56:171

Fall 2002

Operations Research

Quizzes with Solutions

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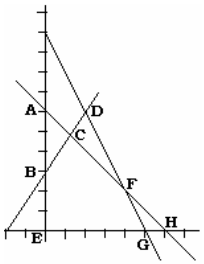
Dept. of Mechanical & Industrial Engineering

Note: In most cases, each quiz is available in several versions!

Section A or B (circle) Name ________________________

56:171 Operations Research
Quiz #1 – 6 September 2002

Consider the following LP:



Minimize $3X_1 + 2X_2$

subject to (1) $2X_1 + X_2 \geq 10$

(2) $-3X_1 + 2X_2 \leq 6$

(3) $X_1 + X_2 \geq 6$

$X_1 \geq 0 \,\&\, X_2 \geq 0$

1. The feasible region has ___ corner points, namely ____________________.
2. At point **F**, the slack (or surplus) variable for constraint # _____ is positive. *(If more than one such variable is positive, only one is required.)*
3. The optimal solution is at point ________

Note: For your convenience, the (X_1, X_2) coordinates of the points labeled above are:

Point	A	B	C	D	E	F	G	H
X_1	0	0	4	2	0	1.2	5	6
X_2	6	3	2	6	0	4.8	0	0

4. Which of the three matrices below (each of which are *row-equivalent* to A) is the result of a "pivot" in matrix A? *(If more than one answer is correct, only one answer is required.)* ________

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -2 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 2 & 1 \\ 0 & 3 & 3 \end{bmatrix}, C = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 1/2 & 1 & 1/2 \\ -1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 2 & 1 \\ -3 & -3 & 0 \end{bmatrix}$$

___5. Which method of solving a system of linear equations requires more row operations?
a. Gauss elimination b. Gauss-Jordan elimination c. Both require same number

56:171 Operations Research
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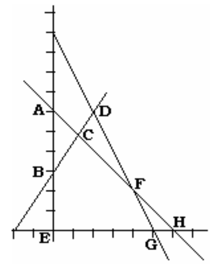
Consider the following LP:



Minimize $3X_1 + 2X_2$

subject to (1) $2X_1 + X_2 \geq 10$

(2) $-3X_1 + 2X_2 \leq 6$

(3) $X_1 + X_2 \geq 6$

$X_1 \geq 0 \,\&\, X_2 \geq 0$

1. The feasible region has _3_ corner points, namely _D, F, & H____.
2. At point **F**, the slack (or surplus) variable for constraint # __2___ is positive. *(If more than one such variable is positive, only one is required.)*
3. The optimal solution is at point __F______

Note: For your convenience, the (X_1, X_2) coordinates of the points labeled above are:

Point	A	B	C	D	E	F	G	H
X_1	0	0	4	2	0	1.2	5	6
X_2	6	3	2	6	0	4.8	0	0
Obj.				18		13.2		18

4. Which of the three matrices below (each of which are *row-equivalent* to A) is the result of a "pivot" in matrix A? *(If more than one answer is correct, only one answer is required.)* __B__

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -2 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 2 & 1 \\ 0 & 3 & 3 \end{bmatrix}, C = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 1/2 & 1 & 1 \\ -1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 2 & 1 \\ -3 & -3 & 0 \end{bmatrix}$$

_b_5. Which method of solving a system of linear equations requires more row operations?
a. Gauss elimination b. Gauss-Jordan elimination c. Both require same number

Section A or B (circle)   Name ___________________________

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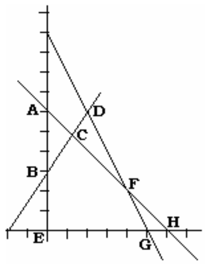
Consider the following LP:



Maximize $3X_1 + 2X_2$

subject to
(1) $2X_1 + X_2 \le 10$
(2) $-3X_1 + 2X_2 \le 6$
(3) $X_1 + X_2 \ge 6$
$X_1 \ge 0 \,\&\, X_2 \ge 0$

1. The feasible region has ___ corner points, namely ______________________.
2. At point **F**, the slack (or surplus) variable for constraint # _____ is positive. *(If more than one such variable is positive, only one is required.)*
3. The optimal solution is at point ________

Note: For your convenience, the (X_1, X_2) coordinates of the points labeled above are:

Point	A	B	C	D	E	F	G	H
X_1	0	0	4	2	0	1.2	5	6
X_2	6	3	2	6	0	4.8	0	0

4. Which of the three matrices below (each of which are *row-equivalent* to A) is the result of a "pivot" in matrix A? *(If more than one answer is correct, only one answer is required.)* ________

$$A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 2 & 1 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 3 & 0 \\ 1 & -2 & -1 \\ 0 & 3 & -1 \end{bmatrix}, C = \begin{bmatrix} 3/2 & 0 & -3/2 \\ -1/2 & 1 & 1/2 \\ -3/2 & 0 & 1/2 \end{bmatrix}, D = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

___5. Which method of solving a system of linear equations requires more row operations?
a. Gauss elimination b. Gauss-Jordan elimination c. Both require same number

56:171 Operations Research
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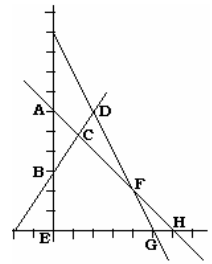
Consider the following LP:



Maximize $3X_1 + 2X_2$

subject to
(1) $2X_1 + X_2 \le 10$
(2) $-3X_1 + 2X_2 \le 6$
(3) $X_1 + X_2 \ge 6$
$X_1 \ge 0 \,\&\, X_2 \ge 0$

1. The feasible region has _3__ corner points, namely __C, D, & F____.
2. At point **F**, the slack (or surplus) variable for constraint # __2___ is positive. *(If more than one such variable is positive, only one is required.)*
3. The optimal solution is at point __D___

Note: For your convenience, the (X_1, X_2) coordinates of the points labeled above are:

Point	A	B	C	D	E	F	G	H
X_1	0	0	4	2	0	1.2	5	6
X_2	6	3	2	6	0	4.8	0	0
Obj.			16	18		13.2		

4. Which of the three matrices below (each of which are *row-equivalent* to A) is the result of a "pivot" in matrix A? *(If more than one answer is correct, only one answer is required.)* __C___

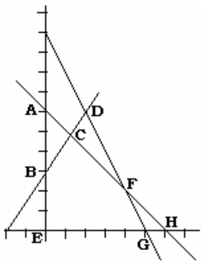
$$A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 2 & 1 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 3 & 0 \\ 1 & -2 & -1 \\ 0 & 3 & -1 \end{bmatrix}, C = \begin{bmatrix} 3/2 & 0 & -3/2 \\ -1/2 & 1 & 1/2 \\ -3/2 & 0 & 1/2 \end{bmatrix}, D = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

_b_5. Which method of solving a system of linear equations requires more row operations?
a. Gauss elimination b. Gauss-Jordan elimination c. Both require same number

Section A or B (circle) Name ____________________

56:171 Operations Research
Quiz #1 – 6 September 2002

Consider the following LP:



$$\begin{aligned} &\text{Maximize } 3X_1 + 2X_2 \\ &\text{subject to } \quad (1) \quad 2X_1 + X_2 \le 10 \\ &\qquad\qquad\quad (2) \quad -3X_1 + 2X_2 \le 6 \\ &\qquad\qquad\quad (3) \quad X_1 + X_2 \le 6 \\ &\qquad\qquad\qquad\quad X_1 \ge 0 \,\&\, X_2 \ge 0 \end{aligned}$$

1. The feasible region has ___ corner points, namely ______________________.
2. At point **C**, the slack (or surplus) variable for constraint # _____ is positive. *(If more than one such variable is positive, only one is required.)*
3. The optimal solution is at point ________

Note: For your convenience, the (X_1, X_2) coordinates of the points labeled above are:

Point	A	B	C	D	E	F	G	H
X_1	0	0	4	2	0	1.2	5	6
X_2	6	3	2	6	0	4.8	0	0

4. Which of the three matrices below (each of which are *row-equivalent* to A) is the result of a "pivot" in matrix A? *(If more than one answer is correct, only one answer is required.)* ________

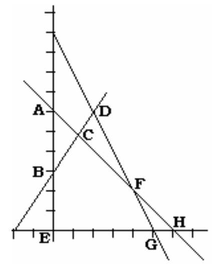
$$A = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 2 & 1 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 & 0 \\ 1 & -2 & -1 \\ 0 & -3 & -1 \end{bmatrix}, C = \begin{bmatrix} 5/2 & 0 & -3/2 \\ -1/2 & 1 & 1/2 \\ -4 & 0 & 2 \end{bmatrix}, D = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

___5. Which method of solving a system of linear equations requires more row operations?
a. Gauss elimination b. Gauss-Jordan elimination c. Both require same number

Version C Solutions

56:171 Operations Research
Quiz #1 Solutions – 6 September 2002

Consider the following LP:



$$\begin{aligned} &\text{Maximize } 3X_1 + 2X_2 \\ &\text{subject to } \quad (1) \quad 2X_1 + X_2 \le 10 \\ &\qquad\qquad\quad (2) \quad -3X_1 + 2X_2 \le 6 \\ &\qquad\qquad\quad (3) \quad X_1 + X_2 \le 6 \\ &\qquad\qquad\qquad\quad X_1 \ge 0 \,\&\, X_2 \ge 0 \end{aligned}$$

1. The feasible region has _5_ corner points, namely _B, C, E, F & G ____
2. At point **C**, the slack (or surplus) variable for constraint # __1_ is positive. *(If more than one such variable is positive, only one is required.)*
3. The optimal solution is at point ___C___

Note: For your convenience, the (X_1, X_2) coordinates of the points labeled above are:

Point	A	B	C	D	E	F	G	H
X_1	0	0	4	2	0	1.2	5	6
X_2	6	3	2	6	0	4.8	0	0
Obj.		6	16		0	13.2	15	

4. Which of the three matrices below (each of which are *row-equivalent* to A) is the result of a "pivot" in matrix A? *(If more than one answer is correct, only one answer is required.)* ___D___

$$A = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 2 & 1 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 & 0 \\ 1 & -2 & -1 \\ 0 & -3 & -1 \end{bmatrix}, C = \begin{bmatrix} 5/2 & 0 & -3/2 \\ -1/2 & 1 & 1/2 \\ -4 & 0 & 2 \end{bmatrix}, D = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

_b_5. Which method of solving a system of linear equations requires more row operations?
a. Gauss elimination b. Gauss-Jordan elimination c. Both require same number

56:171 Operations Research
Quiz #2 (version A) Solution—Fall 2002

Part I. For each statement, indicate "+"=**true** or "o"=**false**.

+ 1. A "pivot" in a nonbasic column of a tableau will make it a basic column.
O 2. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *cannot* pivot in row i.
O 3. A "pivot" in row i of the column for variable X_j will increase the number of basic variables.
+ 4. A basic solution of the problem "maximize cx subject to Ax≤b, x≥0" corresponds to a corner of the feasible region.
+ 5. In a basic LP solution, the nonbasic variables equal zero.

Part II. Below are several simplex tableaus. Assume that the objective in each case is to be **maximized**. Classify each tableau by writing to the right of the tableau a letter **A** through **F**, according to the descriptions below. *Also circle the pivot element when specified.*

(**A**) Nonoptimal, nondegenerate tableau with bounded solution. *Circle a pivot element which would improve the objective.*
(**B**) Nonoptimal, degenerate tableau with bounded solution. *Circle an appropriate pivot element.*
(**C**) Unique optimum.
(**D**) Optimal tableau, with alternate optimum. *Circle a pivot element which would lead to another optimal basic solution.*
(**E**) Objective unbounded (above).
(**F**) Tableau with infeasible basic solution.

(1)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	0	0	1	3	0	0	2	2	-36	
	0	3	0	4	0	0	1	$\boxed{3}$	0	9	__B____
	0	-1	1	-2	$\boxed{5}$	0	0	-2	1	4	(4 correct pivots)
	0	6	0	$\boxed{3}$	-2	1	0	-4	$\boxed{3}$	0	

(2)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	-3	0	-1	-3	0	0	0	-2	-36	
	0	3	0	4	0	0	1	$\boxed{3}$	0	9	__D_____
	0	6	0	3	-2	1	0	-4	3	5	
	0	-1	1	-2	-5	0	0	-2	1	4	

(3)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	-3	0	1	3	0	0	-2	-2	-36	
	0	3	0	4	0	0	1	3	0	9	__E_____
	0	-1	1	-2	-5	0	0	-2	1	4	(variable X4→∞)
	0	6	0	3	-2	1	0	-4	3	5	

(4)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	3	0	-1	3	0	0	2	-2	-36	
	0	3	0	4	1	0	1	$\boxed{3}$	0	9	__A_____
	0	-1	1	2	$\boxed{5}$	0	0	-2	1	2	(3 correct pivots)
	0	$\boxed{6}$	0	3	-2	1	0	-4	3	5	

(5)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	-3	0	-1	-3	0	0	-2	-2	-36	
	0	3	0	4	1	0	1	3	0	9	__F_____
	0	-1	1	-2	-5	0	0	-2	1	-4	
	0	6	0	3	2	1	0	-4	3	5	

56:171 Operations Research
Quiz #2 (version B) Solution—Fall 2002

Part I. For each statement, indicate "+"=**true** or "o"=**false**.

O 1. It may happen that an LP problem has (exactly) two optimal solutions.
O 2. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *must* pivot in row i.
\+ 3. A "pivot" in the simplex method corresponds to a move from one corner point of the feasible region to another.
\+ 4. In the simplex method, every variable of the LP is either basic or nonbasic.
\+ 5. If there is a tie in the "minimum-ratio test" of the simplex method, the next basic solution will be degenerate, i.e., one of the basic variables will be zero.

Part II. Below are several simplex tableaus. Assume that the objective in each case is to be **maximized**. Classify each tableau by writing to the right of the tableau a letter **A** through **F**, according to the descriptions below. *Also circle the pivot element when specified.*

(**A**) Optimal tableau, with alternate optimum. *Circle a pivot element which would lead to another optimal basic solution.*
(**B**) Objective unbounded (above).
(**C**) Nonoptimal, nondegenerate tableau with bounded solution. *Circle a pivot element which would improve the objective.*
(**D**) Nonoptimal, degenerate tableau with bounded solution. *Circle an appropriate pivot element.*
(**E**) Unique optimum.
(**F**) Tableau with infeasible basic solution.

(1)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	-3	0	-1	-3	0	0	-2	-2	-36	
	0	3	0	4	1	0	1	3	0	9	__F_____
	0	-1	1	-2	-5	0	0	-2	1	-4	
	0	6	0	3	2	1	0	-4	3	5	

(2)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	0	0	1	3	0	0	2	2	-36	
	0	3	0	4	0	0	1	$\boxed{3}$	0	9	__D_____
	0	-1	1	-2	$\boxed{5}$	0	0	-2	1	4	
	0	6	0	$\boxed{3}$	-2	1	0	-4	$\boxed{3}$	0	

(3)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	-3	0	-1	-3	0	0	0	-2	-36	
	0	3	0	4	0	0	1	$\boxed{3}$	0	9	__A_____
	0	6	0	3	-2	1	0	-4	3	5	
	0	-1	1	-2	-5	0	0	-2	1	4	

(4)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	-3	0	1	3	0	0	-2	-2	-36	
	0	3	0	4	0	0	1	3	0	9	__B ___
	0	-1	1	-2	-5	0	0	-2	1	4	(X4$\rightarrow\infty$)
	0	6	0	3	-2	1	0	-4	3	5	

(5)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	3	0	-1	3	0	0	2	-2	-36	
	0	3	0	4	1	0	1	$\boxed{3}$	0	9	__C_____
	0	-1	1	2	$\boxed{5}$	0	0	-2	1	2	
	0	$\boxed{6}$	0	3	-2	1	0	-4	3	5	

56:171 Operations Research
Quiz #2 (version C) Solution –Fall 2002

Part I. For each statement, indicate "+"=**true** or "o"=**false**.

O 1. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *cannot* pivot in row i.
\+ 2. A basic solution of the problem "maximize cx subject to Ax≤b, x≥0" corresponds to a corner of the feasible region.
\+ 3. In a basic LP solution, the nonbasic variables equal zero.
O 4. The "minimum ratio test" is used to select the pivot column in the simplex method for linear programming.
\+ 5. In the simplex tableau, all rows, including the objective row, are written in the form of equations.

