56:171
Operations Research
Fall 2000

Quiz Solutions

Dept of Mechanical & Industrial Engineering
University of Iowa

56:171 Operations Research
Quiz #1 Solutions – August 30, 2000

For each statement, indicate "+"=**true** or "o"=**false**.

version a:

__o_ 1. When you enter an LP formulation into LINDO, you must first convert all inequalities to equations.

__o_ 2. When you enter an LP formulation into LINDO, you must include any nonnegativity constraints.

__+_ 3. When you enter an LP formulation into LINDO, you must manipulate your constraints so that all variables appear on the left, and all constants on the right.

__+_ 4. LINDO would interpret the constraint "X1 + 2X2 > 10" as "X1 + 2X2 ≥ 10".

version b:

__+_ 1. LINDO would interpret the constraint "X1 + 2X2 > 10" as "X1 + 2X2 ≥ 10".

__+_ 2. When you enter an LP formulation into LINDO, you need not explicitly include any nonnegativity constraints.

__o_ 3. When you enter an LP formulation into LINDO, you must first convert all inequalities to equations.

__+_ 4. When you enter an LP formulation into LINDO, you must manipulate your constraints so that all variables appear on the left, and all constants on the right.

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Walnut Orchard has two farms that grow wheat and corn. Because of differing soil conditions, there are differences in the yields and costs of growing crops on the two farms. The yields and costs are shown in the table below. Farm #1 has 100 acres available for cultivation, while Farm #2 has 150 acres. The farm has contracted to grow 11,000 bushels of corn and 6000 bushels of wheat. Determine a planting plan that will minimize the cost of meeting these contracts.

	Farm #1	Farm #2
Corn yield/acre	100 bushels	120 bushels
Cost/acre of corn	$90	$115
Wheat yield/acre	40 bushels	35 bushels
Cost/acre of wheat	$90	$80

Decision variables:

C1 = # of acres of Farm 1 planted in corn
W1 = # of acres of Farm 1 planted in wheat
C2 = # of acres of Farm 2 planted in corn
W2 = # of acres of Farm 2 planted in wheat

The model & LINDO output is below:

```
MIN     90 C1 + 115 C2 + 90 W1 + 80 W2
  SUBJECT TO
         2)   C1 + W1 <=   100
         3)   C2 + W2 <=   150
         4)   100 C1 + 120 C2 >=   11000
         5)   40 W1 + 35 W2 >=   6000
  END
```

5. Complete the right-hand-sides above.

```
        OBJECTIVE FUNCTION VALUE
        1)      24096.15

  VARIABLE        VALUE          REDUCED COST
        C1         3.846154          0.000000
        C2        88.461540          0.000000
        W1        96.153847          0.000000
        W2        61.538460          0.000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000        17.692308
        3)         0.000000        14.230769
        4)         0.000000        -1.076923
        5)         0.000000        -2.692308
```

6. *version a:*

The optimal solution is to plant __3.846_ acres of Farm#1 in corn and _96.154_ acres in wheat.

version b:

The optimal solution is to plant 88.461 acres of Farm#2 in corn and 61.438 acres in wheat.

7. *version a:* A total of 92.3077 acres will be planted in corn.
version b: A total of 157.692 acres will be planted in wheat.

8. The total cost of satisfying the grain contracts is $24,096.15 .

Multiple choice:

_e_ 9. The additional restriction that the planted acres of Farm #1 cannot be more than 75% wheat could be stated as the linear inequality:

a. $W1 \leq 75$ | d. $C1 \geq 25$
b. $25W1 - 75C1 \leq 0$ | e. $25W1 - 75C1 \geq 0$
c. $75W1 - 25C1 \geq 0$ | f. $75W1 - 25C1 \leq 0$

Note:

$$\frac{W1}{W1+C1} \leq 0.75 \Rightarrow W1 \leq 0.75(W1+C1) \Rightarrow 0.25W1 - 0.75C1 \leq 0$$

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Quiz #2 Solution – September 13, 2000

Below are several simplex tableaus. Assume that the objective in each case is to be **minimized**. Classify each tableau by writing to the right of the tableau a letter **A** through **G**, according to the descriptions below. *Also answer the question accompanying each classification, if any.*

(**A**) Nonoptimal, nondegenerate tableau with bounded solution. *Circle a pivot element which would improve the objective.*

(**B**) Nonoptimal, degenerate tableau with bounded solution. *Circle an appropriate pivot element.*

(**C**) Unique optimum.

(**D**) Optimal tableau, with alternate optimum. *Circle a pivot element which would lead to another optimal basic solution.*

(**E**) Objective unbounded (below). *Specify a variable which, when going to infinity, will make the objective arbitrarily low.*

(F) Tableau with infeasible primal

Warning: *Some of these classifications might be used for more than one tableau, while others might not be used at all*

(1) -z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	-1	3	0	0	2	2	-36	
0	3	0	4	0	0	1	3	0	9	__B_
0	-1	1	$\boxed{2}$	-5	0	0	-2	1	0	
0	6	0	3	-2	1	0	-4	3	5	

(2) -z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	3	0	0	2	-2	-36	
0	3	0	4	0	0	1	3	0	9	___A__
0	-1	1	-2	-5	0	0	-2	1	4	
0	6	0	3	-2	1	0	-4	$\boxed{3}$	5	

(3) -z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	-1	3	0	0	2	2	-36	
0	3	0	4	0	0	1	3	0	9	__B_
0	-1	1	-2	-5	0	0	-2	1	0	
0	6	0	$\boxed{3}$	-2	1	0	-4	3	5	

(4) -z	X_1	X_2	$\boxed{X_3}$	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	-1	3	0	0	2	2	-36	
0	3	0	-4	0	0	1	3	0	9	___E__
0	-1	1	-2	-5	0	0	-2	1	4	
0	6	0	0	-2	1	0	-4	3	5	

(5) -z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	-1	3	0	0	2	2	-36	
0	3	0	4	1	0	1	3	0	9	__F__
0	-1	1	-2	-5	0	0	-2	1	-4	
0	6	0	3	2	1	0	-4	3	5	

(6) -z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	0	0	1	3	0	0	2	2	-36	
0	3	0	4	0	0	1	3	0	9	__D__
0	-1	1	-2	-5	0	0	-2	1	4	
0	$\boxed{6}$	0	3	-2	1	0	-4	3	5	

(7) -z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	1	3	0	0	2	2	-36	
0	3	0	4	0	0	1	3	0	9	_C__
0	-1	1	-2	-5	0	0	-2	1	4	
0	6	0	3	-2	1	0	-4	3	5	

True (+) or False (o)?

__o__ 8. If, when solving an LP by the simplex method, you make a mistake in choosing a pivot column, the resulting tableau is infeasible. *Note: Objective will get worse, not better!*

_+___9. An LP with 5 variables and 2 equality constraints can have as many as (but no more than) ten basic solutions. *Note: the number of ways to choose 2 basic variables from five possible variables is* $\binom{5}{2} = \frac{5!}{2!3!} = \frac{5\times4\times3\times2\times1}{(2\times1)(3\times2\times1)} = 10$

_+___10. If, when solving an LP by the simplex method, you make a mistake in choosing a pivot row, the resulting tableau is infeasible.

_+___11. In the simplex method, every variable of the LP is either basic or nonbasic..

_+___12. In the simplex tableau, the objective row is written in the form of an equation.

__o _ 13. When you enter an LP formulation into LINDO, you must first convert all inequalities to equations. *Note: LINDO will perform this transformation automatically, before beginning the simplex method.*

__o _ 14. It may happen that an LP problem has (exactly) two optimal solutions. *Note: If two solutions are optimal, then the line segment (containing infinitely many points) joining them is optimal.*

__o _ 15. The restriction that X1 be nonnegative should be entered into LINDO as the constraint X1 >= 0. *Note: LINDO assumes that all variables are nonnegative.*

_+___16. A "pivot" in a nonbasic column of a tableau will make it a basic column.

