.000  0.000  0.000  0.000  0.000 -0.250  0.750  1.000
  7    D      1.000  0.000  0.000  0.000  0.000  0.000 -1.000  3.000  0.000
  8    BD     0.000  0.000  0.000  0.000  1.000  0.000 -0.500  1.500  0.000

ROW    SLK 7     SLK 8      RHS
  1    2.000     2.000     99.000
  2    3.000     2.000     25.000
  3   -4.000    -4.000     20.000
  4   -3.000    -2.000     15.000
  5   -1.000    -2.000      5.000
  6    2.000     1.000     12.500
  7    4.000     4.000     50.000
  8    1.000     2.000     25.000
```

1. How many quarts of "Red Baron" are produced in the optimal solution? __20 quarts__
2. How much total profit does the firm make on these two products? $_99.00__
3. What *additional* amount should the firm be willing to pay to have another quart of ingredient
   B available? $ _2.30_____

What is the *total* amount the firm should be willing to pay for another quart of ingredient B?
$__4.35__
How many quarts should they be willing to buy at this price? __10 quarts__
4. Using one more quart of ingredient B (i.e., a total of 31 quarts) would
(___increase/_X_decrease) the "slack variable" in row #5 by _one_ unit (quart).

5. Using one more quart of ingredient B (i.e., a total of 31 quarts) would result the following
    changes in the variables:
    • the "slack variable" in row #6?   (_X_increase/___decrease) by __0.75_
    • the quantity of A used in producing "Red Baron"? (___increase/_X_decrease) by __1.5_
    • the quantity of A used in producing "Diablo"?   (_X_increase/___decrease) by __1.5_
    • the total quantity of "Diablo" produced?   (_X_increase/___decrease) by __3.0__
    • the total quantity of "Red Baron" produced?   (___increase/_X_decrease) by __2.0_
6. How much can the price of "Red Baron" increase before the composition of the current
    optimal product mix changes? $ ___0.50___
7. What is the optimal value of the objective function of the LP which is the dual of this
    problem? $ _99.00_ This dual LP objective is _X_minimized / ___maximized.

▮▯▮▯▮▯▮▯ Part C ▯▮▯▮▯▮▯▮

(C.1) **Transportation Problem.** Consider the problem with the initial solution specified on the
left below:

1. Is this a basic solution? (*circle:* **Yes** / No)
2. Is this a degenerate solution? (*circle:* Yes / **No**)
3. If the dual variable u_1 (for supply constraint #1) were assigned the value zero, the values of the
other dual variables are: u_2= -2, u_3 = __-3_ , v_1= +5, v_2= _+7_, v_3= +9 , v_4= __+2_.
4. If X_{33} were to be increased, what would be the change in the cost function per unit increase in
    X_{33}? __-1_
5. If X_{33} were to enter the basis, what would be its new value? __1____
6. If X_{33} were to enter the basis, what variable(s) would leave the basis?
            Circle as many as apply: X_{13} X_{14} X_{21} X_{22} **X_{23}** X_{31}
7. If X_{33} were to enter the basis, would the new basic solution be degenerate? (*circle:* Yes / **No**)
8. Which other variable *instead* of X_{33}would result in a degenerate basic solution if it were to
enter the basis? *Circle as many as apply:* X_{11} X_{12} X_{24} X_{32} X_{33} X_{34} **none**
9. What is the cost of the initial solution shown above?   __81_

C.2. Decision Tree. The decision sciences department is trying to determine which of two
copying machines to purchase. Both machines will satisfy the department's needs for the next **ten**
years.
    Machine 1 costs $3000 and has a maintenance agreement, which, for an annual fee of $100,
covers all repairs. Machine 2 costs $2500, and its annual maintenance cost is a random variable.

At present, the decision sciences department believes there is a 40% chance that the annual cost for machine 2 will be $0, a 40% chance it will be $100, and a 20% chance it will be $200.