Part II. Below are several simplex tableaus. Assume that the objective in each case is to be **maximized**. Classify each tableau by writing to the right of the tableau a letter **A** through **F**, according to the descriptions below. *Also circle the pivot element when specified.*

(**A**) Nonoptimal, nondegenerate tableau with bounded solution. *Circle a pivot element which would improve the objective.*
(**B**) Nonoptimal, degenerate tableau with bounded solution. *Circle an appropriate pivot element.*
(**C**) Unique optimum.
(**D**) Optimal tableau, with alternate optimum. *Circle a pivot element which would lead to another optimal basic solution.*
(**E**) Objective unbounded (above).
(F) Tableau with infeasible basic solution.

(1)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	-3	0	1	3	0	0	-2	-2	-36	
	0	3	0	4	0	0	1	3	0	9	_E _______
	0	-1	1	-2	-5	0	0	-2	1	4	(variable X4→∞)
	0	6	0	3	-2	1	0	-4	3	5	

(2)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	3	0	-1	3	0	0	2	-2	-36	
	0	3	0	4	1	0	1	$\boxed{3}$	0	9	_A_______
	0	-1	1	2	$\boxed{5}$	0	0	-2	1	2	(3 correct pivots)
	0	$\boxed{6}$	0	3	-2	1	0	-4	$\boxed{3}$	5	

(3)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	0	0	1	3	0	0	2	2	-36	
	0	3	0	4	0	0	1	$\boxed{3}$	0	9	_B_______
	0	-1	1	-2	$\boxed{5}$	0	0	-2	1	4	(4 correct pivots)
	0	6	0	$\boxed{3}$	-2	1	0	-4	3	0	

(4)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	-3	0	-1	-3	0	0	-2	-2	-36	
	0	3	0	4	1	0	1	3	0	9	_F_______
	0	-1	1	-2	-5	0	0	-2	1	-4	
	0	6	0	3	2	1	0	-4	3	5	

(5)	-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
max	1	-3	0	-1	-3	0	0	0	-2	-36	
	0	3	0	4	0	0	1	$\boxed{3}$	0	9	_D_______
	0	6	0	3	-2	1	0	-4	3	5	
	0	-1	1	-2	-5	0	0	-2	1	4	

56:171 Operations Research
Quiz #3 Solutoins -- 20 September 2002

Part I. For each statement, indicate "+"=**true** or "o"=**false**.

__o__ a. When you enter an LP formulation into LINDO, you must first convert all inequalities to equations.

__o__ b. Unlike the ordinary simplex method, the "Revised Simplex Method" never requires the use of artificial variables.

__+__ c. Whether an LP is a minimization or a maximization problem, the first phase of the two-phase method is exactly the same.

__o__ d. At the beginning of the *first* phase of the two-phase simplex method, the phase-one objective function will have the value 0.

__+__ e. At the end of the *first* phase of the two-phase simplex method, the phase-one objective function must be zero if the LP is feasible.

__o__ f. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *must* pivot in row i.

__+__g. If an LP model has constraints of the form $Ax \leq b$, $x \geq 0$, and b is nonnegative, then there is no need for artificial variables.

__o__ h. If a zero appears on the right-hand-side of row *i* of an LP tableau, then at the next iteration you *cannot* pivot in row *i*.

__o__ i. Every variable in the "primal" problem has a corresponding dual variable.

__o__ j. The *primal* LP is a *minimization* problem, whereas the *dual* problem is a *maximization* problem.

__+__k. If the slack or surplus variable in a constraint is positive, then the corresponding dual variable must be zero.

__+__ l. If the right-hand-side of constraint *i* in the LP problem "Minimize cx st $Ax \leq b$, $x \geq 0$" increases, then the optimal value must either decrease or remain unchanged.

__o__ m. If the right-hand-side of constraint *i* in the LP problem "Maximize cx st $A \leq b$, $x \geq 0$" increases, then the optimal value must either decrease or remain unchanged.

__o__ n. The revised simplex method usually requires fewer iterations than the ordinary simplex method.

__+__ o. The simplex multipliers at the termination of the revised simplex method are always feasible in the dual LP of the problem being solved.

__+__ p. In the two-phase method, the first phase finds a basic feasible solution to the LP being solved, while the second phase finds the optimal solution.

__+__ q. The original objective function is ignored during phase one of the two-phase method.

__+__ r. If a zero appears in row *i* of the column of substitution rates in the pivot column, then then row *i* cannot be the pivot row.

Part II. Sensitivity analysis using LINDO.

Ken and Larry, Inc., supplies its ice cream parlors with three flavors of ice cream: chocolate, vanilla, and banana. Because of extremely hot weather and a high demand for its products, the company has run short of its supply of ingredients: milk, sugar, & cream. Hence, they will not be able to fill all the orders received from their retail outlets, the ice cream parlors. Owing to these circumstances, the company has decided to choose the amount of each product to produce that will maximize total profit, given the constraints on supply of the basic ingredients.

The chocolate, vanilla, and banana flavors generate, respectively, \$1.00, \$0.90, and \$0.95 per profit per gallon sold. The company has only 200 gallons of milk, 150 pounds of sugar, and 60 gallons of cream left in its inventory. The LP formulation for this problem has variables C, V, and B representing gallons of chocolate, vanilla, and banana ice cream produced, respectively.

```
MAXIMIZE  C+0.9V+0.95B
ST

0.45C + 0.50V + 0.40B <= 200  ! milk resource
0.50C + 0.40V + 0.40B <= 150  ! sugar resource
0.10C + 0.15V + 0.20B <= 60   ! cream resource

END
```

```
        OBJECTIVE FUNCTION VALUE
        1)      341.2500

  VARIABLE        VALUE          REDUCED COST
         C         0.000000          0.037500
         V       300.000000          0.000000
         B        75.000000          0.000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)        20.000000          0.000000
        3)         0.000000          1.875000
        4)         0.000000          1.000000
```

```
RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
        C        1.000000         0.037500         INFINITY
        V        0.900000         0.050000         0.012500
        B        0.950000         0.021429         0.050000

                           RIGHTHAND SIDE RANGES
      ROW         CURRENT        ALLOWABLE        ALLOWABLE
                    RHS          INCREASE         DECREASE
        2      200.000000         INFINITY        20.000000
        3      150.000000        10.000000        30.000000
        4       60.000000        15.000000         3.750000
```

True/False (+ or O):

__+__ 1. If the profit per gallon of chocolate increases to $1.02, the basis and the values of the basic variables will be unchanged.

__o__ 2. If the profit per gallon of vanilla drops to $0.88, the basis and the values of the basic variables will be unchanged.

Multiple choice: *(**NSI** = "not sufficient information")*

_d_ 3. If the amount of cream available were to increase to 65 gallons, the increase in profit will be *(choose nearest value):*

a. $0.00 b. $0.50 c. $1 d. $5 e. $10 f. *NSI*

_a_ 4. If the amount of milk available were to increase to 225 gallons, the increase in profit will be *(choose nearest value):*

a. $0.00 b. $0.50 c. $1 d. $5 e. $10 f. *NSI*

_e_ 5. If the profit per gallon of banana ice cream were to drop to $0.93 per gallon, the loss in total profit would be *(choose nearest value):*

a. $0.00 b. $0.50 c. $1 d. $5 e. $10 *($15)* f. *NSI*

56:171 Operations Research
Quiz #4 -- 27 September 2002

"A manufacturer produces two types of plastic cladding. These have the trade names Ankalor and Beslite.

- One yard of Ankalor requires 8 lb of polyamine, 2.5 lb of diurethane and 2 lb of monomer.
- A yard of Beslite needs 10 lb of polyamine, 1 lb of diurethane, and 4 lb of monomer.
- The company has in stock 80,000 lb of polyamine, 20,000 lb of diurethane, and 30,000 lb of monomer.
- Both plastics can be produced by alternate parameter settings of the production plant, which is able to produce cladding at the rate of 12 yards per hour.
- A total of 750 production plant hours are available for the next planning period.
- The contribution to profit on Ankalor is \$10/yard and on Beslite is \$20/yard.
- The company has a contract to deliver at least 3,000 yards of Ankalor.

What production plan should be implemented in order to maximize the contribution to the firm's profit from this product division."

Definition of variables: A = Number of yards of Ankalor produced
B = Number of yards of Beslite produced

LP model:

1)	Maximize 10 A + 20 B	subject to	
2)	8 A + 10 B	≤80,000	(lbs. Polyamine available)
3)	2.5 A + 1 B	≤ 20,000	(lbs. Diurethane available)
4)	2 A + 4 B	≤ 30,000	(lbs. Monomer available)
5)	A + B	≤ 9,000	(lbs. Plant capacity)
6)	A	≥ 3,000	(Contract)
	A ≥ 0, B ≥ 0		

The LINDO solution is:

```
        OBJECTIVE FUNCTION VALUE
            1)   142000.000
        VARIABLE        VALUE     REDUCED COST
            A          3000.000       0.000
            B          5600.000       0.000

       ROW        SLACK OR SURPLUS  DUAL PRICES
        2)                 0.000          2.000
        3)              6900.000          0.000
        4)              1600.000          0.000
        5)               400.000          0.000
        6)                 0.000         -6.000

 RANGES IN WHCH THE BASIS IS UNCHANGED
                               OBJ COEFFICIENT RANGES
     VARIABLE     CURRENT    ALLOWABLE   ALLOWABLE
                   COEF      INCREASE    DECREASE
        A         10.000       6.000    INFINITY
        B         20.000     INFINITY      7.500

                                RIGHTHAND SIDE RANGES
     ROW          CURRENT        ALLOWABLE    ALLOWABLE
                    RHS          INCREASE      DECREASE
       2         80000.000       4000.000     56000.000
       3         20000.000       INFINITY      6900.000
       4         30000.000       INFINITY      1600.000
       5          9000.000       INFINITY       400.000
       6          3000.000       2000.000      1333.333
```

```
THE TABLEAU
 ROW (BASIS)    A      B     SLK 2 SLK 3  SLK 4  SLK 5   SLK 6     RHS
  1    ART     .00    .00    2.00   .00    .00    .00    6.00      0.14E+06
  2    B       .00   1.00     .10   .00    .00    .00     .80    5600.00
  3    SLK 3   .00    .00    -.10  1.00    .00    .00    1.70    6900.00
  4    SLK 4   .00    .00    -.40   .00   1.00    .00   -1.20    1600.00
  5    SLK 5   .00    .00    -.10   .00    .00   1.00     .20     400.00
  6    A      1.00    .00     .00   .00    .00    .00   -1.00    3000.00
```

*Consult the LINDO output above to answer the following questions. If there is not sufficient information in the LINDO output, answer "**NSI**".*

_b_1. Suppose that the company can purchase 2000 pounds of additional polyamine for $2.50 per pound. Should they make the purchase? a. yes $\boxed{\text{b. no}}$ c. *NSI*

(since the dual variable for row [2] is only $2.00/lb.)

_a_2. Regardless of your answer in (4), suppose that they do purchase 2000 pounds of additional polyamine. Increasing the quantity of polyamine used in the model above is equivalent to

$\boxed{\text{a. decreasing the slack in row 2 by 2000}}$ d. decreasing surplus in row 2 by 2000
b. increasing the surplus in row 2 by 2000 e. none of the above
c. increasing the slack in row 2 by 2000 f. NSI

(since 8A+10B+SLK2 = 80000 & 8A+10B=82000 $\Rightarrow$ SLK2 = –2000.)

_c_3. If the company purchases 2000 pounds of additional polyamine, what is the total amount of Ankelor that they should deliver? (Choose nearest value!)

a. 2800 yards $\boxed{\text{c. 3000 yards}}$ *unchanged!* e. 3200 yards
b. 2900 yards d. 3100 yards f. *NSI*

(substitution rate of SLK2 for A is 0, so A is unchanged as SLK2 decreases, up to ALLOWABLE INCREASE for RHS of row#2, i.e., 4000.)

_d_4. If the company purchases 2000 pounds of additional polyamine, what is the total amount of Beslite that they should deliver? (Choose nearest value!)

a. 5500 yards c. 5700 yards e. 5900 yards
b. 5600 yards d. 5800 yards f. *NSI*

(substitution rate of SLK2 for B is 0.10, so B increases by 0.10 for each pound decrease in SLK2)

_c_5. How will the decision to purchase 2000 pounds of additional polyamine change the quantity of diurethane used during the next planning period?

a. increase by 100 pounds $\boxed{\text{c. increase by 200 pounds}}$ e. none of the above
b. decrease by 100 pounds d. decrease by 200 pounds f. *NSI*

(substitution rate of SLK2 for SLK3 is -0.10, so SLK3 decreases by 0.10lb for each pound decrease in SLK2 $\Rightarrow$ 2.5A+1B *(i.e., amount of diurethane used) increases by 200 pounds.)*

_b_6. If the profit contribution from Beslite were to decrease from $20 to $13/yard, will the optimal values of A &/or B change? a. yes $\boxed{\text{b. no}}$ c. *NSI*

(Since decrease of $7 is less than ALLOWABLE DECREASE which is $7.50.)

_a_7. If the profit contribution from Ankelor were to increase from $10 to $17/yard, will the optimal values of A &/or B change? $\boxed{\text{a. yes}}$ b. no c. *NSI*

(Since increase of $7 is greater than ALLOWABLE INCREASE which is $6.)

_a_8. Suppose that the company could deliver 1000 yards less than the contracted amount of Ankalor if they were to pay a penalty of $5/yard shortage.

Should they do so? $\boxed{\text{a. yes}}$ b. no c. *NSI*

(Since dual variable for row [6] is –$6.00, profit would decrease by $6 for every pound increase, or decrease by $6 for every pound decrease in requirements.)

_b_9. Is the optimal solution of this LP *degenerate*? a. yes $\boxed{\text{b. no}}$ c. *NSI*

(No zero appears on RHS of optimal tableau.)

_b_10. Are there multiple optimal solutions of this LP? a. yes $\boxed{\text{b. no}}$ c. *NSI*

(No zero appears in objective row (1) of any nonbasic column of tableau.)

56:171 Operations Research
Quiz #5 Version **A** Solution -- Fall 2002

Part 1. Transportation Simplex Method.

Consider the feasible solution of the transportation problem below:

dstn→ ↓source	1	2	3	4	5	Supply
A	|12 **0 & basic**	|8 **7**	|9	|10	|11 **2**	9
B	|10	|11 *nonbasic*	|12	|11	|14 **7**	7
C	|9 **4**	|7	|11 *nonbasic*	|14 **1**	|8	5
D	|13 *nonbasic*	|12	|13 **2**	|12 **5**	|12	7
E	|8 *nonbasic*	|9	|10 **3**	|9	|10	3
Demand=	4	7	5	6	9	

_a_1. Which additional variable (=0) of those below might be made a basic variable in order to complete the (degenerate) basis?
a. X_{A1} c. X_{B2} e. X_{E1}
b. X_{D1} d. X_{C3} f. None of above

_d_2. If we choose to assign dual variable U_D=5, what must be the value of dual variable V_3?
a. 0 c. 6 e. 11
b. 3 d. 8 f. None of above

_e_3. If, in addition to U_D=5, we determine that V_2= −2 , then the reduced cost of X_{D2} is
a. 0 c. +5 e. +9
b. −3 d. −8 f. None of above

_a_4. If we initially assign U_D=0 (rather than U_D=5) and then compute the remaining dual variables, a different value of the reduced cost of X_{D2} will be obtained, although its sign will remain the same.
a. True b. False c. Cannot be determined

_e_5. Suppose $\mathbf{X}_{E1}$ enters the basis (ignoring whether it would improve the solution or not). Which variable must leave the basis?
a. $\mathbf{X}_{A2}$ c. $\mathbf{X}_{C4}$ e. $\mathbf{X}_{E3}$
b. $\mathbf{X}_{C1}$ d. $\mathbf{X}_{D3}$ f. None of above

Part 2: Assignment Problem

Consider the problem of assigning machines (rows) to jobs (columns), with cost matrix

	1	2	3	4	5
1	**0**	**2**	**0**	**1**	**0**
2	2	**2**	**0**	6	3
3	4	**0**	**5**	5	2
4	7	**0**	**5**	3	4
5	**0**	**2**	**0**	**0**	**4**

By drawing lines in rows 1&5 and in columns 2&3, all the nine zeroes are "covered".

_c_ 1. After the next step of the Hungarian method, the number of zeroes in the cost matrix will be
a. 7 c. 9 e. 11
b. 8 d. 10 f. None of the above

Consider the reduced assignment problem shown below:

\	1	2	3	4	5
1	0	0	3	1	3
2	1	2	0	2	0
3	4	3	5	5	0
4	1	0	5	3	5
5	0	2	1	0	11

_a_ 2. A zero-cost solution of the problem with this cost matrix must have
a. $X_{11}=1$ c. $X_{12}=1$ e. $X_{54}=0$
b. $X_{42}=0$ d. $X_{25}=1$ f. None of the above

Part 3. True(+) or False(o) ?