_+___17. The "minimum ratio test" is used to select the pivot row in the simplex method for linear programming.

_+___ 18. In the simplex method (as described in the lectures, not the textbook), the quantity $-Z$ serves as a basic variable, where Z is the value of the objective function.

_o __ 19. Every optimal solution of an LP is a basic solution. *Note: The midpoint between two optimal corner-point solutions is optimal, but not basic.*

_+___ 20. Basic feasible solutions of an LP with constraints $Ax \leq b,\ x \geq 0$ correspond to "corner" points of the feasible region.

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Quiz #3 Solutions -- Fall 2000

There are three versions of the quiz:

Primal	Dual
Min $w = 4Y_1 + 2Y_2 - Y_3$ s.t. $Y_1 + 2Y_2 \le 10$ $Y_1 - Y_2 + 2Y_3 \ge 8$ $Y_1 \le 0,\ Y_3 \ge 0$ (Y_2 is unrestricted in sign)	Max $10X_1 + 8X_2$ s.t. $X_1 + X_2 \ge 4$ $2X_1 - X_2 = 2$ $2X_2 \le -1$ $X_1 \le 0,\ X_2 \ge 0$
Min $w = 4Y_1 + 2Y_2 - Y_3$ s.t. $Y_1 + 2Y_2 \ge 10$ $Y_1 - Y_2 + 2Y_3 = 8$ $Y_1 \ge 0,\ Y_2 \le 0$ (Y_3 is unrestricted in sign)	Max $10X_1 + 8X_2$ s.t. $X_1 + X_2 \le 4$ $2X_1 - X_2 \ge 2$ $2X_2 = -1$ $X_1 \ge 0$, X_2 unrestricted in sign
Min $w = 4Y_1 + 2Y_2 - Y_3$ s.t. $Y_1 + 2Y_2 = 10$ $Y_1 - Y_2 + 2Y_3 \ge 8$ $Y_1 \le 0,\ Y_2 \ge 0$ (Y_3 is unrestricted in sign)	Max $10X_1 + 8X_2$ s.t. $X_1 + X_2 \ge 4$ $2X_1 - X_2 \le 2$ $2X_2 = -1$ X_1 unrestricted in sign , $X_2 \ge 0$

For each statement, indicate "+"=**true** or "o"=**false**.

__+__ 1. According to the Complementary Slackness Theorem, if constraint #1 of the primal problem is slack, then variable #1 of the dual problem must be zero.

__+__ 2. If you increase the right-hand-side of a "≥" constraint in a <u>min</u>imization LP, the optimal objective value will either increase or stay the same.

__o__ 3. If the increase in the right-hand-side of a "tight" constraint remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.
Note: The basis will not change; however, if the right-hand-side b changes, then the *values* of the basic variables, given by $x_B = \left(A^B\right)^{-1} b$ will change.

__+__ 4. At the completion of the revised simplex method applied to an LP, the simplex multipliers give the optimal solution to the dual of the LP.

__o__ 5. If a minimization LP problem is feasible and unbounded below, then its dual problem has an objective (to be maximized) which must be unbounded above.
Note: In this case, the dual problem cannot be feasible.

__o__ 6. According to the Complementary Slackness Theorem, if variable #1 of the primal problem is zero, then constraint #1 of the dual problem must be tight.
Note: No conclusion can be drawn in this case-- the constraint may be either slack or (in the case of degeneracy) tight.

__+__ 7. If the increase in the cost of a nonbasic variable remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.

__o__ 8. The "reduced cost" in an LP solution provides an estimate of the change in the objective value when a basic variable increases.
Note: The reduced cost of a *basic* variable is zero, and gives no information about the effect of changing that basic variable.

__o__ 9. In the two-phase simplex method, Phase One computes the optimal dual variables, followed by Phase Two in which the optimal primal variables are computed.
Note: The purpose of Phase One is to get an initial basic feasible (primal) solution, usually by introducing artificial variables and then forcing them to be zero.

__o__ 10. If a minimization LP problem has a cost which is infeasible, then its dual problem cannot be feasible.
Note: In this case, the dual maximization problem might be infeasible or feasible (but if feasible, its profit must be unbounded above.)

FYI:

Maximize	Minimize
Type of constraint i:	Sign of variable i:
≤	nonnegative
=	unrestricted in sign
≥	nonpositive
Sign of variable j:	Type of constraint j:
nonnegative	≥
unrestricted in sign	=
nonpositive	≤

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Quiz #4 Solutions – Fall 2000

Consider again the LP model of PAR, Inc., which manufactures standard and deluxe golf bags:

X1 = number of **STANDARD** golf bags manufactured per quarter
X2 = number of **DELUXE** golf bags manufactured per quarter

Four operations are required, with the time per golf bag as follows:

	STANDARD	DELUXE	Available
Cut-&-Dye	0.7 hr	1 hr	630 hrs.
Sew	0.5 hr	0.8666 hr	600 hrs.
Finish	1 hr	0.6666 hr	708 hrs.
Inspect-&-Pack	0.1 hr	0.25 hr	135 hrs.
Profit ($/bag)	$10	$9	

LINDO provides the following output:

```
MAX     10 X1 + 9 X2
 SUBJECT TO
        2)   0.7 X1 + X2 <=   630
        3)   0.5 X1 + 0.86666 X2 <=   600
        4)   X1 + 0.66666 X2 <=   708
        5)   0.1 X1 + 0.25 X2 <=   135
 END
        OBJECTIVE FUNCTION VALUE
        1)    7668.01200

  VARIABLE        VALUE          REDUCED COST
        X1       540.003110           .000000
        X2       251.997800           .000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)          .000000          4.375086
        3)       111.602000           .000000
        4)          .000000          6.937440
        5)        18.000232           .000000

RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE          CURRENT          ALLOWABLE         ALLOWABLE
                    COEF            INCREASE          DECREASE
       X1        10.000000          3.500135          3.700000
       X2         9.000000          5.285715          2.333400

                          RIGHTHAND SIDE RANGES

      ROW          CURRENT          ALLOWABLE         ALLOWABLE
                    RHS             INCREASE          DECREASE
        2       630.000000          52.364582        134.400000
        3       600.000000           INFINITY        111.602000
        4       708.000000         192.000010        128.002800
        5       135.000000           INFINITY         18.000232
```

THE TABLEAU

ROW	(BASIS)	X1	X2	SLK 2	SLK 3	SLK 4	SLK 5	
1	ART	.00	.00	4.375	.00	6.937	.00	7668.012
2	X2	.00	1.00	1.875	.00	-1.312	.00	251.998
3	SLK 3	.00	.00	-1.000	1.00	.200	.00	111.602
4	X1	1.00	.00	-1.250	.00	1.875	.00	540.003
5	SLK 5	.00	.00	-.344	.00	.141	1.00	18.000

Enter the correct answer into each blank or check the correct alternative answer, as appropriate. If not sufficient information, write "NSI" in the blank:

a. If the profit on STANDARD bags were to decrease from $10 each to $7 each, the number of STANDARD bags to be produced would

|__| increase |__| decrease |_X_| remain the same |__| not sufficient info.

Note: $3 is less than the ALLOWABLE DECREASE in the OBJ COEFFICIENT RANGES.

b. If the profit on DELUXE bags were to increase from $9 each to $15 each, the number of DELUXE bags to be produced would

|_X_| increase |__| decrease |__| remain the same |__| not sufficient info.

Note: $6 exceeds the ALLOWABLE INCREASE in the OBJ COEFFICIENT RANGES, which means that the basis will change. It seems clear that the basis change would result in an increase in production.

c. The LP problem above has

|_X_| exactly one optimal solution |__|exactly two optimal solutions

|__| an infinite number of optimal solutions

Note: there is no nonbasic variable (SLK2 nor SLK4) with a zero reduced cost.

d. If an additional 10 hours were available in the inspect-&-pack department, PAR would be able to obtain an additional $_0___ in profits.