For a fee of $100 the department can have Joe, a trained repairman, evaluate the quality of machine 2 before the purchase decision is made. If Joe believes that machine 2 is *satisfactory* , there is a 70% chance that its annual maintenance cost will be $0 and a 30% chance that it will be $100. If Joe believes that machine 2 is *unsatisfactory* , there is a 10% chance that the annual maintenance cost will be $0, a 40% chance it will be $100, and a 50% chance it will be $200. The department believes that there is a 70% probability that Joe will give a satisfactory report after evaluating machine 2. Based upon this data, the decision tree below is drawn:
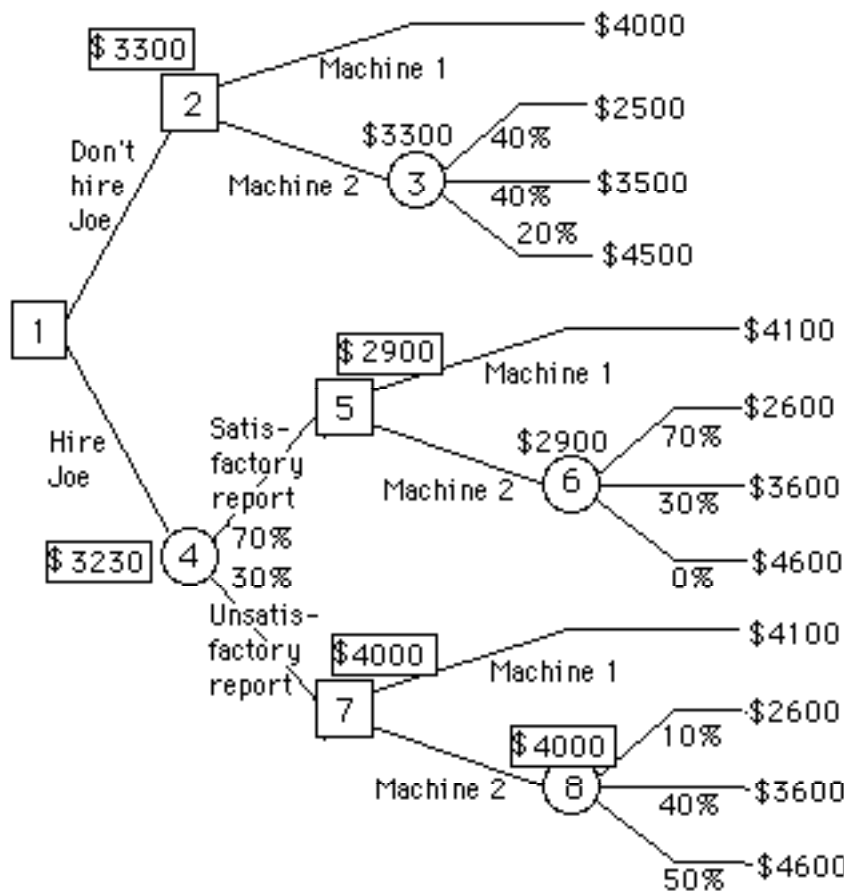
Fill the blank boxes in the decision tree above and answer the following questions:

__d_ 1. If Joe's evaluation of machine 2 is "unsatisfactory" and they were to select machine 2, what would be their expected cost (including the cost of Joe's evaluation)?

a. $ 2600	c. $ 3300	e. $ 4500
b. $ 2900	d. $ 4000	f. *NOTA*

__e_ 2. This computation is referred to as "folding back" which node in the tree above?

a. Node # 4	c. Node #6	e. Node #8
b. Node # 5	d. Node #7	f. *NOTA*

__f_ 3. What is EVSI (i.e., the expected value of Joe 's evaluation)?

a. *negative* $ 30	c. $ 40	e. $ 120
b. *negative* $ 20	d. $ 70	f. *NOTA*

Question #4 was disregarded... the answer depends upon the report of the repairman!