_+_ 1. The transportation problem is a special case of a linear programming problem.
_o_2. The Hungarian algorithm can be used to solve a transportation problem.
_+_3. Every basic feasible solution of an assignment problem must be degenerate.
_o_ 4. If the current basic solution of a transportation problem is degenerate, no improvement of the objective function will occur in the next iteration of the (transportation) simplex method.
_o_ 5. The Hungarian algorithm is the simplex method specialized to the assignment problem.
_+_6. At each iteration of the Hungarian method, the original cost matrix is replaced with a new cost matrix having the same optimal assignment.

56:171 Operations Research
Quiz #5 Version B -- Fall 2002

Part 1. Transportation Simplex Method.

Consider the feasible solution of the transportation problem below:

dstn→ ↓source	1	2	3	4	5	Supply
A	12 **0 & basic**	8 **7**	9	10	11 **2**	9
B	10	11 *nonbasic*	12	11	14 **7**	7
C	9 **4**	7	11 *nonbasic*	14 **1**	8	5
D	13 *nonbasic*	12	13 **2**	12 **5**	12	7
E	8 *nonbasic*	9	10 **3**	9	10	3
Demand=	4	7	5	6	9	

_c_1. Which additional variable (=0) of those below might be made a basic variable in order to complete the (degenerate) basis?
a. XB2 c. XA1 e. XE1
b. XC3 d. XD1 f. None of above

_d_2. If we choose to assign dual variable U_D=5, what must be the value of dual variable V_3?
a. 0 c. 6 e. 11
b. 3 d. 8 f. None of above

_e_3. If, in addition to U_D=5, we determine that V_2= −2 , then the reduced cost of X_{D2} is
a. 0 c. +5 e. +9
b. −3 d. −8 f. None of above

_a_4. If we initially assign U_D=0 (rather than U_D=5) and then compute the remaining dual variables, a different value of the reduced cost of X_{D2} will be obtained, although its sign will remain the same.
a. True b. False c. Cannot be determined

_e_5. Suppose X_{E1} enters the basis (ignoring whether it would improve the solution or not). Which variable must leave the basis?
a. XC1 c. XA2 e. XE3
b. XC4 d. XD3 f. None of above

Part 2: Assignment Problem

Consider the problem of assigning machines (rows) to jobs (columns), with cost matrix

	1	2	3	4	5
1	**0**	**2**	**0**	**1**	**0**
2	2	**2**	**0**	6	3
3	4	**0**	**5**	5	2
4	7	**0**	**5**	3	4
5	**0**	**2**	**1**	**0**	**4**

By drawing lines in rows 1&5 and in columns 2&3, all the nine zeroes are "covered".

_c_ 1. After the next step of the Hungarian method, the number of zeroes in the cost matrix will be

a. 7   c. 9   e. 11
b. 8   d. 10   f. None of the above

Consider the reduced assignment problem shown below:

```
\| 1  2   3  4  5
1| 0  0  3  1  3
2| 1  2  0  2  0
3| 4  3  5  5  0
4| 1  0  5  3  5
5| 0  2  1  0 11
```

_a_ 2. A zero-cost solution of the problem with this cost matrix must have

a. $X_{11}=1$ c. $X_{12}=1$ e. $X_{54}=0$
b. $X_{42}=0$ d. $X_{25}=1$ f. None of the above

Part 3. True(+) or False(o) ?

_+_ 1. The assignment problem is a special case of an transportation problem.
_o_ 2. The number of basic variables in a n×n assignment problem is 2n.
_o_ 3. The dual variables of the transportation problem are uniquely determined at each iteration of the simplex method.
_+_ 4. The transportation simplex method can be applied to solution of an assignment problem.
_o_ 5. The optimal dual variables of the transportation problem obtained at the final iteration must be nonnegative.
_o_ 6. A "balanced" transportation problem has an equal number of sources and destinations.

56:171 Operations Research
Quiz #5 Version C -- Fall 2002

Part 1. Transportation Simplex Method.

Consider the feasible solution of the transportation problem below:

dstn→ ↓source	1	2	3	4	5	Supply
A	|12 **0 & basic**	|8 **7**	|9	|10	|11 **2**	9
B	|10	|11 *nonbasic*	|12	|11	|14 **7**	7
C	|9 **4**	|7	|11 *nonbasic*	|14 **1**	|8	5
D	|13 *nonbasic*	|12	|13 **2**	|12 **5**	|12	7
E	|8 *nonbasic*	|9	|10 **3**	|9	|10	3
Demand=	4	7	5	6	9	

_d_1. Which additional variable (=0) of those below might be made a basic variable in order to complete the (degenerate) basis?

a. XD1 b. XC3 c. XE1
d XA1 e. XB2 f. None of above

_e_2. If we choose to assign dual variable $U_D=5$, what must be the value of dual variable V_3?

a. 0 b. 6 c. 11
d. 3 e. 8 f. None of above

_c_3. If, in addition to $U_D=5$, we determine that $V_2=-2$, then the reduced cost of XD2 is

a. 0 b. +5 c. +9
d. −3 e. −8 f. None of above

_a_4. If we initially assign $U_D=0$ (rather than $U_D=5$) and then compute the remaining dual variables, a different value of the reduced cost of X_{D2} will be obtained, although its sign will remain the same.

a. True b. False c. Cannot be determined

_c_5. Suppose X_{E1} enters the basis (ignoring whether it would improve the solution or not). Which variable must leave the basis?

a. XA2 b. XC4 c. XE3
d. XC1 e. XD3 f. None of above

Part 2: Assignment Problem

Consider the problem of assigning machines (rows) to jobs (columns), with cost matrix

	1	2	3	4	5
1	**0**	**2**	**0**	**1**	**0**
2	2	**2**	**0**	6	3
3	4	**0**	**5**	5	2
4	7	**0**	**5**	3	4
5	**0**	**2**	**1**	**0**	**4**

By drawing lines in rows 1&5 and in columns 2&3, all the **eight** zeroes are "covered".

_b_ 1. After the next step of the Hungarian method, the number of zeroes in the cost matrix will be

a. 7 b. 9 c. 11
d. 8 e. 10 f. None of the above

Consider the reduced assignment problem shown below:

\\	1	2	3	4	5
1	0	0	3	1	3
2	1	2	0	2	0
3	4	3	5	5	0
4	1	0	5	3	5
5	0	2	1	0	11

_a_ 2. A zero-cost solution of the problem with this cost matrix must have

a. $X_{11}=1$ b. $X_{12}=1$ c. $X_{54}=0$
d. $X_{42}=0$ e. $X_{25}=1$ f. None of the above

Part 3. True(+) or False(o) ?

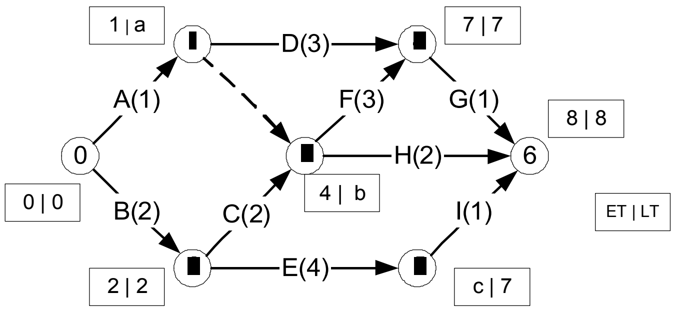
_o_ 1. The Hungarian algorithm can be used to solve a transportation problem.
_o_ 2. The number of basic variables in a n×n assignment problem is 2n.
_o_ 3. If the current basic solution of a transportation problem is degenerate, no improvement of the objective function will occur in the next iteration of the (transportation) simplex method.
_o_ 4. The Hungarian algorithm is the simplex method specialized to the assignment problem.
_+_ 5. At each iteration of the Hungarian method, the original cost matrix is replaced with a new cost matrix having the same optimal assignment.
_+_ 6. A "balanced" transportation problem has an equal number of supply and demand.

56:171 Operations Research
Quiz #6 Solutions – Fall 2002

Part I: ***True***(+) or ***False***(o)?

_+__ 1. The critical path in a project network is the *longest* path from a specified source node (beginning of project) to a specified destination node (end of project).
_o__ 2. There is at most one critical path in a project network.
_o__ 3. The latest times of the events in a project schedule must be computed before the earliest times of those events.
_+__ 4. In PERT, the total completion time of the project is assumed to have a normal distribution.
_+__ 5. All tasks on the critical path of a project schedule have their latest finish time equal to their earliest finish time.
_+__ 6. In the LP formulation of the project scheduling problem, the constraints include
$Y_B - Y_A \geq d_A$ if activity A must precede activity B, where d_A = duration of activity A.
_o__ 7. In CPM, the "forward pass" is used to determine the latest time (LT) for each event (node).
_+__ 8. The A-O-N project network does not require any "dummy" activities (except for the "begin" and "end" activities).
_o__ 9. The PERT method assumes that the completion time of the project has a *beta* distribution.
_o__ 10. A "dummy" activity in an A-O-A project network always has duration zero and cannot be a "critical" activity.
_o(?) 11.The Hungarian algorithm can be used to solve a transportation problem. *(If the supplies & demands are integers, one could, however, replace each source i with Si rows and each destination j with Dj columns, and apply the Hungarian algorithm.)*
_o__ 12.The number of basic variables in a n×n assignment problem is n.
_+__ 13. At each iteration of the Hungarian method, the original cost matrix is replaced with a new cost matrix having the same optimal assignment.
_+__ 14. Every basic feasible solution of an assignment problem must be degenerate.
_+__ 15. In order to apply the Hungarian algorithm, the assignment cost matrix must be square.
_+__ 16. The transportation problem is a special case of a linear programming problem.
_o__ 17. The PERT method assumes that the completion time of the project has a *beta* distribution.
_o__ 18. If the current basic solution of a transportation problem is degenerate, no improvement of the objective function will occur in the next iteration of the (transportation) simplex method.
_o__ 19. The PERT method assumes that the duration of each activity has a *normal* distribution.
_+__ 20. A "dummy" activity in an A-O-A project network always has duration zero.
_o__ 21. At each iteration of the Hungarian method, the number of zeroes in the cost matrix will increase.
_+__ 22. If at some iteration of the Hungarian method, the zeroes of a n×n assignment cost matrix cannot be covered with fewer than n lines, this cost matrix must have a zero-cost assignment.
_+__ 23. Every A-O-A project network has at least one critical path.

Part II: Project Scheduling. Consider the project with the A-O-A (activity-on-arrow) network below. The activity durations are given on the arrows. The Earliest (event) Times (ET) and Latest (event) Times (LT) for each node are written in the box beside each node. Note: There are three different versions, each having different durations of activity A:



1. Complete the labeling of the nodes on the network.
__Note__: The labeling above is one of several such that an arrow always goes from lower-numbered node to a higher-numbered node.

_c_ 2. The number of activities (i.e., tasks), <u>not</u> including "dummies", which are required to complete this project is

a. six	c. eight	e. ten	g.
b. seven	d. nine	f. eleven	h. *NOTA*

_d_ 3. The latest event time (LT) indicated by **a** in the network above is:

a. one	c. three	e. five	g. seven
b. two	d. four	f. six	h. *NOTA*

_d_ 4. The latest event time (LT) indicated by **b** in the network above is:

a. one	c. three	e. five	g. seven
b. two	d. four	f. six	h. *NOTA*

_f_ 5. The earliest event time (ET) indicated by **c** in the network above is:

a. one	c. three	e. five	g. seven
b. two	d. four	f. six	h. *NOTA*

_d_ 6. The slack ("total float") for activity D is: *solution: depends upon ET(1) which depends upon duration of A. For network shown above, answer is 3.*

a. zero	c. two	e. four	g. six
b. one	d. three	f. five	h. *NOTA*

7. Which activities are critical? (*circle:* A $\boxed{B}$ $\boxed{C}$ D E $\boxed{F}$ $\boxed{G}$ H I)

Suppose that the non-zero durations are *random*, with each value in the above network being the *expected* values and each *standard deviation* equal to 1.00. Then...

_c_ 8. The expected earliest completion time for the project is

a. six	c. eight	e. ten	g. twelve
b. seven	d. nine	f. eleven	h. *NOTA*

_e_ 9. The *variance* σ^2 of the earliest completion time for the project is

a. zero	c. two	e. four	g. six
b. one	d. three	f. five	h. *NOTA*

Note: variance of sum of durations of four activities along critical path is sum of variances of those activities.

56:171 Operations Research
Quiz #7 Version **A** Solution – Fall 2002

Part I:

Consider a decision problem whose payoffs are given by the following payoff table:

Decision	A	B
one	80	25
two	30	50
three	60	70
Prior Probability	**0.4**	**0.6**

_c_ 1. Which alternative should be chosen under the maximin payoff criterion?
a. one b. two $\boxed{\text{c. three}}$ d. *NOTA*

_a_ 2. Which alternative should be chosen under the maximax payoff criterion?
$\boxed{\text{a. one}}$ b. two c. three d. *NOTA*

_c_ 3. Which alternative should be chosen under the maximum expected payoff criterion?
a. one b. two $\boxed{\text{c. three}}$ d. *NOTA*

_c_ 4. What will be the entry in the "regret" table for decision **three** & State-of-Nature **A**?
a. zero b. 10 $\boxed{\text{c. 20}}$ d. 30 e. 40 f. *NOTA*

Suppose that you perform an experiment to predict the state of nature (**A** or **B**) above. The experiment has two possible outcomes which we label as **positive** and **negative**. If the state of nature is **A**, there is a 60% probability that the outcome will be **positive**, whereas if the state of nature is **B**, there is a 20% probability that the outcome will be **positive**.

According to Bayes' rule,

$$P\{A \mid positive\} = \frac{P\{\alpha \mid \beta\} P\{\gamma\}}{P\{\delta\}}$$

In this equation, …

_c_ 5. $\alpha =$
a. state of nature is A $\boxed{\text{c. experimental outcome is positive}}$ e. NOTA
b. state of nature is B d. experimental outcome is negative

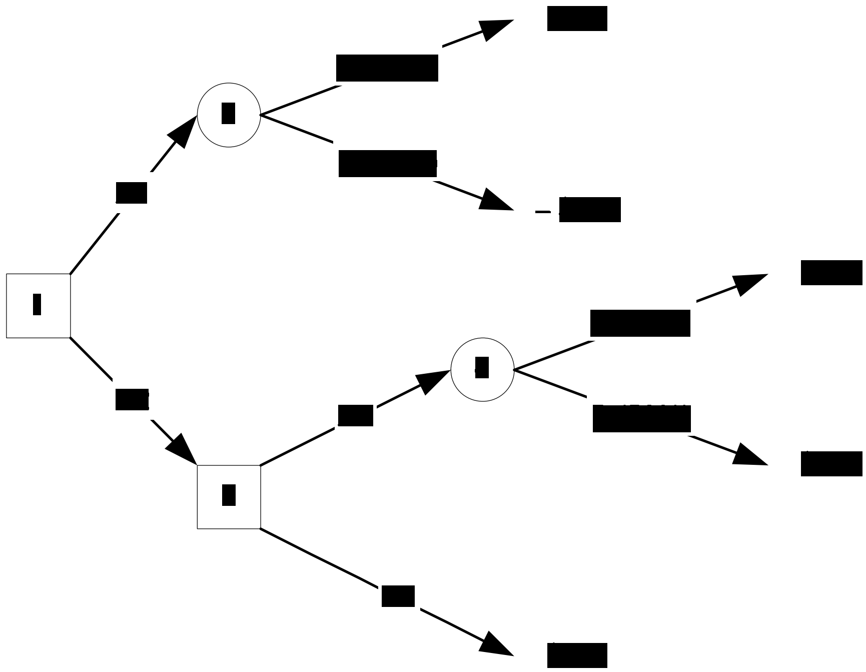
_a_ 6. $\beta =$
$\boxed{\text{a. state of nature is A}}$ c. experimental outcome is positive e. NOTA
b. state of nature is B d. experimental outcome is negative

_a_ 7. $\gamma =$
$\boxed{\text{a. state of nature is A}}$ c. experimental outcome is positive e. NOTA
b. state of nature is B d. experimental outcome is negative

_c_ 8. $\delta =$
a. state of nature is A $\boxed{\text{c. experimental outcome is positive}}$ e. NOTA
b. state of nature is B d. experimental outcome is negative

_c_ 9. Suppose that the outcome of the experiment is **positive**. Then the probability that the state of nature is **A** is revised to … *(choose nearest value)*:
a. 0.5 b. 0.6 $\boxed{\text{c. 0.7}}$ $\approx 2/3$ d. 0.8 e. 0.9 f. *NOTA*

Part II.