Note: The dual price is zero, and the 10 hour increase is less than the ALLOWABLE INCREASE in the RHS RANGES table.

e. If an additional 10 hours were available in the cut-&-dye department, PAR would be able to obtain an additional $43.75 in profits.

Note: The dual price is $4.375086/hour, and the 10 hour increase is less than the ALLOWABLE INCREASE in the RHS RANGES table.

f. If the variable "SLK 2" were increased, this would be equivalent to

_X_ decreasing the hours used in the cut-&-dye department

___ increasing the hours used in the cut-&-dye department

___ none of the above

Note: SLK2 is the unused hours in the cut-&-dye department-- more unused hours means fewer hours used.

g. If the variable "SLK 2" were increased by 10, X1 would |X| increase |__| decrease by _12.5__ STANDARD golf bags/quarter.

Note: the substitution rate (from the TABLEAU) is –1.25.

h. If the variable "SLK 2" were increased by 10, X2 would |__| increase |X| decrease by _18.75__ DELUXE golf bags/quarter.

Note: the substitution rate (from the TABLEAU) is _1.875.

i. If a pivot were to be performed to enter the variable SLK2 into the basis, then according to the "minimum ratio test", the value of SLK2 in the resulting basic solution would be approximately

_X 252/1.875 __ 111.6 __540/1.25 __18/0.344

__1.875/252 __1/111.6 __1.25/540 __0.344/18

__ not sufficient information

j. If the variable SLK2 were to enter the basis, then the variable _X2___ will leave the basis.

Note: since there is only one positive substitution rate (in the row in which X2 is basic), that row must be the pivot row. The ratio of RHS/substitution rate is approximately 252/1.875 = 134.4.

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Quiz #5 Solutions-- Fall 2000

PART ONE: Data Envelopment Analysis (Note: *DMU* = "decision-making-unit")

_c_ 1. In the *maximization* problem of the primal-dual pair of LP models, the decision variables are:
a. The amount of each input and output to be used by the DMU
b. The multipliers for the DMUs which are used to form a linear combination of DMUs in the reference set.
c. The "prices" assigned to the inputs and outputs.
c. None of the above

_a_ 2. The "prices" or weights assigned to the input & output variables in the maximization problem must
a. be nonnegative
b. sum to 1.0
c. Both a & b
d. Neither a nor b.

True (+) or false (o)?

_+_ 3. To perform a complete DEA analysis, an LP must be solved for *every* DMU.
_o_ 4. In the maximization LP form of the problem, all constraints have non-zero right-hand-sides.
Note: One constraint states that the value of the denominator in the efficiency ratio equals 1.0.
_+_ 5. There is a constraint for *every* DMU (in the maximization LP form of the problem).
_+_ 6. The optimal value of the LP cannot exceed 1.0.
_o_ 7. The number of input and output variables must be equal.

✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲

Version A: *Assignment Problem.* Three machines are to be assigned to three jobs (one machine per job), so that total machine hours used is minimized. The hours required by each machine for the jobs is given in the table below.

	Job 1	Job 2	Job 3
Machine A	4	2	9
Machine B	2	1	5
Machine C	5	2	10

a. Perform the row reduction step of the Hungarian method so that every row contains at least one zero. (Write the updated matrix below.) *Subtract 2 from each time in row 1, 1 from every time in row 2, and 2 from every time in row 3 above:*

	Job 1	Job 2	Job 3
Machine A	2	0	7
Machine B	1	0	4
Machine C	3	0	8

b. Perform the column reduction step so that every column contains at least one zero, and write the updated matrix below:
Subtract 1 from every time in column 1 and 4 from every time in column 3 above:

	Job 1	Job 2	Job 3
Machine A	1	0	3
Machine B	0	0	0
Machine C	2	0	4

c. What is the smallest number of (horizontal & vertical) lines required to cover all the zeroes? _2__ *(A line through row 2 and a line through column 2)*

d. Are any further steps required? If so, perform the next step, and write the resulting matrix below:
Subtract 1 from every time without a line through it and add 1 to the intersection of the two lines:

	Job 1	Job 2	Job 3
Machine A	0	0	2
Machine B	0	1	0
Machine C	1	0	3

e. What is now the smallest number of (horizontal & vertical) lines required to cover all the zeroes? _3__

f. Find the optimal assignment: Machine A performs job _1_. Machine B performs job _3_. Machine C performs job _2_. Total machine hours required is _11_.

g. The assignment problem can be modeled as a transportation problem with _3_ sources and __3_ destinations, with the supplies available at the sources equal to __1__ and the demands at the destinations equal to ___1___. The number of basic variables will be __5__(=m+n-1) , while the number of positive variables in a basic solution will be __3__(<5). Every basic solution is therefore classified as "degenerate".

✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲ ✲

Version B: ***Assignment Problem.*** **Three machines are to be assigned to three jobs (one machine per job),** so that total machine hours used is minimized. The hours required by each machine for the jobs is given in the table below.

	Job 1	Job 2	Job 3
Machine A	5	2	2
Machine B	9	4	5
Machine C	10	5	7

a. Perform the row reduction step of the Hungarian method so that every row contains at least one zero. (Write the updated matrix below.) *Subtract 2 from every time in row 1, 4 from every time in row 2, and 5 from every time in row 3:*

	Job 1	Job 2	Job 3
Machine A	3	0	0
Machine B	5	0	1
Machine C	5	0	2

b. Perform the column reduction step so that every column contains at least one zero, and write the updated matrix below.
Subtract 3 from every time in column 1:

	Job 1	Job 2	Job 3
Machine A	0	0	0
Machine B	2	0	1
Machine C	2	0	2

c. What is the smallest number of (horizontal & vertical) lines required to cover all the zeroes? _2__

d. Are any further steps required? If so, perform the next step, and write the resulting matrix below:
Subtract 1 from every time without a line through it, and add 1 to the intersections of the two lines:

	Job 1	Job 2	Job 3
Machine A	0	1	0
Machine B	1	0	0
Machine C	1	0	1

e. What is now the smallest number of (horizontal & vertical) lines required to cover all the zeroes? _3__

f. Find the optimal assignment: Machine A performs job _1_. Machine B performs job _3_. Machine C performs job _2_. Total machine hours required is _15_.

g. *Same as above.*

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Quiz #6 Solution -- Fall 2000

Consider the following payoff table in which you must choose among three possible alternatives, after which two different states of nature may occur. (Nothing is known about the probability distribution of the state of nature.)

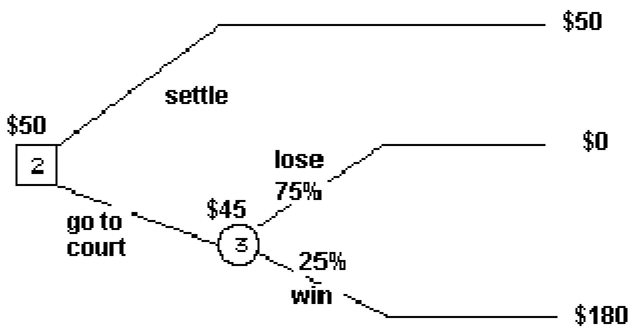
	State of Nature			
Decision	1	2	Min	Max
1	4	2	2	4
2	6	4	4	6
3	3	8	3	8

1. What is the optimal decision if the maximin criterion is used? __2__
2. What is the optimal decision if the maximax criterion is used? __3__
3. Create the regret table:

	State of Nature		
Decision	1	2	Max
1	2	6	6
2	0	4	4
3	3	0	3

4. What is the optimal decision if the minimax regret is used? __3__

General Custard Corporation is being sued by Sue Smith. Sue must decide whether to accept an offer of $50,000 by the corporation to settle out of court, or she can go to court. If she goes to court, there is a 25% chance that she will win the case *(event W)* and a 75% chance she will lose *(event L)*. If she wins, she will receive $180,000, but if she loses, she will net $0. A decision tree representing her situation appears below, where payoffs are in thousands of dollars:



5. What is the decision which maximizes the expected value? _X_ settle ___go to court

For $20,000, Sue can hire a consultant who will predict the outcome of the trial, i.e., either he predicts a loss of the suit *(event PL)*, or he predicts a win *(event PW)*. The consultant is correct 80% of the time, e.g., if the suit will win, the probability that the consultant predicts the win is 80%.