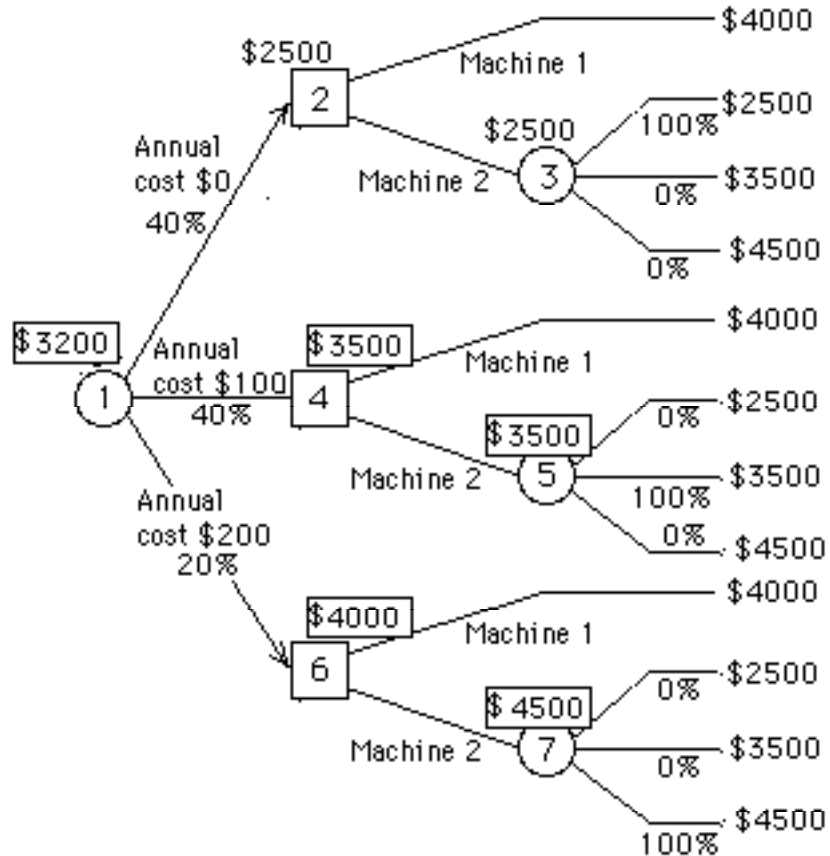_____ 4. Which machine should the department buy?

a. Machine #1	c. a "toss-up"

b. Machine #2   d. *NOTA*

Suppose that the department were able to get "perfect information", i.e., a perfect prediction of the annual maintenance costs. Based upon this supposition, the department head drew the decision tree below:

Fill the blank boxes in the decision tree above and answer the following questions:

__c__ 5. What is the minimum expected cost to the department if they were able to obtain this perfect prediction?

a. $ 2600   c. $ 3200   e. $ 4500
b. $ 2900   d. $ 4000   f. *NOTA*

__b__ 6. What is EVPI (expected value of perfect information)?

a. $ 70   c. $ 150   e. $ 300
b. $ 100   d. $ 200   f. *NOTA*

C.3. Simplex Algorithm for LP: At an intermediate step of the simplex algorithm, the tableau is:

^-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS
1	2	0	0	0	0	-3	1	-2	-15
0	-4	2	1	0	0	0	1	1	4
0	1	-1	0	1	0	-1	0	-1	2
0	2	0	0	0	1	-3	3	2	6

1. What are the basic variables for this tableau? (*circle*: X_1 X_2 **X_3 X_4 X_5** X_6 X_7 X_8 RHS)

2. What are the current values of the variables?

X_1 = _0_, X_2 = _0_, X_3 = _4_ , X_4 = _2_ , X_5 = _6_ , X_6 = _0_, X_7 = _0_, X_8 = _0_

3. Increasing X_1 would (*circle*: **increase** / decrease) the objective function.

4. Increasing X_6 would (circle: increase / **decrease**) the objective function.

5. What is the substitution rate of X_1 for X_3? ___-4___

That is, if X_1 is increased by 1 unit, X_3 (*circle*: **increases** / decreases) by a quantity _4_ .

6. Suppose that X_6 and X_7 are slack variables in the first 2 constraints, and X_8 a surplus variable in the the last constraint. (That is, the first two constraints were originally   constraints, and the third was originally a   constraint, all converted to equations.) What are the values of the simplex multipliers for this tableau? Π_1 = ___+3_ , Π_2 = ___-1_ , Π_3 = ___-2_

7. If the objective is to (*circle:* maximize / **minimize**) the objective z, the optimal solution is unbounded.

8. If the objective is <u>not</u> that which you specified in (7), perform a pivot to improve the objective function, and write the new tableau below. *(You need only complete the unshaded portion.)*

If the objective is the maximize, then either X_1 or X_7 could be selected to enter the basis.

If X_1 is entered into the basis:

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS
			0	-2	0				-19
			1	4	0				
			0	1	0				
			0	-2	1				

If X_7 is entered into the basis:

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS
			-1	0	0				-19
			1	0	0				
			0	1	0				
			-3	0	1				