Consider the decision tree above.

Fold back the branches and write the values of each node in the table below:

Node	#1	#2	#3	#4
Value	**150**	**140**	**150**	**150**

_b_ 5. What is the optimal decision at node #1?

a. A1 $\boxed{\text{b. A2}}$

56:171 Operations Research
Quiz #7 Version B Solution – Fall 2002

Part I:

Consider a decision problem whose payoffs are given by the following payoff table:

Decision	A	B
one	50	30
two	40	50
three	30	70
Prior Probability	**0.4**	**0.6**

_b_ 1. Which alternative should be chosen under the maximin payoff criterion?
a. one $\quad$ $\boxed{\text{b. two}}$ $\quad$ c. three $\quad$ d. *NOTA*

_c_ 2. Which alternative should be chosen under the maximax payoff criterion?
a. one $\quad$ b. two $\quad$ $\boxed{\text{c. three}}$ $\quad$ d. *NOTA*

_c_ 3. Which alternative should be chosen under the maximum expected payoff criterion?
a. one $\quad$ b. two $\quad$ $\boxed{\text{c. three}}$ $\quad$ d. *NOTA*

_c_ 4. What will be the entry in the "regret" table for decision **three** & State-of-Nature **A**?
a. zero $\quad$ b. 10 $\quad$ $\boxed{\text{c. 20}}$ $\quad$ d. 30 $\quad$ e. 40 $\quad$ f. *NOTA*

Suppose that you perform an experiment to predict the state of nature (**A** or **B**) above. The experiment has two possible outcomes which we label as **positive** and **negative**. If the state of nature is **A**, there is a 60% probability that the outcome will be **positive**, whereas if the state of nature is **B**, there is a 20% probability that the outcome will be **positive**.

According to Bayes' rule,

$$P\{A \mid negative\} = \frac{P\{\alpha \mid \beta\} P\{\gamma\}}{P\{\delta\}}$$

In this equation, …

_d_ 5. $\alpha =$
a. state of nature is A $\quad$ c. experimental outcome is positive $\quad$ e. NOTA
b. state of nature is B $\quad$ d. experimental outcome is negative

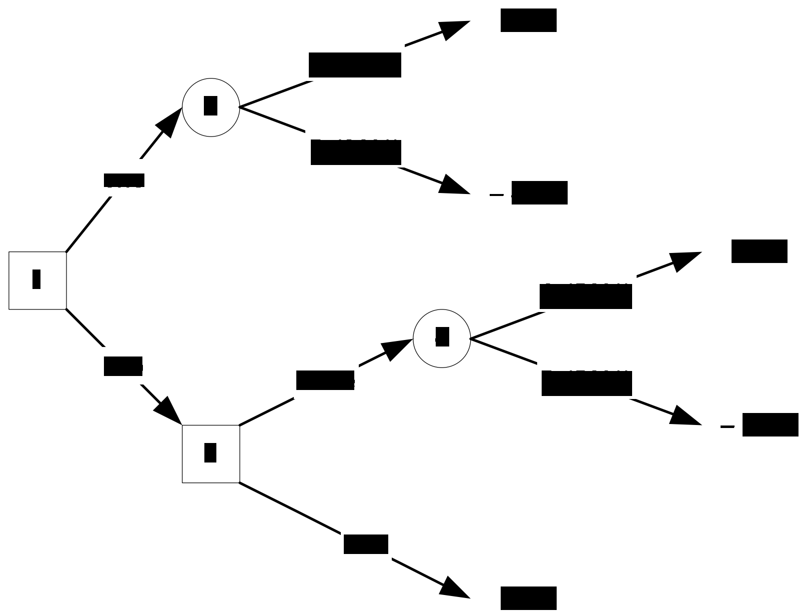
_a_ 6. $\beta =$
a. state of nature is A $\quad$ c. experimental outcome is positive $\quad$ e. NOTA
b. state of nature is B $\quad$ d. experimental outcome is negative

_a_ 7. $\gamma =$
a. state of nature is A $\quad$ c. experimental outcome is positive $\quad$ e. NOTA
b. state of nature is B $\quad$ d. experimental outcome is negative

_d_ 8. $\delta =$
a. state of nature is A $\quad$ c. experimental outcome is positive $\quad$ e. NOTA
b. state of nature is B $\quad$ d. experimental outcome is negative

_d_ 9. Suppose that the outcome of the experiment is **negative**. Then the probability that the state of nature is **A** is revised to … *(choose nearest value)*:
a. 0.1 $\quad$ b. 0.15 $\quad$ c. 0.2 $\quad$ d. 0.25 $\quad$ e. 0.3 $\quad$ f. *NOTA*

Part II.



Consider the decision tree above.

Fold back the branches and write the values of each node in the table below:

Node	#1	#2	#3	#4
Value	$140	$140	$125	$50

_a_ 5. What is the optimal decision at node #1?
a. A1 b. A2

56:171 Operations Research
Quiz #7 Version C Solution –Fall 2002

Part I:

Consider a decision problem whose payoffs are given by the following payoff table:

Decision	A	B
one	70	50
two	40	80
three	60	30
Prior Probability	**0.4**	**0.6**

_a_ 1. Which alternative should be chosen under the maximin payoff criterion?
a. one b. two c. three d. *NOTA*

_b_ 2. Which alternative should be chosen under the maximax payoff criterion?
a. one b. two c. three d. *NOTA*

_b_ 3. Which alternative should be chosen under the maximum expected payoff criterion?
a. one b. two c. three d. *NOTA*

_b_ 4. What will be the entry in the "regret" table for decision **three** & State-of-Nature **A**?
a. zero b. 10 c. 20 d. 30 e. 40 f. *NOTA*

Suppose that you perform an experiment to predict the state of nature (**A** or **B**) above. The experiment has two possible outcomes which we label as **positive** and **negative**. If the state of nature is **A**, there is a 60% probability that the outcome will be **positive**, whereas if the state of nature is **B**, there is a 20% probability that the outcome will be **positive**.

According to Bayes' rule,

$$P\{B \mid positive\} = \frac{P\{\alpha \mid \beta\} P\{\gamma\}}{P\{\delta\}}$$

In this equation, …

_c_ 5. $\alpha =$
a. state of nature is A c. experimental outcome is positive e. NOTA
b. state of nature is B d. experimental outcome is negative

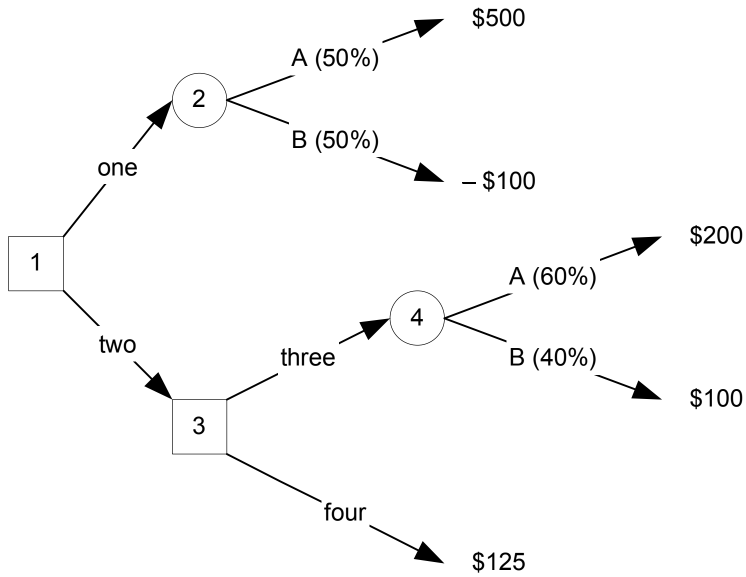
_b_ 6. $\beta =$
a. state of nature is A c. experimental outcome is positive e. NOTA
b. state of nature is B d. experimental outcome is negative

_b_ 7. $\gamma =$
a. state of nature is A c. experimental outcome is positive e. NOTA
b. state of nature is B d. experimental outcome is negative

_c_ 8. $\delta =$
a. state of nature is A c. experimental outcome is positive e. NOTA
b. state of nature is B d. experimental outcome is negative

_b_ 9. Suppose that the outcome of the experiment is **positive**. Then the probability that the state of nature is **B** is revised to … *(choose nearest value)*:
a. 0.2 b. 0.3 ≈1/3 c. 0.4 d. 0.5 e. 0.9 f. *NOTA*

Part II.



Consider the decision tree above.

Fold back the branches and write the values of each node in the table below:

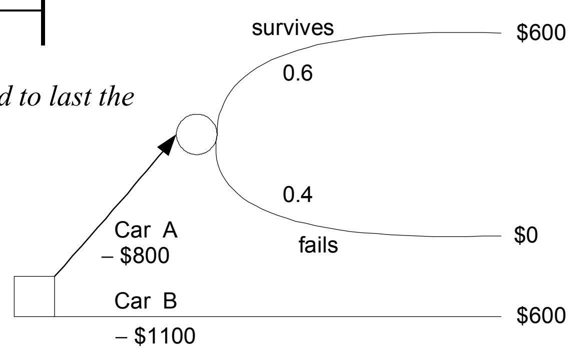
Node	#1	#2	#3	#4
Value	$200	$200	$160	$160

_a_ 5. What is the optimal decision at node #1?
a. A1 b. A2

56:171 Operations Research
Quiz #8 Solution –Fall 2002

Suppose that you are in the position of having to buy a used car, and you have narrowed down your choices to two possible models: one car (A) a private sale and the other (B) is from a dealer. You must now choose between them. The cars are similar, and the only criterion is to minimize expected cost. The dealer car is more expensive, but it comes with a one-year warranty which would cover all costs of repairs. You decide that, if car A will last for 1 year, you can sell it again and recover a large part of your investment. If it falls apart, it will not be worth fixing. After test driving both cars and checking for obvious flaws, you make the following evaluation of probable resale value:

Car	Purchase price	Probability of lasting one year	Estimated resale price
A: Private	$800	0.6	$600
B: Dealer	$1100	1.0	$600



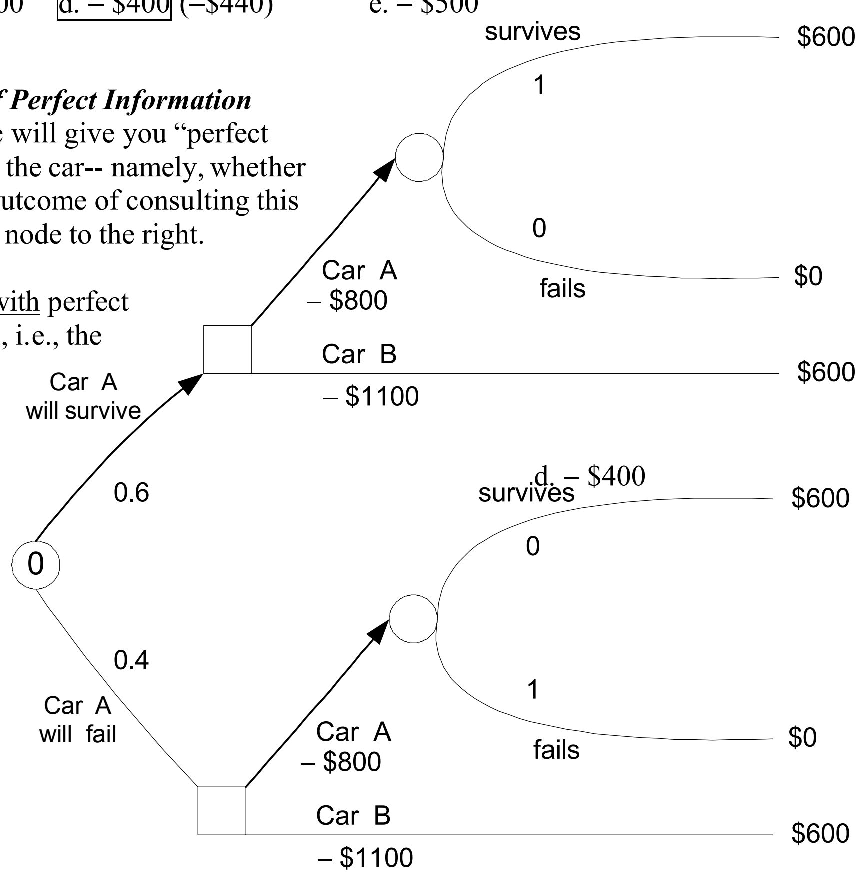
Note that car B, because of the warranty, is guaranteed to last the year and bring full resale price!

_a 1. Which car should you buy?
a. Car A b. Car B

_d 2. What is the expected cost for the year, after resale of the car? This is the expected value without information (about the future), denoted **EVWOI**. *(choose nearest value)*
a. – $100 b. – $200 c. – $300 d. – $400 (–$440) e. – $500

Calculation of Expected Value of Perfect Information

Suppose that a "seer" of the future will give you "perfect information" before you purchase the car-- namely, whether car A will survive the year. The outcome of consulting this seer is the initial (left-most) event node to the right.



_c 3. What is the expected value with perfect information (denoted **EVWPI**), i.e., the expected value at node 0? *(nearest value)*
a. – $100 b. – $200
c. – $300(–$320) d. – $400

_a 4. What is the expected value of perfect information (**EVPI**)? *(nearest value)*
a. $100 (= $120) b. $200
c. $300 d. $400
e. $500

For \$50, you may take car A to an independent mechanic, who will do a complete inspection and offer you an opinion as to whether the car will last 1 year. For various subjective reasons, you assign the following probabilities to the accuracy of the mechanic's opinion:

Given:	Mechanic says Yes	Mechanic says No
A car that will survive 1 year	90%	10%
A car that will fail in next year	30%	70%

That is, the mechanic is 90% likely to correctly identify a car that will survive the year, but only 70% likely to correctly identify a car that will fail.

Let **AS** and **AF** be the "states of nature", namely "car **A S**urvives" and "car **A F**ails" during the next year, respectively.
Let **PR** and **NR** be the outcomes of the mechanic's inspection, namely "**P**ositive **R**eport" and "**N**egative **R**eport", respectively.