Bayes' Rule states that if S_i is one of the *n* states of nature and O_j is the outcome of an experiment,

$$P\{S_i|O_j\}=\frac{P\{O_j|S_i\}P\{S_i\}}{P\{O_j\}},\ \text{where}\ P\{O_j\}=\sum_k P\{O_j|S_k\}P\{S_k\}$$

6. The probability that the consultant will predict a win, i.e. P{PW} is 35%

$$P\{PW\}=P\{PW|W\}P\{W\}+P\{PW|L\}P\{L\}$$
$$=0.8\times0.25+0.2\times0.75=0.35$$

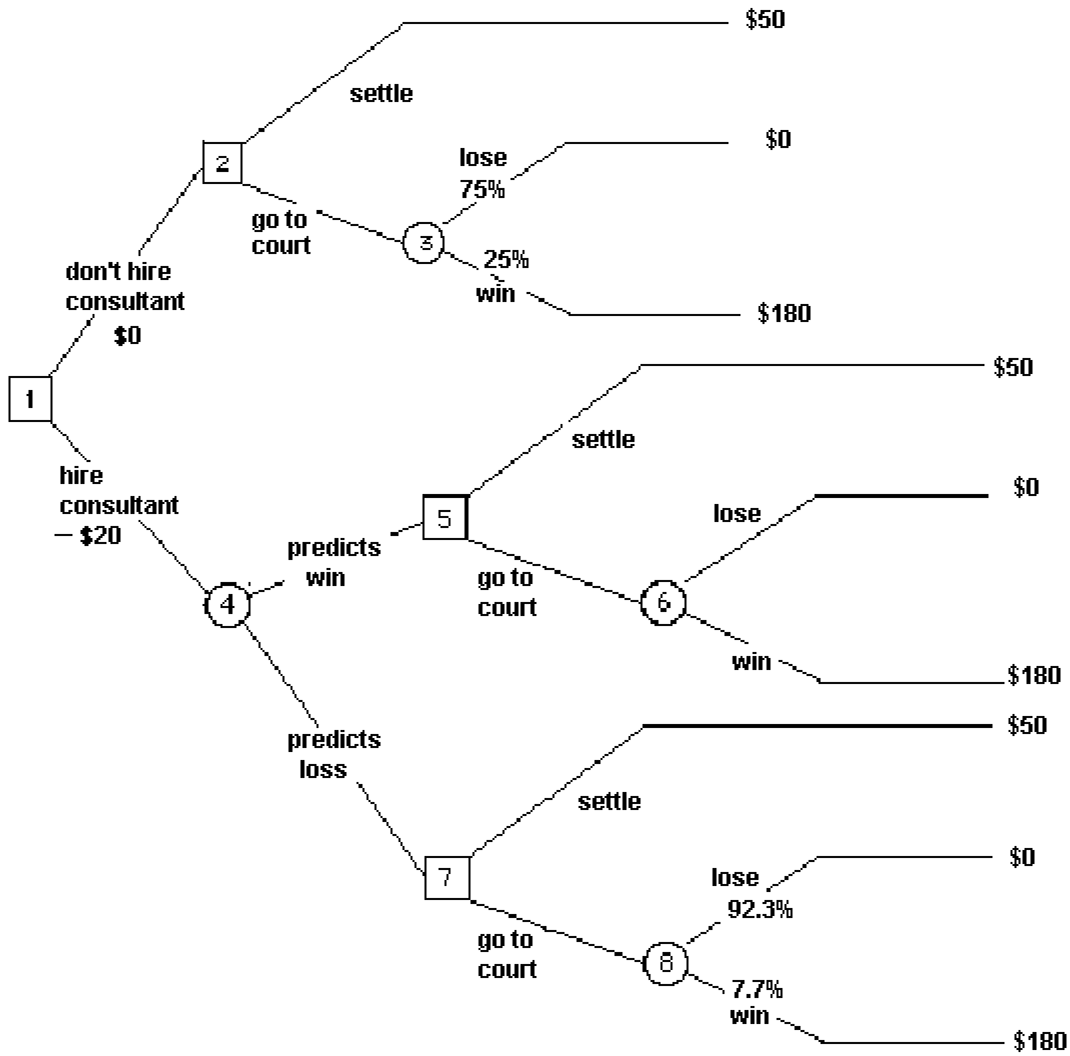
The probability that the consultant will predict a loss, i.e. P{PL} is 65%

$$P\{PL\} = P\{PL|L\}P\{L\} + P\{PW|W\}P\{W\}$$
$$= 0.8\times 0.75 + 0.2\times 0.25 = 0.65$$

7. According to Bayes' theorem, the probability that Sue *will* win, given that the consultant predicts a win, is

$$P\{W|PW\} = \frac{P\{PW|W\}P\{W\}}{P\{PW\}} = \frac{0.8\times 0.25}{0.35} = 0.5714$$

The decision tree below includes Sue's decision as to whether or not to hire the consultant. Insert the probability that you computed in (7) above, together with its complement $P\{L|PW\}$.



Note that in this tree, the cost of the consultant is not included in the "payoffs" at the ends of the tree, but is included only when comparing the payoffs at nodes 2 & 4.

8. "Fold back" nodes 2 through 8, and write the value of each node below:

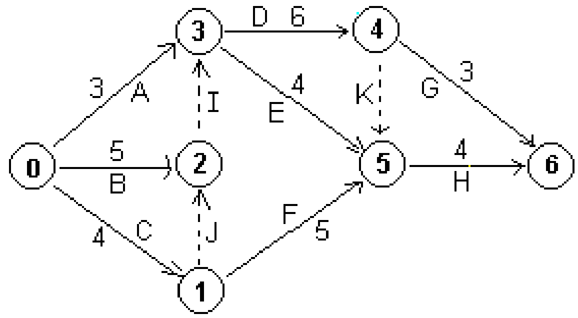
Node	Value	Node	Value	Node	Value
8	13.86	5	102.85	2	50
7	50	4	68.5	1	50
6	102.85	3	45		

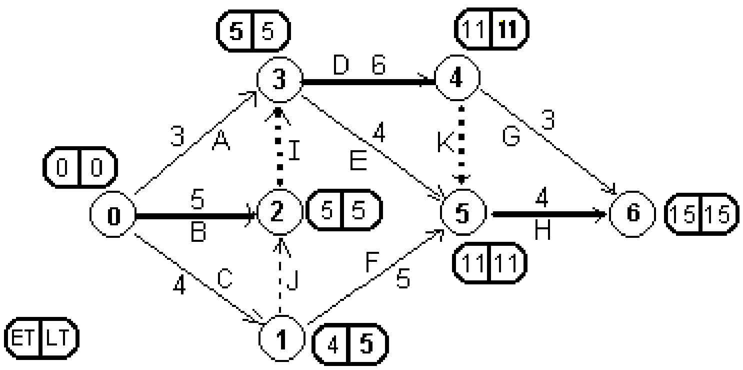
9. Should Sue hire the consultant? *Circle:* Yes $\boxed{\text{No}}$

10. The expected value of the consultant's opinion is (in thousands of $) $*18,500*

56:171 Operations Research
Quiz #7 Solutions -- Version A -- Fall 2000

a. Complete the labeling of the nodes on the A-O-A project network (so that if arrow goes from node **i** to node **j**, then **i**<**j**). *Note that I, J, & K are "dummy" activities.*





b. The activity durations are given on the arrows. Finish computing the Early Times (ET) and Late Times (LT) for each node, writing them in the box (with rounded corners) beside each node.

c. Complete the table:

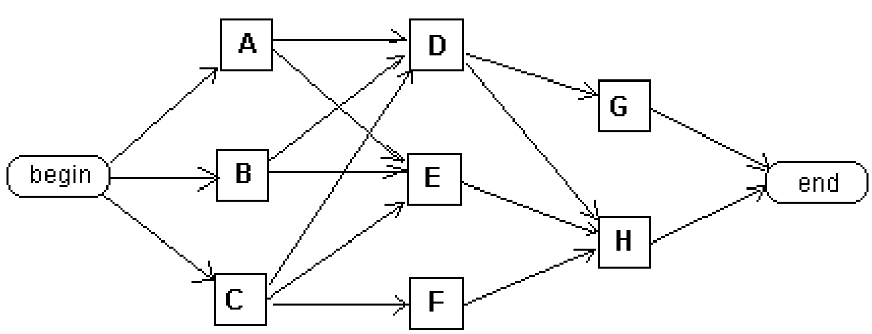
Activity	Duration	Early Start	Early Finish	*Late Start*	Late Finish	Total Slack
A	3	0	3	*2*	5	2
B	5	0	5	*0*	5	0
C	4	0	4	*1*	5	1
D	6	5	11	*5*	11	0
E	4	5	9	*7*	11	2
F	5	4	9	*6*	11	2
G	3	11	14	*12*	15	1
H	4	11	15	*11*	15	0

Note: Find Early Start from ET at beginning node, Early Finish = Early Start + duration, Late Finish = LT at end node. Total Slack = difference between early finish & late finish.

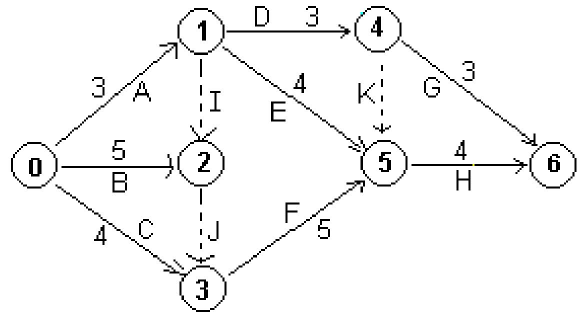
d. Which activities are critical? (circle: A $\boxed{B}$ C $\boxed{D}$ E F G $\boxed{H}$ $\boxed{I}$ J $\boxed{K}$) *Note: those with slack=0!*

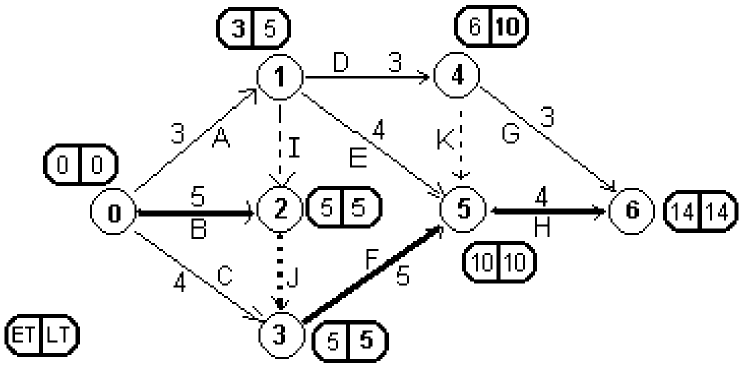
e. Suppose that the durations are random, with the expected value as given, but with standard deviations all equal to 1.00. What is the standard deviation of the project completion time? _√3 = 1.7322_
Note: Variance of sum of three durations = sum of variances! So variance = 3 and std deviation = Ö3

f. Complete the A-O-N (activity-on-node) network for this same project. (Add any "dummy" activities which are necessary.) *Note: A-O-N network does not require any "dummy activities" (other than "begin" and "end").*



a. Complete the labeling of the nodes on the A-O-A project network above (so that if arrow goes from node **i** to node **j**, then **i<j**). *Note that I, J, & K are "dummy" activities.*





b. The activity durations are given below on the arrows. Finish computing the Early Times (ET) and Late Times (LT) for each node, writing them in the box (with rounded corners) beside each node.

c. Complete the table:

Activity	Duration	Early Start	Early Finish	*Late Start*	Late Finish	Total Slack
A	3	0	3	*2*	5	2
B	5	0	5	*0*	5	0
C	4	0	4	*1*	5	1
D	3	3	6	*7*	10	4
E	4	3	7	*6*	10	3
F	5	5	10	*5*	10	0
G	3	6	9	*11*	14	5
H	4	10	14	*10*	14	0

Note: Find Early Start from ET at beginning node, Early Finish = Early Start + duration, Late Finish = LT at end node. Total Slack = difference between early finish & late finish.

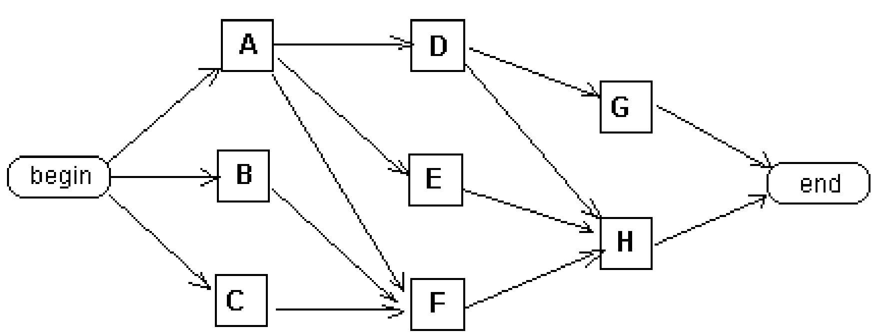
d. Which activities are critical? (circle: A $\boxed{B}$ C D E $\boxed{F}$ G $\boxed{H}$ I $\boxed{J}$ K) *Note: those with slack=0!*

e. Suppose that the durations are random, with the expected value as given, but with standard deviations all equal to 1.00. What is the standard deviation of the project completion time? _√3 = 1.7322_

Note: Variance of sum of three durations = sum of variances! So variance = 3 and std deviation = Ö3

f. Complete the A-O-N (activity-on-node) network for this same project. (Add any "dummy" activities which are necessary.)

Note: A-O-N network does not require any "dummy activities" (other than "begin" and "end").



56:171 Operations Research
Quiz #8 Solutions -- Fall 2000

1. The Cubs are trying to determine which of the following free agent pitchers should be signed: Rick Sutcliffe (RS), Bruce Sutter (BS), Dennis Eckersley (DE), Steve Trout (ST), Tim Stoddard (TS).

Define five binary variables, one for each pitcher: for example: **RS** = 1 if Rick Sutcliffe is signed, 0 otherwise
Find the appropriate constraint corresponding to each restriction. If none apply, write "z"

If DE and ST are signed, then BS cannot be signed:	DE + ST + BS ≤ 2
The Cubs cannot sign both ST and TS:	ST + TS ≤ 1
If TS is signed, then DE must also be signed:	TS ≤ DE
If DE is signed, then TS cannot be signed:	(z) *None of the above* (would need DE + TS ≤ 1)
If ST is not signed, then DE must be signed:	TS + DE ≥ 1
The Cubs must sign either BS or RS (or both):	RS + BS ≥ 1

a. ST + TS = 1	b. ST + TS ≤ 1	c. ST ≤ TS	d. TS ≤ ST
e. DE + ST ≥ 1	f. DE + ST = 1	g. DE + ST ≤ 1	h. DE ≤ ST
i. TS ≤ DE	j. TS + DE = 1	k. TS + DE ≥ 1	l. ST ≤ DE
m. DE + ST + BS ≤ 2	n. DE + ST + BS = 2	o. DE + ST + BS ≥ 2	p. BS ≤ DE+ ST
q. DE + ST–1 ≥ BS	r. ST + DE ≥ 1	s. TS ≥ DE	t. TS ≤ DE
u. DE + ST–1 ≤ BS	v. RS + BS ≥ 1	w. RS + BS ≤ 1	x. RS + BS = 1

z. *None of the above*

2. Four trucks are available to deliver milk to five grocery stores. Capacities and daily operating costs vary among the trucks. Trucks are to make a single trip per day. Each grocery may be supplied by only one truck, but a truck may deliver to more than one grocery in one trip. No more than $100 may be spent on the trucks.