Bayes' Rule states that if S_i are the states of nature and O_j are the outcomes of an experiment,

$$P\{S_i \mid O_j\} = \frac{P\{O_j \mid S_i\} \times P\{S_i\}}{P\{O_j\}}$$

$$\text{where } P\{O_j\} = \sum_i P\{O_j \mid S_i\} \times P\{S_i\}$$

The probability that the mechanic will give a postive report, i.e., $P\{PR\}$ is

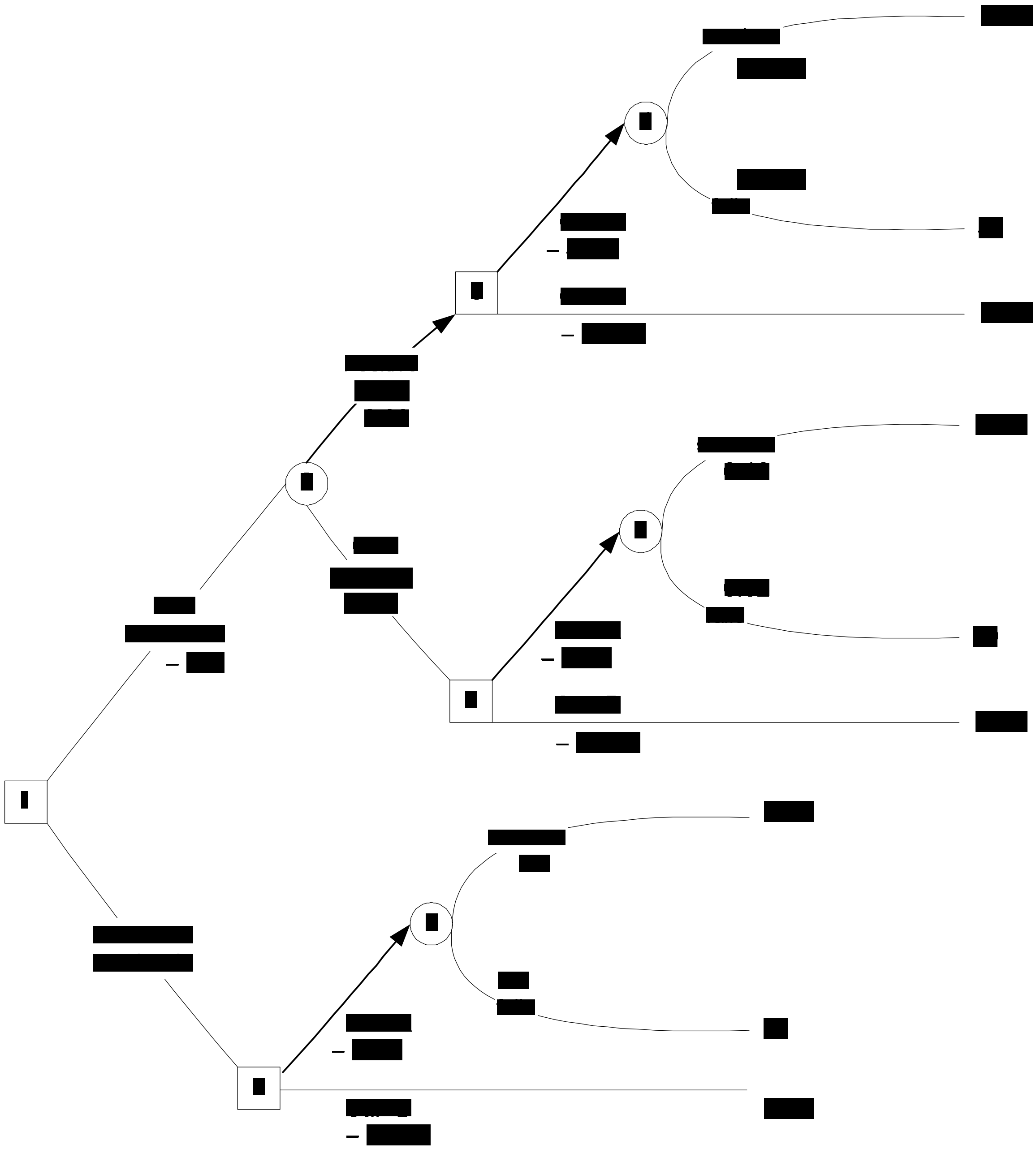
$$P\{PR\} = P\{PR \mid AS\}P\{AS\} + P\{PR \mid AF\}P\{AF\} = (0.9)(0.6) + (0.3)(0.4) = 0.66$$

_e 5. According to Bayes' theorem, the probability that car A will survive, given that the mechanic gives a positive report, i.e., $P\{AS \mid PR\}$, is *(choose nearest value)*:

a. 0.6 b. 0.65 c. 0.7 d. 0.75 $\boxed{\text{e. 0.8}}$ (0.818) f. 0.85 g. 0.9 h. 0.95

The decision tree on the next page includes your decision as to whether or not to hire the mechanic.

6. Insert P{AS|PR}, i.e., the probability that car A survives if the mechanic gives a positive report, and P{AF|PR} on the appropriate branches of the tree.



7. "Fold back" nodes 2 through 8, and write the value of each node below:

Node	Value		Node	Value		Node	Value
1	**– $424**		4	**– $492**		7	**– $440**
2	– $374		5	– $500		8	$360
3	**– $308**		6	$108			

8. Should you hire the mechanic? *Circle:* Yes No

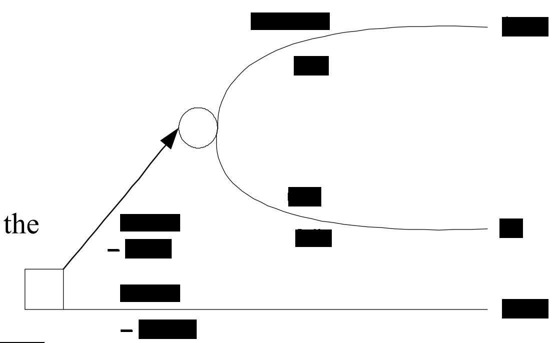
__e_ 9. The expected value of the mechanic's opinion is *(Choose nearest value)*:
a. $0 b. $15 c. $30 d. $45 e. $60 ($66) f. $75 g. $90 h. $105

56:171 Operations Research
Quiz #8 – 1 November 2002

Suppose that you are in the position of having to buy a used car, and you have narrowed down your choices to two possible models: one car (A) a private sale and the other (B) is from a dealer. You must now choose between them. The cars are similar, and the only criterion is to minimize expected cost. The dealer car is more expensive, but it comes with a one-year warranty which would cover all costs of repairs. You decide that, if car A will last for 1 year, you can sell it again and recover a large part of your investment. If it falls apart, it will not be worth fixing. After test driving both cars and checking for obvious flaws, you make the following evaluation of probable resale value:

Car	Purchase price	Probability of lasting one year	Estimated resale price
A: Private	$900	0.5	$600
B: Dealer	$1100	1.0	$600

Note that car B, because of the warranty, is guaranteed to last the year and bring full resale price!



_b 1. Which car should you buy?

a. Car A   b. Car B

_e 2. What is the expected cost for the year, after resale of the car? This is the expected value without information (about the future), denoted **EVWOI**.

(choose nearest value)

a. – $100   b. – $200   c. – $300   d. – $400   e. – $500

Calculation of Expected Value of Perfect Information

Suppose that a "seer" of the future will give you "perfect information" before you purchase the car-- namely, whether car A will survive the year. The outcome of consulting this seer is the initial event node to the right.

_d 3. What is the expected value with perfect information (denoted **EVWPI**)? *(nearest value)*

a. – $100   b. – $200

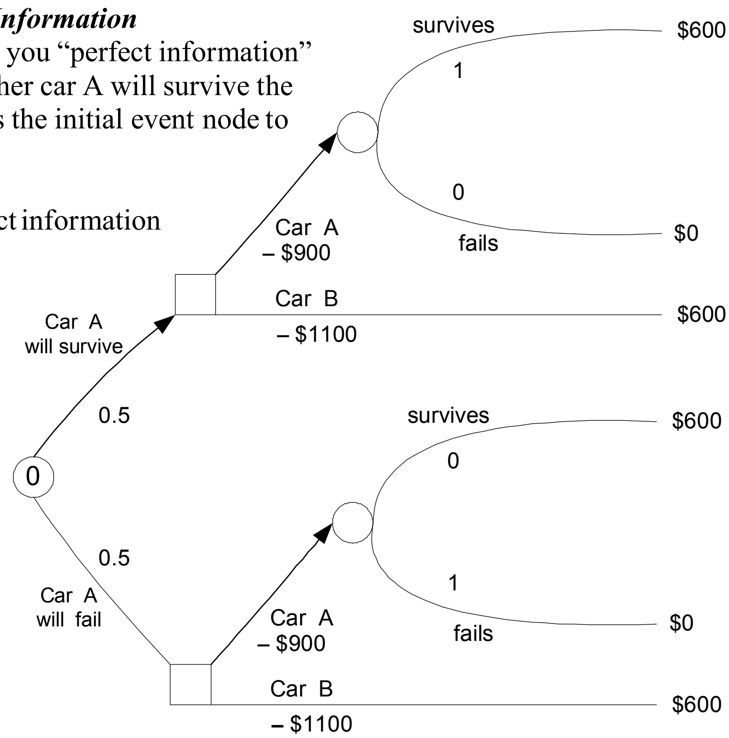
c. – $300   d. – $400

_a 4. What is the expected value of perfect information (**EVPI**)?

a. – $100   b. – $200

c. – $300   d. – $400

e. – $500



For $50, you may take car A to an independent mechanic, who will do a complete inspection and offer you an opinion as to whether the car will last 1 year. For various subjective reasons, you assign the following probabilities to the accuracy of the mechanic's opinion:

Given:	Mechanic says Yes	Mechanic says No
A car that will survive 1 year	90%	10%
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That is, the mechanic is 90% likely to correctly identify a car that will survive the year, but only 70% likely to correctly identify a car that will fail.

Let **AS** and **AF** be the "states of nature", namely "car **A S**urvives" and "car **A F**ails" during the next year, respectively.
Let **PR** and **NR** be the outcomes of the mechanic's inspection, namely "**P**ositive **R**eport" and "**N**egative **R**eport", respectively.

Bayes' Rule states that if S_i are the states of nature and O_j are the outcomes of an experiment,

$$P\{S_i \mid O_j\} = \frac{P\{O_j \mid S_i\} \times P\{S_i\}}{P\{O_j\}}$$

$$\text{where } P\{O_j\} = \sum_i P\{O_j \mid S_i\} \times P\{S_i\}$$

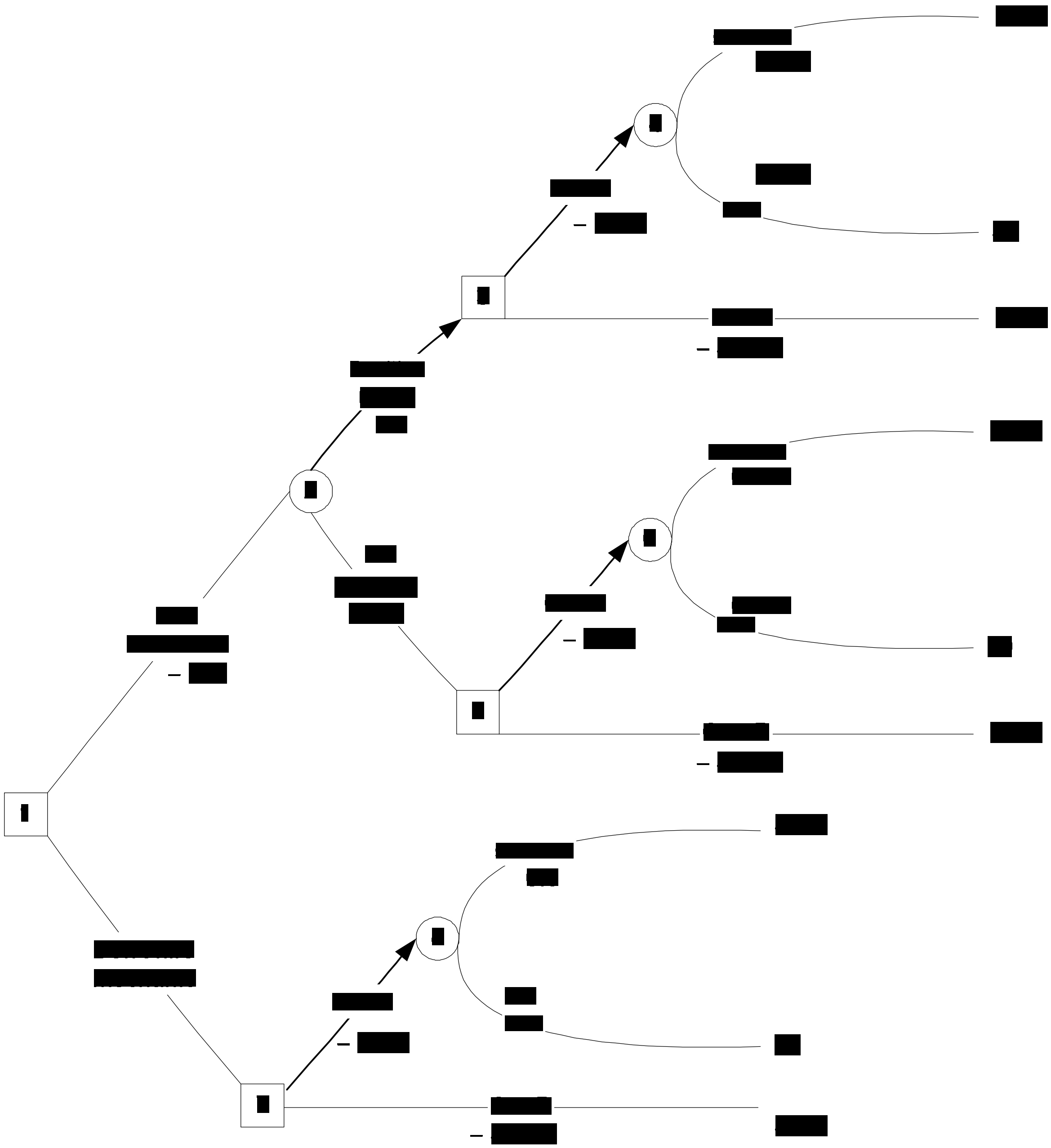
The probability that the mechanic will give a postive report, i.e., $P\{PR\}$ is

$$P\{PR\} = P\{PR \mid AS\}P\{AS\} + P\{PR \mid AF\}P\{AF\} = (0.9)(0.5) + (0.3)(0.5) = 0.6$$

_d 5. According to Bayes' theorem, the probability that car A will survive, given that the mechanic gives a positive report, i.e., $P\{AS \mid PR\}$, is *(choose nearest value)*:

a. 0.6 b. 0.65 c. 0.7 $\boxed{\text{d. 0.75}}$ e. 0.8 f. 0.85 g. 0.9

6. The decision tree below includes your decision as to whether or not to hire the mechanic. Insert P{AS|PR} and P{AF|PR} on the appropriate branches.



7. "Fold back" nodes 2 through 8, and write the value of each node below:

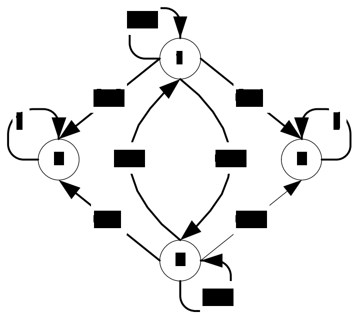
Node	Value	Node	Value	Node	Value
1	**– $500**	4	**– $450**	7	**– $500**
2	– $470	5	– $500	8	$300
3	**– $450**	6	$75		

8. Should you hire the mechanic? *Circle:* Yes $\boxed{\text{No}}$

____ 9. The expected value of the mechanic's opinion is *(Choose nearest value)*:
a. $0 b. $15 c. $30 d. $45 e. $60 f. $75 g. $90 h. $105

56:171 Operations Research
Quiz #9 – November 8, 2002

Markov Chains. Consider the discrete-time Markov chain diagrammed below:



P	1	2	3	4
1	0.3	0.4	0.2	0.1
2	0.2	0.5	0.1	0.2
3	0	0	1	0
4	0	0	0	1

A	3	4
1	0.518	0.482
2	0.407	0.593

E	1	2	row sum
1	1.851	1.482	3.333
2	0.741	2.592	3.333

n	$f_{13}^{(n)}$	$f_{14}^{(n)}$
1	0.2	0.1
2	0.1	0.11
3	0.066	0.081
4	0.0458	0.0571
5	0.03202	0.04001
6	0.02241	0.028011
7	0.0156866	0.0196081
8	0.0109806	0.0137257
9	0.0076864	0.009608
10	0.00538048	0.0067256

_b_ 1. Which is the matrix Q (used in computation of E)?

a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$ b. $\begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.5 \end{bmatrix}$ c. $\begin{bmatrix} 0.3 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ f. *None of the above (NOTA)*

_a_ 2. Which is the matrix R (used in computation of A)?

a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$ b. $\begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.5 \end{bmatrix}$ c. $\begin{bmatrix} 0.3 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ f. *NOTA*

_e_ 3. If the system begins in state #1, what is the probability that it is absorbed into state #4? (Choose nearest value)

a. 30% *or less* b. 35% c. 40% d. 45%
e. 50% f. 55% g. 60% h. 65% *or more*

_c_ 4. If the system begins in state #1, what is the expected number of stages (including the initial stage) that the system exists before it is absorbed into one of the two absorbing states? *(Choose nearest value)*

a. 1 b. 2 c. 3 d. 4
e. 5 f. 6 g. 7 h. 8 *or more*

__c_ 5. For a discrete-time Markov chain, let P be the matrix of transition probabilities. The sum of each...

a. column is 1 c. row is 1
b. column is 0 d. row is 0 e. *NOTA*

__a_ 6. An absorbing state of a Markov chain is one in which the probability of
a. moving out of that state is zero c. moving out of that state is one.
b. moving into that state is one. d. moving into that state is zero e. *NOTA*

__g_ 7. The minimal closed set(s) in the above Markov chain =
a. {1,2,3,4} b. {1,2} c. {3.4} d. {1,2,3,4} & {3,4}
e. {1} & {2} f. {1,2} & {3,4} g. {3} & {4} h. *NOTA*

_h__ 8. The probability that the system reaches an absorbing state, beginning in state 1, is *(choose nearest value):*
a. 0.3 b. 0.4 c. 0.5 d. 0.6 e. 0.7 f. 0.8 g. 0.9 h. 1.0