Truck #	Capacity (gallons)	Daily operating cost ($)
1	400	45
2	500	50
3	600	55
4	1100	60

Grocery #	Daily Demand (gallons)
1	100
2	200
3	300
4	500
5	800

Define binary variables
Y_i = 1 if truck i is used, 0 otherwise
X_{ij} = 1 if truck i delivers to grocery j, 0 otherwise

Check each of the constraints below which would be valid in the integer LP model.

_√ $X_{13} + X_{23} + X_{33} + X_{43} = 1$
__ $X_{41} + X_{42} + X_{43} + X_{44} + X_{45} = 1$
__ $X_{31} + X_{32} + X_{33} + X_{34} + X_{35} \le 600Y_3$
_√ $100X_{31} + 200X_{32} + 300X_{33} + 500X_{34} + 800X_{35} \le 600Y_3$
__ $X_{41} + X_{42} + X_{43} + X_{44} + X_{45} \le Y_4$
__ $400X_{14} + 500X_{24} + 600X_{34} + 1100X_{44} \le 500Y_4$
_* $300X_{43} \le 1100Y_4$
_√ $45Y_1 + 50Y_2 + 55Y_3 + 60Y_4 \le 100$

_√ $X_{43} \le Y_4$
__ $Y_4 \le X_{43}$
__ $X_{13} + X_{23} + X_{33} + X_{43} \le 300Y_3$
__ $X_{13} + X_{23} + X_{33} + X_{43} \le 4Y_3$
_√ $X_{41} + X_{42} + X_{43} + X_{44} + X_{45} \le 5Y_4$
_* $X_{41} + X_{42} + X_{43} + X_{44} + X_{45} \le 1100$
__ $300X_{43} \ge 1100Y_4$
__ $45Y_1 + 50Y_2 + 55Y_3 + 60Y_4 = 100$

__

Note: Constraints indicated by "√" should be included in the integer LP model. Those indicated by "*" are valid inequalities, but are redundant and not needed.

56:171 Operations Research
Quiz #9 Solution – Fall 2000

Consider an **(s,S) inventory system** in which the number of items on the shelf is checked at the end of each day. The demand distribution is as follows:

n=	0	1	2
P{D=n}	0.2	0.5	0.3

To avoid shortages, the current policy is to restock the shelf at the end of each day (after spare parts have been removed) so that the shelf is again filled to its limit (i.e., 4) **if** there are fewer than 2 parts on the shelf. *(That is, it is an (s,S) inventory system, with s=2 and S=4.)*

The inventory system has been modeled as a Markov chain, with the state of the system defined as the end-of-day inventory level (before restocking). Refer to the computer output which follows to answer the following questions:

P =

\	0	1	2	3	4
0	0	0	0.3	0.5	0.2
1	0	0	0.3	0.5	0.2
2	0.3	0.5	0.2	0	0
3	0	0.3	0.5	0.2	0
4	0	0	0.3	0.5	0.2

P^2 =

\	0	1	2	3	4
0	0.09	0.3	0.37	0.2	0.04
1	0.09	0.3	0.37	0.2	0.04
2	0.06	0.1	0.28	0.4	0.16
3	0.15	0.31	0.29	0.19	0.06
4	0.09	0.3	0.37	0.2	0.04

P^3 =

\	0	1	2	3	4
0	0.111	0.245	0.303	0.255	0.086
1	0.111	0.245	0.303	0.255	0.086
2	0.084	0.26	0.352	0.24	0.064
3	0.087	0.202	0.309	0.298	0.104
4	0.111	0.245	0.303	0.255	0.086

P^4 =

\	0	1	2	3	4
0	0.0909	0.228	0.3207	0.272	0.0884
1	0.0909	0.228	0.3207	0.272	0.0884
2	0.1056	0.248	0.3128	0.252	0.0816
3	0.0927	0.2439	0.3287	0.2561	0.0786
4	0.0909	0.228	0.3207	0.272	0.0884

P^5 =

\	0	1	2	3	4
0	0.0962	0.2419	0.3223	0.2580	0.0814
1	0.0962	0.2419	0.3223	0.2580	0.0814
2	0.0938	0.2320	0.3191	0.2680	0.0870
3	0.0986	0.2412	0.3183	0.2588	0.0830
4	0.0962	0.2419	0.3223	0.2580	0.0814

$\sum_{n=1}^{5} P^n =$

\	0	1	2	3	4
0	0.388	1.015	1.616	1.485	0.495
1	0.388	1.015	1.616	1.485	0.495
2	0.643	1.340	1.463	1.160	0.392
3	0.428	1.297	1.746	1.202	0.325
4	0.388	1.015	1.616	1.485	0.495

First Passage Probabilities

n	$f_{4,0}^{(n)}$
1	0.0
2	0.09
3	0.111
4	0.0828
5	0.07623

Mean First Passage Times

\	0	1	2	3	4
0	10.408	4.192	2.653	2.75	11.953
1	10.408	4.192	2.653	2.75	11.953
2	7.755	2.822	3.122	4	13.203
3	10	3.014	2.245	3.83	13.984
4	10.408	4.192	2.653	2.75	11.953

Steady State Distribution

i	name	πi
0	SOH 0	0.09607
1	SOH 1	0.23856
2	SOH 2	0.32026
3	SOH 3	0.26144
4	SOH 4	0.08366

__c_ 1. the value $P_{4,2}$ is

a. P{demand=0} b. P{demand=1} c. P{demand=2}
d. P{demand≤1} e. P{demand≥1} f. *none of the above*

__b_ 2. the value $P_{0,3}$ is

a. P{demand=0} b. P{demand=1} c. P{demand=2}
d. P{demand≤1} e. P{demand≥1} f. *none of the above*

__c_ 3. the value $P_{2,0}$ is

a. P{demand=0} b. P{demand=1} c. P{demand=2}
d. P{demand≤1} e. P{demand≥1} f. *none of the above*

__d_ 4. If the shelf is full Monday morning, the expected number of days until a stockout occurs is *(select nearest value):*

a. 2.5 b. 5 c. 7.5 $\boxed{\text{d. 10}}$ (m_{40}=10.4) e. 12.5
f. 15 g. 17.5 h. 20 i. more than 20

__e_ 5. If the shelf is full Monday morning, the probability that the shelf is full Thursday night (i.e., after 4 days of sales) is *(select nearest value):*

a. 5% b. 6% c. 7% d. 8% $\boxed{\text{e. 9\%}}$ ($p_{44}^{(4)} =$ 0.0884)
f. 10% g. 11% h. 12% i. 13% j. ≥14%

__f_ 6. If the shelf is full Monday morning, the probability that the shelf is restocked Thursday night is *(select nearest value):*

a. 5% b. 10% c. 15% d. 20% e. 25%
$\boxed{\text{f. 30\%}}$ ($p_{4,0}^{(4)} + p_{4,1}^{(4)} = 0.0909 + 0.228 = 0.3189$) g. 35% h. 40% i. 45% j. ≥50%

__f__ 7. If the shelf is full Monday morning, the *expected* number of nights that the shelf is restocked during the next five nights is *(select nearest value):*

a. 0.25 b. 0.5 c. 0.75 d. 1 e. 1.25
$\boxed{\text{f. 1.5}}$ ($\sum_{n=1}^{5}\left(p_{40}^{(n)} + p_{41}^{(n)}\right) = 0.388 + 1.015 = 1.403$) g. 1.75 h. 2 i. 2.25 j. ≥2.5

__f_ 8. How frequently will the shelf be restocked? *(select nearest value): once every* ______ *days*

a. 0.5 days b. 1 days c. 1.5 days d. 2 days
e. 2.5 days $\boxed{\text{f. 3 days}}$ g. 3.5 days h. 4 days i. 4.5 days j. ≥5 days

Note: $\dfrac{1}{p_0 + p_1} = \dfrac{1}{0.09607 + 0.23856} = \dfrac{1}{0.33463} = 2.988$

__d_ 9. If the shelf is full Monday morning, what is the probability that the next stockout is Thursday night? *(select nearest value):*

a. 5% b. 6% c. 7% $\boxed{\text{d. 8\%}}$ ($f_{4,0}^{4} = 0.0828$) e. 9%
f. 10% g. 11% h. 12% i. 13% j. ≥14%

10. Circle (one or more) of the following equations which are among those solved to compute the steady state probability distribution:

$\boxed{\text{a.}}$ $p_0 = 0.3p_2$

b. $p_3 = 0.3p_0 + 0.5p_1 + 0.2p_2$

$\boxed{\text{c.}}$ $p_2 = 0.3p_0 + 0.3p_1 + 0.2p_2 + 0.5p_3 + 0.3p_4$

d. $p_4 = 0.3p_2 + 0.5p_3 + 0.2p_4$

e. $p_4 = 0.2p_2 + 0.5p_3 + 0.3p_4$

$\boxed{\text{f.}}$ $p_0 + p_1 + p_2 + p_3 + p_4 = 1$

56:171 Operations Research
Quiz #10 – Fall 2000

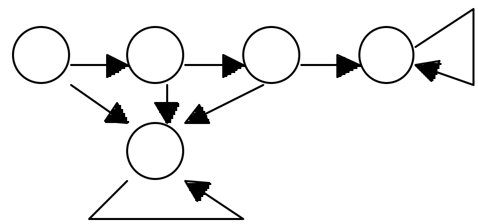
Part I. Coldspot manufactures refrigerators. The company has issued a warranty on all refrigerators that requires free replacement of any refrigerator that fails before it is three years old.

- 3% of all new refrigerators fail during their first year of operation.
- 5% of all 1-year-old refrigerators fail during their second year of operation.
- 7% of all 2-year-old refrigerators fail during their third year of operation.

Replacement refrigerators are not covered by the warranty.

Define a discrete-time Markov chain, with states

(0.) new refrigerators
(1.) 1-year-old refrigerators
(2.) 2-year-old refrigerators
(3.) refrigerators that have passed their third anniversary
(4.) replacement refrigerators



Define the stages to be years, with the refrigerator observed on the anniversaries of its purchase.

P=

	0	1	2	3	4
0	0	0.97	0	0	0.03
1	0	0	0.95	0	0.05
2	0	0	0	0.93	0.07
3	0	0	0	1	0
4	0	0	0	0	1

Q=

\	0	1	2
0	0	0.97	0
1	0	0	0.95
2	0	0	0

R=

\	3	4
0	0	0.03
1	0	0.05
2	0.93	0.07

E =

\	0	1	2
0	1	0.97	0.9215
1	0	1	0.95
2	0	0	1

A=

\	3	4
0	0.857	0.14301
1	0.8835	0.1165
2	0.93	0.07

_c_ 1. Which states are transient, and which are absorbing?

a. All are transient & none are absorbing
b. All are absorbing & none are transient
c. States {0, 1, 2} are transient & {3, 4} are absorbing
d. States {0, 1, 2} are absorbing & {3, 4} are transient
e. None of the above

_e_ 2. What fraction of the refrigerators will Coldspot expect to replace? *(Choose nearest value!)*

a. 6% b. 8% c. 10% d. 12% e. 14% f. 16% g. 18% h. 20%

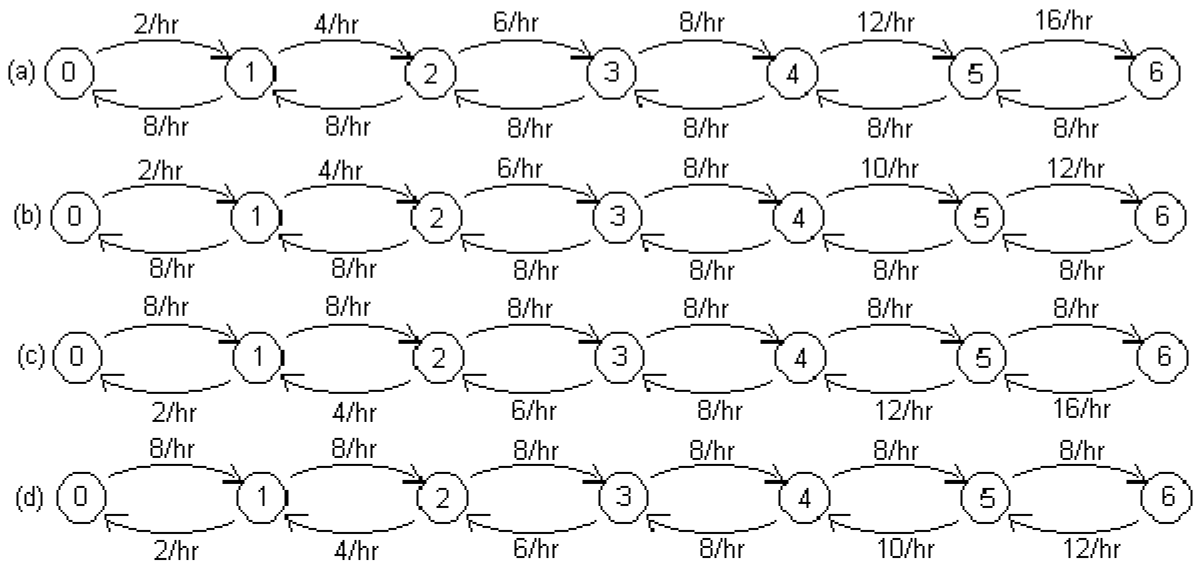
_a_ 3. What fraction of one-year-old refrigerators are expected to survive past the warranty period? *(Choose nearest value!)*

a. 88% b. 89% c. 90% d. 91% e. 92% f. 93% g. 94% h. 95%

Part II. A parking lot consists of four spaces. Cars making use of these spaces arrive according to a Poisson process at the rate of *eight cars per hour*. Parking time is exponentially distributed with mean of *30 minutes*. Cars who cannot find an empty space immediately on arrival may temporarily wait inside the lot until a parked car leaves, but may get impatient and leave before a parking space opens up. Assume that the time that a driver is willing to wait has exponential distribution with an average of *15 minutes*. The temporary space can hold only two cars. All other cars that cannot park or find a temporary waiting space must go elsewhere. Model this system as a birth-death process, with states 0, 1, … 6.

Name________________________________

_c_ 1. Which is are the correct transition rates?



The steadystate probability distribution of the number of cars in the system is:

n	0	1	2	3	4	5	6
π_n	0.02	0.08	0.18	0.24	0.24	0.16	0.08

_e_ 2. What is the fraction of the time that there is at least one empty space? *(Choose nearest value!)*

a. 10% c. 30% e. 50% g. 70%
b. 20% d. 40% f. 60% h. 80%

Solution: $\pi_0 + \pi_1 + \pi_2 + \pi_3 = 52\%$

_f_ 3. What is the average number of cars in the lot? *(Choose nearest value!)*

a. 1 c. 2 e. 3 g. 4
b. 1.5 d. 2.5 f. 3.5 h. 4.5

Solution: $0\pi_0 + 1\pi_1 + 2\pi_2 + 3\pi_3 + 4\pi_4 + 5\pi_5 + 6\pi_6 = 3.4$

_c_ 4. What is the average number of cars waiting? *(Choose nearest value!)*

a. 0.1 c. 0.3 e. 0.5 g. 0.7
b. 0.2 d. 0.4 f. 0.6 h. 0.8

Solution: $0\pi_0 + 0\pi_1 + 0\pi_2 + 0\pi_3 + 0\pi_4 + 1\pi_5 + 2\pi_6 = 0.32$

_c_ 5. What is the average arrival rate (keeping in mind that the arrival rate is zero when n=6)? *(Choose nearest value!)*

a. 5/hr c. 7/hr e. 9/hr g. 11/hr
b. 6/hr d. 8/hr f. 10/hr h. 12/hr

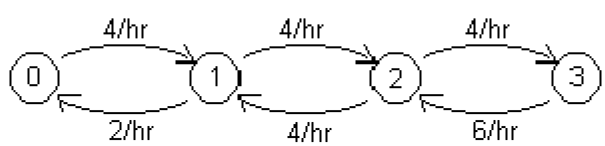
Solution: $8\pi_0 + 8\pi_1 + 8\pi_2 + 8\pi_3 + 8\pi_4 + 8\pi_5 + 0\pi_6 = 8\times(1-\pi_6) = 7.36$ /hr

_b_ 6. According to Little's Law, what is the average time that a car waits for a parking space? *(Choose nearest value!)*

a. 0.025 hr c. 0.075 hr e. 0.25 hr g. 0.75 hr
b. 0.05hr d. 0.1 hr f. 0.5 hr h. 1 hr.