__b_ 9. The recurrent state(s) in the above Markov chain =
a. 1 & 2 b. 3 & 4 c. 1, 2, 3, & 4 d. NOTA

__a_ 10. The transient state(s) in the above Markov chain =
a. 1 & 2 b. 3 & 4 c. 1, 2, 3, & 4 d. NOTA

__e_ 11. The quantity $f_{13}^{(4)}$ is
a. the probability that the system, beginning in state 1, is in state 3 at stage n
b. the probability that the system first visits state 3 before state 4.
c. the expected number of visits to state 3 during the first 4 stages, beginning in state1
d. the stage in which the system, beginning in state 1, visits state 3 for the fourth time
e. the probability that the system, beginning in state 1, first reaches state 3 in stage 4
f. *NOTA*

_b__ 12. From which state is the system more likely to eventually reach state 4?
a. 1 b. 2 c. equally likely d. *NOTA*

_g_ 13. What is the probability that the system is absorbed into state 4, starting in state 1, if the first transition is to state 2? *(choose nearest value)*
a. 30% *or less* b. 35% c. 40% d. 45%
e. 50% f. 55% g. 60% h. 65% *or more*

__g_ 14. The probability that the system, starting in state 1, is in state 1 after 2 stages? *(choose nearest value)*
a. 5% *or less* b. 10% c. 15% d. 20%
e. 25% f. 30% g. 35% h. 40% *or more*

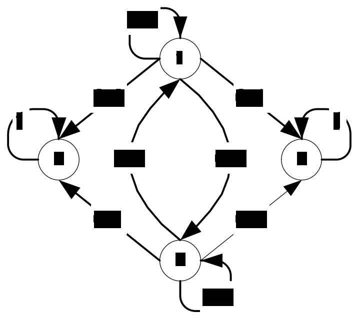
__c_ 15. The probability that the system, starting in state 1, is in state 4 after 3 stages? *(choose nearest value)*
a. 5% *or less* b. 10% c. 15% d. 20%
e. 25% f. 30% g. 35% h. 40% *or more*
a. 0.1 b. 0.2 c. 0.3 d. 0.4 e. 0.5 f. 0.6 g. 0.7 h. 0.8

__d_ 16. In general, the steadystate probability distribution π (if it exists) must satisfy
a. Pπ=0 b. Pπ=1 c. πP=1 d. πP=π
e. Pπ=π f. πP=0 g. *NOTA*

Version B--SOLUTIONS

56:171 Operations Research
Quiz #9 – November 8, 2002

Markov Chains. Consider the discrete-time Markov chain diagrammed below:



P	1	2	3	4
1	0.4	0.3	0.2	0.1
2	0.2	0.5	0.1	0.2
3	0	0	1	0
4	0	0	0	1

A	3	4
1	0.542	0.458
2	0.417	0.583

E	1	2	row sum
1	2.083	1.25	3.333
2	0.833	2.5	3.333

n	$f_{13}^{(n)}$	$f_{14}^{(n)}$
1	0.2	0.1
2	0.11	0.1
3	0.071	0.076
4	0.0485	0.0544
5	0.03371	0.03832
6	0.023549	0.026872
7	0.0164747	0.01882
8	0.0115304	0.0131759
9	0.00807088	0.00922353
10	0.00564954	0.00645655

_a_ 1. Which is the matrix R (used in computation of A)?

a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$ b. $\begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.5 \end{bmatrix}$ c. $\begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ f. *None of the above (NOTA)*

_b_ 2. Which is the matrix Q (used in computation of E)?

a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$ b. $\begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.5 \end{bmatrix}$ c. $\begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ f. *None of the above (NOTA)*

_d_ 3. If the system begins in state #1, what is the probability that it is absorbed into state #4? (Choose nearest value)

a. 30% *or less* b. 35% c. 40% d. 45%
e. 50% f. 55% g. 60% h. 65% *or more*

_c_ 4. If the system begins in state #1, what is the expected number of stages (including the initial stage) that the system exists before it is absorbed into one of the two absorbing states? *(Choose nearest value)*

a. 1 b. 2 c. 3 d. 4
e. 5 f. 6 g. 7 h. 8 *or more*

_b_ 5. For a discrete-time Markov chain, let P be the matrix of transition probabilities. The sum of each...

a. column is 1 b. row is 1
c. column is 0 d. row is 0 e. *NOTA*

__d_ 6. An absorbing state of a Markov chain is one in which the probability of
a. moving into that state is zero   b. moving out of that state is one.
c. moving into that state is one.   d. moving out of that state is zero   e. *NOTA*

__c_ 7. The minimal closed set(s) in the above Markov chain =
a. {1,2,3,4}   b. {1,2}   c. {3} & {4}   d. {1,2,3,4} & {3,4}
e. {1} & {2}   f. {1,2} & {3,4}   g. {3.4}   h. *NOTA*

__h_ 8. The probability that the system reaches an absorbing state, beginning in state 1, is *(choose nearest value):*
a. 0.3   b. 0.4   c. 0.5   d. 0.6   e. 0.7   f. 0.8   g. 0.9   h. 1.0

__a_ 9. The transient state(s) in the above Markov chain =
a. 1 & 2   b. 3 & 4   c. 1, 2, 3, & 4   d. NOTA

_b__ 10. The recurrent state(s) in the above Markov chain =
a. 1 & 2   b. 3 & 4   c. 1, 2, 3, & 4   d. NOTA

_a__ 11. The quantity $f_{13}^{(4)}$ is
a. the probability that the system, beginning in state 1, first reaches state 3 in stage 4
b. the probability that the system, beginning in state 1, is in state 3 at stage 4
c. the expected number of visits to state 3 during the first 4 stages, beginning in state1
d. the probability that the system first visits state 3 before state 4.
e. the stage in which the system, beginning in state 1, visits state 3 for the fourth time
f. *NOTA*

__b_ 12. From which state is the system more likely to eventually reach state 4?
a. 1   b. 2   c. equally likely   d. *NOTA*

__c_ 13. What is the probability that the system is absorbed into state 3, starting in state 1, if the first transition is to state 2? *(choose nearest value)*
a. 30% *or less*   b. 35%   c. 40%   d. 45%
e. 50%   f. 55%   g. 60%   h. 65% *or more*

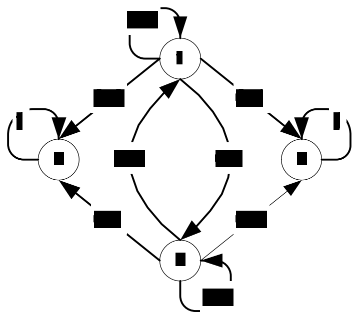
__b_ 14. The probability that the system, starting in state 1, is in state 1 after 2 stages? *(choose nearest value)*
a. 5% *or less*   b. 10%   c. 15%   d. 20%
e. 25%   f. 30%   g. 35%   h. 40% *or more*

__f_ 15. The probability that the system, starting in state 1, is in state 4 after 3 stages? *(choose nearest value)*
a. 5% *or less*   b. 10%   c. 15%   d. 20%
e. 25%   f. 30%   g. 35%   h. 40% *or more*

__e_ 16. In general, the steadystate probability distribution π (if it exists) must satisfy
a. $P\pi=0$ b. $P\pi=1$ c. $\pi P=1$ d. $P\pi=\pi$
e. $\pi P=\pi$ f. $\pi P=0$ g. *NOTA*

56:171 Operations Research
Quiz #9 – November 8, 2002

Markov Chains. Consider the discrete-time Markov chain diagrammed below:



A	3	4
1	0.611	0.389
2	0.444	0.556

E	1	2	row sum
1	2.778	0.555	3.333
2	1.111	2.222	3.333

n	$f_{13}^{(n)}$	$f_{14}^{(n)}$
1	0.2	0.1
2	0.13	0.08
3	0.087	0.06
4	0.0593	0.0436
5	0.04087	0.03116
6	0.028353	0.022068
7	0.0197447	0.01555
8	0.0137803	0.010926
9	0.00962985	0.00766456
10	0.00673434	0.00537174

P	1	2	3	4
1	0.6	0.1	0.2	0.1
2	0.2	0.5	0.1	0.2
3	0	0	1	0
4	0	0	0	1

_b_ 1. Which is the matrix Q (used in computation of E)?

a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$ b. $\begin{bmatrix} 0.6 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}$ c. $\begin{bmatrix} 0.6 & 0.1 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ f. *None of the above (NOTA)*

_a_ 2. Which is the matrix R (used in computation of A)?

a. $\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$ b. $\begin{bmatrix} 0.6 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}$ c. $\begin{bmatrix} 0.6 & 0.1 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ f. *None of the above (NOTA)*

_c_ 3. If the system begins in state #1, what is the probability that it is absorbed into state #4? (Choose nearest value)

a. 30% *or less* b. 35% c. 40% d. 45%
e. 50% f. 55% g. 60% h. 65% *or more*

_c_ 4. If the system begins in state #1, what is the expected number of stages (including the initial stage) that the system exists before it is absorbed into one of the two absorbing states? *(Choose nearest value)*

a. 1 b. 2 c. 3 d. 4
e. 5 f. 6 g. 7 h. 8 *or more*

_c_ 5. For a discrete-time Markov chain, let P be the matrix of transition probabilities. The sum of each...

a. column is 1 c. row is 1
b. column is 0 d. row is 0 e. *NOTA*

__c_ 6. An absorbing state of a Markov chain is one in which the probability of
a. moving out of that state is one c. moving out of that state is zero.
b. moving into that state is one. d. moving into that state is zero e. *NOTA*

__e_ 7. The minimal closed set(s) in the above Markov chain =
a. {1,2,3,4} c. {1,2} e. {3} & {4} g. {1,2,3,4} & {3,4}
b. {1} & {2} d. {1,2} & {3,4} f. {3.4} h. *NOTA*

__b_ 8. The recurrent state(s) in the above Markov chain =
a. 1 & 2 b. 3 & 4 c. 1, 2, 3, & 4 d. NOTA

__a_ 9. The transient state(s) in the above Markov chain =
a. 1 & 2 b. 3 & 4 c. 1, 2, 3, & 4 d. NOTA

__h_ 10. The probability that the system reaches an absorbing state, beginning in state 1, is *(choose nearest value):*
a. 0.3 b. 0.4 c. 0.5 d. 0.6 e. 0.7 f. 0.8 g. 0.9 h. 1.0

_d__ 11. The quantity $f_{13}^{(4)}$ is
a. the probability that the system, beginning in state 1, is in state 3 at stage 4
b. the probability that the system first visits state 3 before state 4.
c. the expected number of visits to state 3 during the first 4 stages, beginning in state1
d. the probability that the system, beginning in state 1, first reaches state 3 in stage 4
e. the stage in which the system, beginning in state 1, visits state 3 for the fourth time
f. *NOTA*

_b__ 12. From which state is the system more likely to eventually reach state 4?
a. 1 b. 2 c. equally likely d. *NOTA*

_f__ 13. What is the probability that the system is absorbed into state 4, starting in state 1, if the first transition is to state 2? *(choose nearest value)*
a. 30% *or less* b. 35% c. 40% d. 45%
e. 50% f. 55% g. 60% h. 65% *or more*

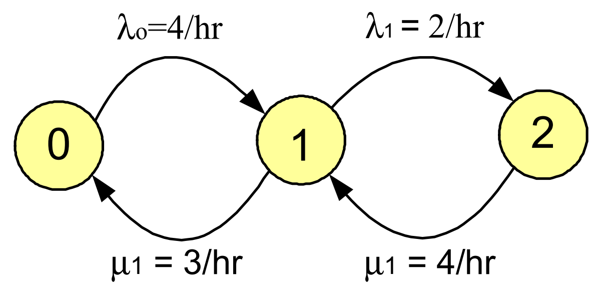
_h__ 14. The probability that the system, starting in state 1, is in state 1 after 2 stages? *(choose nearest value)*
a. 5% *or less* b. 10% c. 15% d. 20%
e. 25% f. 30% g. 35% h. 40% *or more*

_e__ 15. The probability that the system, starting in state 1, is in state 4 after 3 stages? *(choose nearest value)*
a. 5% *or less* b. 10% c. 15% d. 20%
e. 25% f. 30% g. 35% h. 40% *or more*

_f__ 16. In general, the steadystate probability distribution π (if it exists) must satisfy
a. $P\pi=0$ c. $P\pi=1$ e. $\pi P=1$
b. $P\pi=\pi$ d. $\pi P=0$ f. $\pi P=\pi$ g. *NOTA*

56:171 Operations Research
Quiz #10 Versions A, B, & C --Fall 2002

Version A: A machine operator has the task of keeping two machines running. Each machine runs for an average of **30** minutes before it becomes jammed, requiring the operator's attention. When a machine jams, he spends an average of **20** minutes restoring the machine to running condition, unless both machines are jammed, in which case he works faster, clearing the jam in an average of **15** minutes. Define a continuous-time Markov chain, with the state of the system being *the number of machines which are jammed.*



1-4. Specify (by letter) each of the transition rates:

λ_0= 4/hr $\qquad$ λ_1 = 2/hr

μ_1=3/hr $\qquad$ μ_2= 4/hr

_b_5. Which equation is used to compute the steady-state probability π_0?

(a.) $\pi_0 = \frac{\lambda_0}{\mu_1}\times\frac{\lambda_1}{\mu_2}$

(e). $\pi_0 = \left(1+\frac{\lambda_0}{\mu_1}+\frac{\lambda_1}{\mu_2}\right)^{-1}$

(b.)

$$\boxed{\pi_0 = \left(1+\frac{\lambda_0}{\mu_1}+\frac{\lambda_0}{\mu_1}\times\frac{\lambda_1}{\mu_2}\right)^{-1} = \left(1+\frac{4}{3}+\frac{4}{3}\times\frac{1}{2}\right)^{-1} = (3)^{-1} = \frac{1}{3} \Rightarrow \pi_1 = \frac{4}{3}\pi_0 = \frac{4}{9}, \pi_2 = \frac{2}{3}\pi_0 = \frac{2}{9}}$$

(c.) $\pi_0 = \dfrac{1+\dfrac{\lambda_0}{\lambda_1}}{1+\dfrac{\mu_1}{\mu_2}}$

(f.) $\pi_0 = \frac{\lambda_0}{\mu_1}\times\left(\frac{\lambda_1}{\mu_2}\right)^2$

(g.) $\pi_0 = \frac{1+\lambda_0+\lambda_0\times\lambda_1}{1+\mu_1+\mu_1\times\mu_2}$

(h.) *None of the above*

__f_6. What is the relationship between π_0 and π_1 *for this system?*

a. $\pi_1 = \pi_0$ $\quad$ b. $\pi_1 = \frac{1}{4}\pi_0$ $\quad$ c. $\pi_1 = \frac{1}{3}\pi_0$ $\quad$ d. $\pi_1 = \frac{1}{2}\pi_0$

e. $\pi_1 = \frac{2}{3}\pi_0$ $\quad$ f. $\boxed{\pi_1 = \frac{4}{3}\pi_0}$ $\quad$ g. $\pi_1 = 2\pi_0$ $\quad$ h. *None of the above*

__b_7. The value of the steady-state probability π_0 is *(choose nearest value)*:

a. 30%	b. $\boxed{\text{35\% (33\%)}}$	c. 40%	d. 45%	e. 50%
f 55%	g. 60%	h. 65%	i. 70%	j. 75%

__f_ 8. The average number of machines which are running is *(choose nearest value)*:

a. 0.2	b. 0.4	c. 0.6	d. 0.8	e. 1.0
f. $\boxed{\text{1.2 (1.111)}}$	g. 1.4	h. 1.6	i. 1.8	j. 2.0

_J__ 9. The utilization of the machine operator (i.e., the fraction of the time he is busy clearing jams) is *(choose nearest value)*:

a. 20%	b. 25%	c. 30%	d. 35%	e. 40%
f. 45%	g. 50%	h. 55%	i. 60%	j. $\boxed{\text{65\% (67\%)}}$

_h_ 10. The average number of jams per hour which the machine operator must clear is *(choose nearest value):*

a. 0.5	b. 0.75	c. 1.0	d. 1.25	e. 1.5
f. 1.75	g. 2.0	h. $\boxed{\text{2.25 (2.222)}}$	i. 2.5	j. 2.75

_c_ 11. The average time (in minutes) between the jamming of a machine until it is back in operation is *(choose nearest value):*

a. 15	b. 20	$\boxed{\text{c. 25}}$	d. 30	e. 35
f. 40	g. 45	h. 50	i. 55	j. 60

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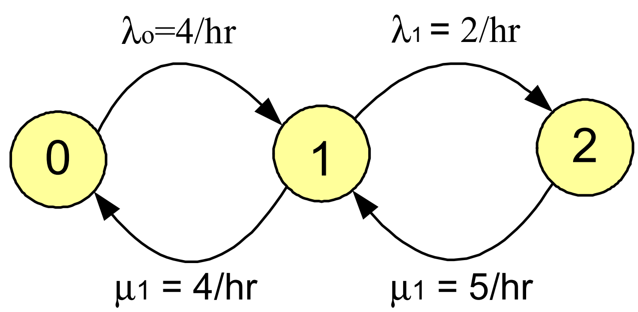
Version B: A machine operator has the task of keeping two machines running. Each machine runs for an average of **30** minutes before it becomes jammed, requiring the operator's attention. When a machine jams, he spends an average of **15** minutes restoring the machine to running condition, unless both machines are jammed, in which case he works faster, clearing the jam in an average of **12** minutes. Define a continuous-time Markov chain, with the state of the system being *the number of machines which are jammed.*



1-4. Specify (by letter) each of the transition rates:

λ_0 =4/hr $\quad$ λ_1= 2/hr

μ_1 =4/hr $\quad$ μ_2 =5/hr

_a__5. Which equation is used to compute the steady-state probability π_0?

(a.) $$\boxed{\pi_0 = \left(1+\frac{\lambda_0}{\mu_1}+\frac{\lambda_0}{\mu_1}\times\frac{\lambda_1}{\mu_2}\right)^{-1} = \left(1+\frac{4}{4}+\frac{4}{4}\times\frac{2}{5}\right)^{-1} = \left(\frac{12}{5}\right)^{-1} = \frac{5}{12} \Rightarrow \pi_1 = \frac{5}{12}, \pi_2 = \frac{1}{6}}$$

(b.) $$\pi_0 = \frac{\lambda_0}{\mu_1}\times\left(\frac{\lambda_1}{\mu_2}\right)^2$$

(c.) $$\pi_0 = \frac{1+\dfrac{\lambda_0}{\lambda_1}}{1+\dfrac{\mu_1}{\mu_2}}$$

(d.) $$\pi_0 = \frac{1+\lambda_0+\lambda_0\times\lambda_1}{1+\mu_1+\mu_1\times\mu_2}$$

(e.) $$\pi_0 = \frac{\lambda_0}{\mu_1}\times\frac{\lambda_1}{\mu_2}$$

(f.) $$\pi_0 = \left(1+\frac{\lambda_0}{\mu_1}+\frac{\lambda_1}{\mu_2}\right)^{-1}$$

(h.) *None of the above*

_e_6. What is the relationship between π_0 and π_1 *for this system?*

a. $\pi_1 = \frac{1}{4}\pi_0$	b. $\pi_1 = \frac{1}{3}\pi_0$	c. $\pi_1 = \frac{1}{2}\pi_0$	d. $\pi_1 = \frac{2}{3}\pi_0$
e. $\boxed{\pi_1 = \pi_0}$	f. $\pi_1 = 2\pi_0$	g. $\pi_1 = 3\pi_0$	h. *None of the above*

_c_7. The value of the steady-state probability π_0 is *(choose nearest value)*:

a. 30%	b. 35%	c. $\boxed{\text{40\% (41.66\%)}}$	d. 45%	e. 50%

f 55% g. 60% h. 65% i. 70% j. 75%

__f_ 8. The average number of machines which are running is *(choose nearest value)*:

a. 0.2 b. 0.4 c. 0.6 d. 0.8 e. 1.0
f. $\boxed{1.2\ (1.25)}$ g. 1.4 h. 1.6 i. 1.8 j. 2.0

_i_ 9. The utilization of the machine operator (i.e., the fraction of the time he is busy clearing jams) is *(choose nearest value)*:

a. 20% b. 25% c. 30% d. 35% e. 40%
f. 45% g. 50% h. 55% i. $\boxed{60\%\ (58.33\%)}$ j. 65%

_i_ 10. The average number of jams per hour which the machine operator must clear is *(choose nearest value):*

a. 0.5 b. 0.75 c. 1.0 d. 1.25 e. 1.5
f. 1.75 g. 2.0 h. 2.25 i. $\boxed{2.5}$ j. 2.75

_b_ 11. The average time (in minutes) between the jamming of a machine until it is back in operation is *(choose nearest value):*

a. 15 b. $\boxed{20\ (18)}$ c. 25 d. 30 e. 35
f. 40 g. 45 h. 50 i. 55 j. 60

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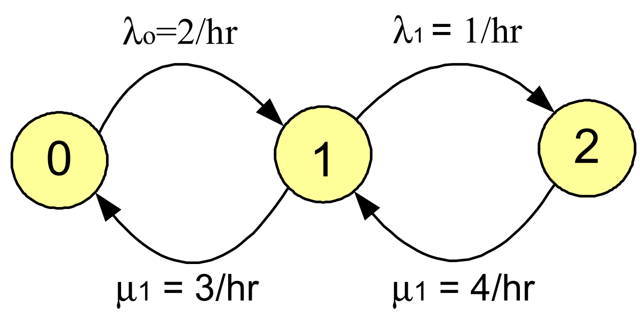
Version C: A machine operator has the task of keeping two machines running. Each machine runs for an average of **60** minutes before it becomes jammed, requiring the operator's attention. When a machine jams, he spends an average of **20** minutes restoring the machine to running condition, unless both machines are jammed, in which case he works faster, clearing the jam in an average of **15** minutes. Define a continuous-time Markov chain, with the state of the system being *the number of machines which are jammed.*



1-4. Specify (by letter) each of the transition rates:

λ_0 =2/hr λ_1 =1/hr
μ_1 =3/hr μ_2=4/hr

_f_5. Which equation is used to compute the steady-state probability π_0?

(a.) $\pi_0 = \left(1+\dfrac{\lambda_0}{\mu_1}+\dfrac{\lambda_1}{\mu_2}\right)^{-1}$

(b.) $\pi_0 = \dfrac{1+\dfrac{\lambda_0}{\lambda_1}}{1+\dfrac{\mu_1}{\mu_2}}$

(c.) $\pi_0 = \dfrac{\lambda_0}{\mu_1}\times\dfrac{\lambda_1}{\mu_2}$

(d.) $\pi_0 = \dfrac{\lambda_0}{\mu_1}\times\left(\dfrac{\lambda_1}{\mu_2}\right)^2$

(e.) $\pi_0 = \dfrac{1+\lambda_0+\lambda_0\times\lambda_1}{1+\mu_1+\mu_1\times\mu_2}$

(f). $\pi_0 = \left(1+\dfrac{\lambda_0}{\mu_1}+\dfrac{\lambda_0}{\mu_1}\times\dfrac{\lambda_1}{\mu_2}\right)^{-1} = \left(1+\dfrac{2}{3}+\dfrac{2}{3}\times\dfrac{1}{4}\right)^{-1} = \left(\dfrac{11}{6}\right)^{-1} = \dfrac{6}{11} \Rightarrow \pi_1 = \dfrac{2}{3}\pi_0 = \dfrac{4}{11}, \pi_2 = \dfrac{1}{6}\pi_0 = \dfrac{1}{11}$

(h.) *None of the above*

__e_6. What is the relationship between π_0 and π_1 *for this system?*

a. $\pi_1 = \pi_0$ b. $\pi_1 = \frac{1}{4}\pi_0$ c. $\pi_1 = \frac{1}{3}\pi_0$ d. $\pi_1 = \frac{1}{2}\pi_0$

e. $\boxed{\pi_1 = \frac{2}{3}\pi_0}$ f. $\pi_1 = 2\pi_0$ g. $\pi_1 = 3\pi_0$ h. *None of the above*

_f__7. The value of the steady-state probability π_0 is *(choose nearest value)*:

a. 30% b. 35% c. 40% d. 45% e. 50%

f $\boxed{55\% \ (54.54\%)}$ g. 60% h. 65% i. 70% j. 75%

_f__ 8. The average number of machines which are running is *(choose nearest value)*:

a. 1.0 b. 1.1 c. 1.2 d. 1.3 e. 1.4

f. $\boxed{1.5 \ (1.4545)}$ g. 1.6 h. 1.7 i. 1.8 j. 1.9

_f__ 9. The utilization of the machine operator (i.e., the fraction of the time he is busy clearing jams) is *(choose nearest value)*:

a. 20% b. 25% c. 30% d. 35% e. 40%

f. $\boxed{45\% \ (45.45\%)}$ g. 50% h. 55% i. 60% j. 65%

_e_ 10. The average number of jams per hour which the machine operator must clear is *(choose nearest value):*

a. 0.5 b. 0.75 c. 1.0 d. 1.25 e. $\boxed{1.5 \ (1.4545)}$

f. 1.75 g. 2.0 h. 2.25 i. 2.5 j. 2.75

b or c 11. The average time (in minutes) between the jamming of a machine until it is back in operation is *(choose nearest value):*

a. 15 $\boxed{\text{b. } 20 \ (22.5) \quad \text{c. } 25}$ d. 30 e. 35

f. 40 g. 45 h. 50 i. 55 j. 60

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The following are common to versions A, B, & C:

__d_ 12. For a continuous-time Markov chain, let Λ be the matrix of transition probabilities. The sum of each...

a. column is 1 c. row is 1

b. column is 0 $\boxed{\text{d. row is 0}}$ e. *NOTA*

__d_ 13. In a birth/death process model of a queue, the time between departures is assumed to

a. have the Beta dist'n c. be constant e. have the uniform dist'n

b. have the Poisson dist'n d. have the exponential dist'n f. *NOTA*

__c_ 14. In an M/M/1 queue, if the arrival rate = $\lambda < \mu$= service rate, then

a. $\pi_0 = 1$ in steady state c. $\pi_i > 0$ for all i e. the queue is not a birth-death process

b. no steady state exists d. $\pi_0 = 0$ in steady state f. *NOTA*

True (+) or false (o)?

_+_ 15. The continuous-time Markov chain on the previous page is a birth/death process.

_o_ 16. Little's Law for queues is valid only if the queue is a birth/death process.

_+_ 17. According to Little's Law, the average arrival rate is the ratio of average number of customers in the system to the average time per customer, i.e., $\underline{\lambda}$ = L/W.

_+_ 18. Little's Law for queues is valid for *every* queue which is a continuous-time Markov chain. *Note: it is valid for other queues as well!*

56:171 Operations Research
Quiz #11 Solutions -- Fall 2002

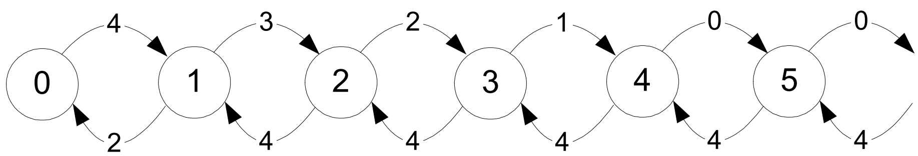
Part I: For each diagram of a Markov model of a queue in (1) through (5) below, indicate the correct Kendall's classification from among the following choices :

a. M/M/1	f. M/M/2	k. M/M/3	p. M/M/4
b. M/M/1/4	g. M/M/2/3	l. M/M/3/3	q. M/M/4/3
c. M/M/1/4/4	h. M/M/2/3/4	m. M/M/3/3/3	r. M/M/4/2/4
d. M/M/1/3/4	i. M/M/2/4/4	n. M/M/3/3/4	s. M/M/4/4/4
e. M/M/1/3/3	j. M/M/2/4	o. M/M/3/4/	t. M/M/4/4

u. *None of the above*

\~~~~~~

M/M/2/4/4



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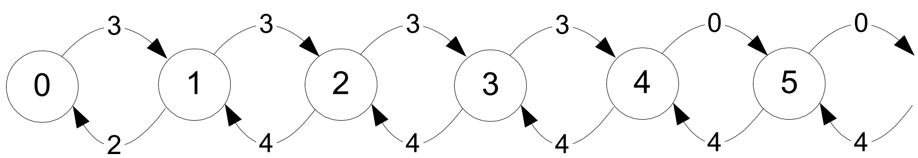
M/M/2/4



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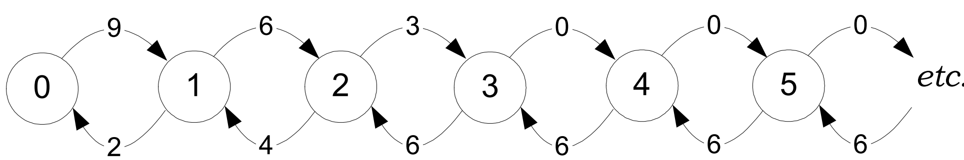
M/M/3/3/3



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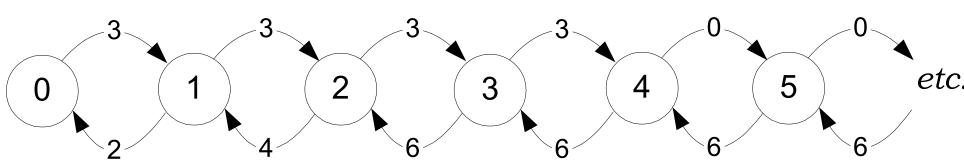
M/M/3/4



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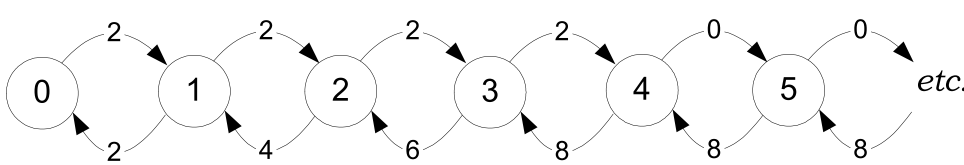
M/M/4/4



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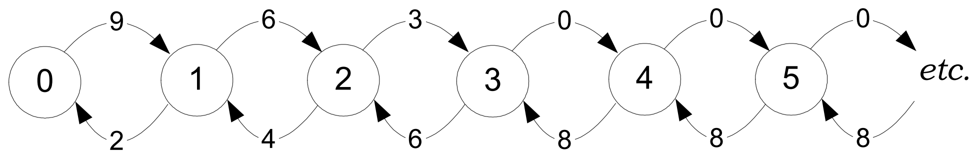
M/M/4/3/3



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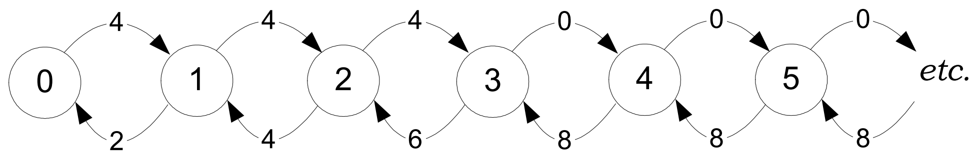
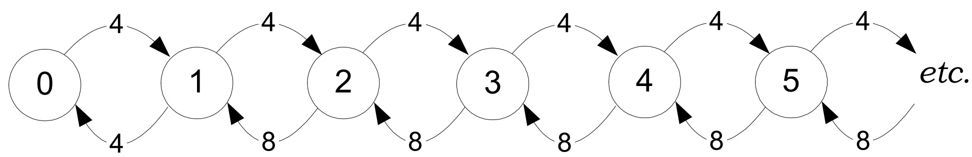


M/M/2



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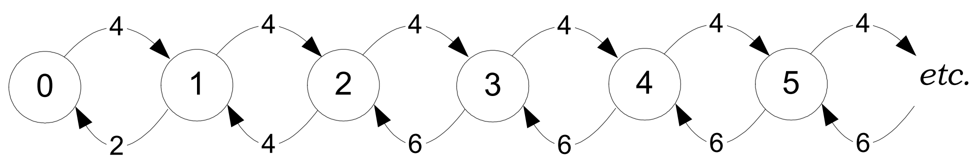
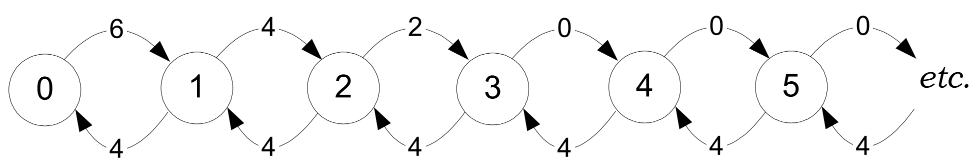
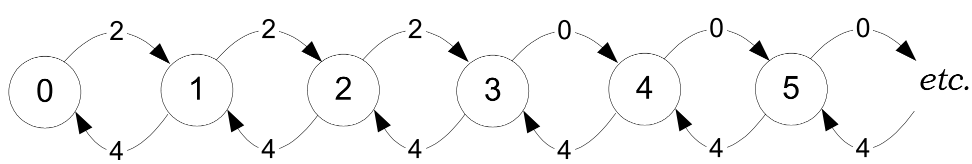


M/M/1/3/3



M/M/1/3



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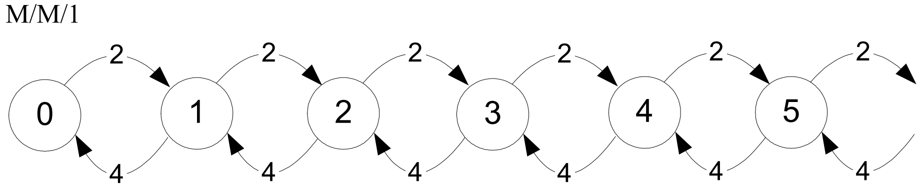

Note: Kendall's notation: **A/B/s/m/n** where
- **A** indicates arrival process (M="Markovian" or "Memoryless")
- **B** indicates service process (M="Markovian" or "Memoryless")
- **s** indicates # of servers
- **m** indicates capacity of system (including those being served)
- **n** indicates size of source population

56:171 Operations Research
Quiz #12 – Fall 2002

Determinstic Production Planning

We wish to plan next week's production (Monday through Saturday) of an expensive, low-demand item.

- the cost of *production* is **$7** for setup, plus **$3** per unit produced, up to a maximum of 4 units.
- the *storage* cost for inventory is **$1** per unit, based upon the level at the beginning of the day.
- a maximum of **6** units may be kept in inventory at the end of each day; any excess inventory is simply discarded.
- the demand D is random, with the same probability distribution each day:

Stage	1	2	3	4	5	6
Day	Mon.	Tues	Wed	Thurs	Fri	Sat
Demand	3	1	1	3	2	1
Produce	3	0	0	3	3	0

- no shortages are allowed.
- the initial inventory is **2**.
- a *salvage* value of **$2** per unit is received for any inventory remaining at the end of the last day (Saturday).

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 1= Monday, stage 2= Tuesday, etc.** We define

S_n = stock on hand at stage n.

$f_n(S_n)$ = minimum total cost for the days n, n+1, …6, if at the beginning of day n the stock on hand is S_n.

Thus, we seek the value of $f_1(2)$, i.e., the minimum expected cost for six days, beginning with two units in inventory.

(a.) What is the value of $f_1(2)$? $_**54.00**__

(b.) What should be the production quantity for Monday? __3___

(c.) What is the total cost (production + storage – salvage value) of the optimal production schedule for all six days? _**$54.00**__

(d.) Three values have been blanked out in the computer output, What are they?

- the cost associated with the decision to produce 1 unit on Friday when the inventory is 1 at the end of Thursday. **(A)__$21.00**_ *(Note: this may or may not be the optimal decision!)*
- the optimal value $f_2(1)$, i.e., the minimum total cost of the last 5 days (Tuesday through Saturday) if there is one unit of stock on hand Tuesday morning. **(B) $__40.00**_____
- the cost associated with the decision to produce 3 units on Monday, when there is initially one unit in stock. **(C.) $__57.00**____

(e.) Complete the last row of the table above, indicating the optimal production quantity each day.

```
  s \ x: 0      1      2      3      4  | Minimum
  0  | 999.99  10.00  11.00  12.00  13.00|  10.00
  1  |   1.00   9.00  10.00  11.00  12.00|   1.00
  2  |   0.00   8.00   9.00  10.00  11.00|   0.00
  3  |  ¯1.00   7.00   8.00   9.00  10.00|  ¯1.00
  4  |  ¯2.00   6.00   7.00   8.00 999.99|  ¯2.00
  5  |  ¯3.00   5.00   6.00 999.99 999.99|  ¯3.00
  6  |  ¯4.00   4.00 999.99 999.99 999.99|  ¯4.00
```

Stage 6 (Saturday)

Day 5 (Friday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	999.99	23.00	17.00	19.00	17.00
1	999.99	___A___	15.00	17.00	19.00	15.00
2	12.00	13.00	15.00	17.00	19.00	12.00
3	4.00	13.00	15.00	17.00	19.00	4.00
4	4.00	13.00	15.00	17.00	19.00	4.00
5	4.00	13.00	15.00	17.00	999.99	4.00
6	4.00	13.00	15.00	999.99	999.99	4.00

Day 4 (Thursday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	999.99	999.99	33.00	34.00	33.00
1	999.99	999.99	31.00	32.00	32.00	31.00
2	999.99	29.00	30.00	30.00	25.00	25.00
3	20.00	28.00	28.00	23.00	26.00	20.00
4	19.00	26.00	21.00	24.00	27.00	19.00
5	17.00	19.00	22.00	25.00	28.00	17.00
6	10.00	20.00	23.00	26.00	999.99	10.00

Day 3 (Wednesday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	43.00	44.00	41.00	39.00	39.00
1	34.00	42.00	39.00	37.00	39.00	34.00
2	33.00	37.00	35.00	37.00	38.00	33.00
3	28.00	33.00	35.00	36.00	32.00	28.00
4	24.00	33.00	34.00	30.00	999.99	24.00
5	24.00	32.00	28.00	999.99	999.99	24.00
6	23.00	26.00	999.99	999.99	999.99	23.00

Day 2 (Tuesday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	49.00	47.00	49.00	47.00	47.00
1	40.00	45.00	47.00	45.00	44.00	__B__
2	36.00	45.00	43.00	42.00	45.00	36.00
3	36.00	41.00	40.00	43.00	45.00	36.00
4	32.00	38.00	41.00	43.00	999.99	32.00
5	29.00	39.00	41.00	999.99	999.99	29.00
6	30.00	39.00	999.99	999.99	999.99	30.00

Day 1 (Monday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	999.99	999.99	63.00	59.00	59.00
1	999.99	999.99	61.00	___C__	56.00	56.00
2	999.99	59.00	55.00	54.00	57.00	54.00
3	50.00	53.00	52.00	55.00	54.00	50.00
4	44.00	50.00	53.00	52.00	52.00	44.00
5	41.00	51.00	50.00	50.00	54.00	41.00
6	42.00	48.00	48.00	52.00	999.99	42.00

56:171 Operations Research
Quiz #12 – Fall 2002

Determinstic Production Planning

We wish to plan next week's production (Monday through Saturday) of an expensive, low-demand item.

- the cost of *production* is **$5** for setup, plus **$4** per unit produced, up to a maximum of 4 units.
- the *storage* cost for inventory is **$1** per unit, based upon the level at the beginning of the day.
- a maximum of **6** units may be kept in inventory at the end of each day; any excess inventory is simply discarded.
- the demand D is random, with the same probability distribution each day:

Stage	1	2	3	4	5	6
Day	Mon.	Tues	Wed	Thurs	Fri	Sat
Demand	2	3	3	1	2	1
Produce	**0**	**3**	**4**	**0**	**3**	**0**

- no shortages are allowed.
- the initial inventory is **2**.
- a *salvage* value of **$3** per unit is received for any inventory remaining at the end of the last day (Saturday).

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 1= Monday, stage 2= Tuesday, etc.** We define

S_n = stock on hand at stage n.

$f_n(S_n)$ = minimum total cost for the days n, n+1, …6, if at the beginning of day n the stock on hand is S_n.

Thus, we seek the value of $f_1(2)$, i.e., the minimum expected cost for six days, beginning with two units in inventory.

(a.) What is the value of $f_1(2)$? $_**65.00**___

(b.) What should be the production quantity for Monday? __**0**___

(c.) What is the total cost (production + storage – salvage value) of the optimal production schedule for all six days? _**$65.00**____

(d.) Three values have been blanked out in the computer output, What are they?

- the cost associated with the decision to produce 1 unit on Friday when the inventory is 1 at the end of Thursday. **(A)**__**$19.00**__ *(Note: this may or may not be the optimal decision!)*
- the optimal value $f_2(1)$, i.e., the minimum total cost of the last 5 days (Tuesday through Saturday) if there is one unit of stock on hand Tuesday morning. **(B) $**__**54.00**____
- the cost associated with the decision to produce 3 units on Monday, when there is initially one unit in stock. **(C.) $**__**69.00**__

(e .) Complete the last row of the table above, indicating the optimal production quantity each day.

```
  s \ x: 0      1      2      3      4   | Minimum
  0  | 999.99   9.00  10.00  11.00  12.00|   9.00
  1  |   1.00   7.00   8.00   9.00  10.00|   1.00
  2  |  ¯1.00   5.00   6.00   7.00   8.00|  ¯1.00
  3  |  ¯3.00   3.00   4.00   5.00  12.00|  ¯3.00
  4  |  ¯5.00   1.00   2.00   9.00 999.99|  ¯5.00
  5  |  ¯7.00  ¯1.00   6.00 999.99 999.99|  ¯7.00
  6  |  ¯9.00   3.00 999.99 999.99 999.99|  ¯9.00
```

Stage 6 (Saturday)

Day 5 (Friday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	999.99	22.00	18.00	20.00	18.00
1	999.99	__A___	15.00	17.00	19.00	15.00
2	11.00	12.00	14.00	16.00	18.00	11.00
3	4.00	11.00	13.00	15.00	17.00	4.00
4	3.00	10.00	12.00	14.00	16.00	3.00
5	2.00	9.00	11.00	13.00	999.99	2.00
6	1.00	8.00	10.00	999.99	999.99	1.00

Day 4 (Thursday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	27.00	28.00	28.00	25.00	25.00
1	19.00	25.00	25.00	22.00	25.00	19.00
2	17.00	22.00	19.00	22.00	25.00	17.00
3	14.00	16.00	19.00	22.00	25.00	14.00
4	8.00	16.00	19.00	22.00	999.99	8.00
5	8.00	16.00	19.00	999.99	999.99	8.00
6	8.00	16.00	999.99	999.99	999.99	8.00

Day 3 (Wednesday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	999.99	999.99	42.00	40.00	40.00
1	999.99	999.99	39.00	37.00	39.00	37.00
2	999.99	36.00	34.00	36.00	37.00	34.00
3	28.00	31.00	33.00	34.00	32.00	28.00
4	23.00	30.00	31.00	29.00	33.00	23.00
5	22.00	28.00	26.00	30.00	34.00	22.00
6	20.00	23.00	27.00	31.00	999.99	20.00

Day 2 (Tuesday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	999.99	999.99	57.00	58.00	57.00
1	999.99	999.99	54.00	55.00	56.00	__B__
2	999.99	51.00	52.00	53.00	51.00	51.00
3	43.00	49.00	50.00	48.00	47.00	43.00
4	41.00	47.00	45.00	44.00	47.00	41.00
5	39.00	42.00	41.00	44.00	46.00	39.00
6	34.00	38.00	41.00	43.00	999.99	34.00

Day 1 (Monday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	999.99	70.00	71.00	72.00	70.00
1	999.99	67.00	68.00	__C__	65.00	65.00
2	59.00	65.00	66.00	62.00	64.00	59.00
3	57.00	63.00	59.00	61.00	63.00	57.00
4	55.00	56.00	58.00	60.00	59.00	55.00
5	48.00	55.00	57.00	56.00	999.99	48.00
6	47.00	54.00	53.00	999.99	999.99	47.00

Version C -- Solution

56:171 Operations Research
Quiz #12 – Fall 2002

Determinstic Production Planning

We wish to plan next week's production (Monday through Saturday) of an expensive, low-demand item.

- the cost of *production* is **$6** for setup, plus **$4** per unit produced, up to a maximum of 4 units.
- the *storage* cost for inventory is **$1** per unit, based upon the level at the beginning of the day.
- a maximum of **6** units may be kept in inventory at the end of each day; any excess inventory is simply discarded.
- the demand D is random, with the same probability distribution each day:

Stage	1	2	3	4	5	6
Day	Mon.	Tues	Wed	Thurs	Fri	Sat
Demand	3	4	1	2	1	2
Produce	**2**	**4**	**0**	**3**	**0**	**2**

- no shortages are allowed.
- the initial inventory is **2**.
- a *salvage* value of **$3** per unit is received for any inventory remaining at the end of the last day (Saturday).

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 1= Monday, stage 2= Tuesday, etc.** We define

S_n = stock on hand at stage n.

$f_n(S_n)$ = minimum total cost for the days n, n+1, …6, if at the beginning of day n the stock on hand is S_n.

Thus, we seek the value of $f_1(2)$, i.e., the minimum expected cost for six days, beginning with two units in inventory.

(a.) What is the value of $f_1(2)$? $ **73.00**

(b.) What should be the production quantity for Monday? **2**

(c.) What is the total cost (production + storage – salvage value) of the optimal production schedule for all six days? $ **73.00**

(d.) Three values have been blanked out in the computer output, What are they?

- the cost associated with the decision to produce 1 unit on Friday when the inventory is 1 at the end of Thursday. **(A)** **19.00** *(Note: this may or may not be the optimal decision!)*
- the optimal value $f_2(1)$, i.e., the minimum total cost of the last 5 days (Tuesday through Saturday) if there is one unit of stock on hand Tuesday morning. **(B) $** **57.00**
- the cost associated with the decision to produce 2 units on Monday, when there is initially one unit in stock. **(C.) $** **76.00**

(e.) Complete the last row of the table above, indicating the optimal production quantity each day.

```
  s \ x: 0      1      2      3      4   | Minimum
  0   | 999.99 999.99  14.00  15.00  16.00|  14.00
  1   | 999.99  11.00  12.00  13.00  14.00|  11.00
  2   |   2.00   9.00  10.00  11.00  12.00|   2.00
  3   |   0.00   7.00   8.00   9.00  10.00|   0.00
  4   |  ¯2.00   5.00   6.00   7.00  14.00|  ¯2.00
  5   |  ¯4.00   3.00   4.00  11.00 999.99|  ¯4.00
  6   |  ¯6.00   1.00   8.00 999.99 999.99|  ¯6.00
```

Stage 6 (Saturday)

Day 5 (Friday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	24.00	25.00	20.00	22.00	20.00
1	15.00	$\boxed{\_\_A\_\_}$	17.00	19.00	21.00	15.00
2	13.00	14.00	16.00	18.00	20.00	13.00
3	5.00	13.00	15.00	17.00	19.00	5.00
4	4.00	12.00	14.00	16.00	999.99	4.00
5	3.00	11.00	13.00	999.99	999.99	3.00
6	2.00	10.00	999.99	999.99	999.99	2.00

Day 4 (Thursday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	999.99	34.00	33.00	35.00	33.00
1	999.99	31.00	30.00	32.00	28.00	28.00
2	22.00	27.00	29.00	25.00	28.00	22.00
3	18.00	26.00	22.00	25.00	28.00	18.00
4	17.00	19.00	22.00	25.00	28.00	17.00
5	10.00	19.00	22.00	25.00	999.99	10.00
6	10.00	19.00	22.00	999.99	999.99	10.00

Day 3 (Wednesday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	43.00	42.00	40.00	40.00	40.00
1	34.00	39.00	37.00	37.00	40.00	34.00
2	30.00	34.00	34.00	37.00	34.00	30.00
3	25.00	31.00	34.00	31.00	35.00	25.00
4	22.00	31.00	28.00	32.00	999.99	22.00
5	22.00	25.00	29.00	999.99	999.99	22.00
6	16.00	26.00	999.99	999.99	999.99	16.00

Day 2 (Tuesday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	999.99	999.99	999.99	62.00	62.00
1	999.99	999.99	999.99	59.00	57.00	$\boxed{\_\_B\_\_}$
2	999.99	999.99	56.00	54.00	54.00	54.00
3	999.99	53.00	51.00	51.00	50.00	50.00
4	44.00	48.00	48.00	47.00	48.00	44.00
5	39.00	45.00	44.00	45.00	49.00	39.00
6	36.00	41.00	42.00	46.00	44.00	36.00

Day 1 (Monday)

s \ x:	0	1	2	3	4	Minimum
0	999.99	999.99	999.99	80.00	79.00	79.00
1	999.99	999.99	$\boxed{\_\_C\_\_}$	76.00	77.00	76.00
2	999.99	74.00	73.00	74.00	74.00	73.00
3	65.00	70.00	71.00	71.00	69.00	65.00
4	61.00	68.00	68.00	66.00	65.00	61.00
5	59.00	65.00	63.00	62.00	63.00	59.00
6	56.00	60.00	59.00	60.00	999.99	56.00