Solution: $L_q = \lambda W_q \Rightarrow W_q = L_q/\lambda = 0.32/7.36/hr = 0.0434\,hr$

Consider the birth-death process:



_d_ 7. The probability of state 0 in steady-state is found by

a. $\frac{1}{p_0} = 1 + \frac{2}{4} + \frac{4}{4} + \frac{6}{4} = 4$

c. $\frac{1}{p_0} = \frac{4}{2}\times\frac{4}{4}\times\frac{4}{6} = \frac{4}{3}$

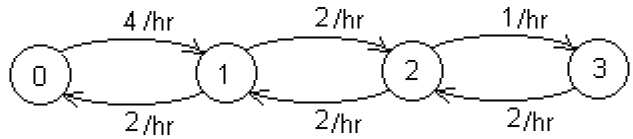
b. $\frac{1}{p_0} = 1 + \frac{4}{2} + \frac{4}{4} + \frac{4}{6} = \frac{14}{3}$

d. $\frac{1}{p_0} = 1 + \frac{4}{2} + \frac{4}{2}\times\frac{4}{4} + \frac{4}{2}\times\frac{4}{4}\times\frac{4}{6} = 6$

e. None of the above

56:171 Operations Research
Quiz #11 Solutions – Fall 2000

Consider the single-server queue with the birth-death model shown below:



_b_ 1. The probability distribution of the time between arrivals is

a. Markov b. Exponential c. Poisson
d. Normal e. Binomial f. Bernouilli

_b_ 2. The steadystate probability that the queue is empty is π_0, where

a. $\frac{1}{p_0} = 1 + \frac{4}{2} + \frac{2}{2} + \frac{1}{2} = \frac{11}{2}$ b. $\frac{1}{p_0} = 1 + \frac{4}{2} + \frac{4}{2}\frac{2}{2} + \frac{4}{2}\frac{2}{2}\frac{1}{2} = 6$

c. $\frac{1}{p_0} = \frac{4}{2} + \frac{4}{2}\frac{2}{2} + \frac{4}{2}\frac{2}{2}\frac{1}{2} = 5$ d. $\frac{1}{p_0} = \frac{4}{2} + \frac{2}{2} + \frac{1}{2} = \frac{9}{2}$

e. $\frac{1}{p_0} = \frac{4 \times 2 \times 1}{2 \times 2 \times 2} = 1$ f. *None of the above*

_d_ 3. The steadystate probability π_1 that the server is busy with no customers waiting is equal to

a. p_0 b. $\frac{1}{2}p_0$ c. $\frac{1}{4}p_0$
d. $2p_0$ e. $4p_0$ f. *None of the above*

_e_ 4. The average number of customers in the system (including the one being served) is denoted by

a. λ b. μ c. M
d. N e. L f. *None of the above*

_b_ 5. If the average number of customers in the system is 1.5, and the average arrival rate is 5/3 per hour, then the average time spent by a customer in the system is (choose nearest value)

a. 0.5 hr b. 1 hr $\approx L/l = \frac{1.5}{5/3hr} = 0.9hr$ c. 1.5 hr
d. 2 hr e. 2.5 hr f. None of the above

Deterministic Dynamic Programming Model: Power Plant Capacity Planning:
This DP model schedules the construction of powerplants over a six-year period, given
R[t] = cumulative number of plants required at the end of year t (t=1,2,…6)
C[t] = cost per plant (in $millions) during year t, where

Year t	C_t	R_t
1	4	1
2	4	2
3	5	4
4	5	5
5	6	6
6	6	8

A total of eight plants are to be constructed during the six-year period, with a restriction that no more than three may be built during each one-year period. Any year in which a plant is built, a cost of $2 million is incurred (independent of number of plants built). Future costs will *not* be discounted, i.e., the time value of money is being ignored. As in the homework assignment, the stages are numbered in *increasing* order, i.e., t=1 is the first year and t=6 is the final year.

Consult the computer output to answer the questions below.

1. The minimum total construction cost is $__40___ million
2. Several values are missing in the tables-- compute them:
   A. _15__ B. __17_ C. __8__
3. The optimal number of plants to be built in the first year is __3_
4. The optimal number of plants to be built in the third year is __2_

---Stage 6---

s \x:	0	1	2
6	999	999	14
7	999	8	999
8	0	999	999

---Stage 5---

s \x:	0	1	2	3
5	999	22	22	20
6	14	16	14	999
7	8	8	999	999
8	0	999	999	999

---Stage 4---

s \x:	0	1	2	3
4	999	27	26	25
5	20	21	20	17
6	14	_A	12	999
7	8	7	999	999
8	0	999	999	999

---Stage 3---

s \x:	0	1	2	3
2	999	999	37	34
3	999	32	29	29
4	25	24	24	24
5	17	19	19	17
6	12	14	12	999

---Stage 2---

s \x:	0	1	2	3
1	999	40	39	38
2	34	35	34	31
3	29	30	27	26

---Stage 1---

s \x:	0	1	2	3
0	999	44	41	40

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Stage 6:

State	Optimal Values	Optimal Decisions	Resulting State
6	14	2	8
7	8	1	8
8	0	0	8

Stage 5:

State	Optimal Values	Optimal Decisions	Resulting State
5	20	3	8
6	14	0	6
7	8	0	7
8	0	0	8

Stage 4:

State	Optimal Values	Optimal Decisions	Resulting State
4	25	3	7
5	_B	3	_C
6	12	2	8
7	7	1	8
8	0	0	8

Stage 3:

State	Optimal Values	Optimal Decisions	Resulting State
2	34	3	5
3	29	2	5
4	24	1	5
5	17	0	5
6	12	0	6

Stage 2:

State	Optimal Values	Optimal Decisions	Resulting State
1	38	3	4
2	31	3	5
3	26	3	6

Stage 1:

State	Optimal Values	Optimal Decisions	Resulting State
0	40	3	3

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Name_______________________________

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Quiz #12 Solution (Version A)– Fall 2000

Stochastic Production Planning with Backordering

We wish to plan next week's production (Monday through Saturday) of an expensive, low-demand item.

- the cost of *production* is **$6** for setup, plus **$4** per unit produced, up to a maximum of 4 units.
- the *storage* cost for inventory is **$1** per unit, based upon the level at the beginning of the day.
- a maximum of 5 units may be kept in inventory at the end of each day; any excess inventory is simply discarded.
- the demand D is random, with the same probability distribution each day:

demand d	0	1	2	3
P{D=d}	0.2	0.3	0.3	0.2

- there is a *penalty* of $10 per unit for any demand which cannot be satisfied. Any customer whose demand cannot be met takes his business elsewhere.
- the initial inventory is **1**.
- a *salvage* value of **$3** per unit is received for any inventory remaining at the end of the last day (Saturday).

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 6= Monday, stage 5= Tuesday, etc.** (i.e., n = # days remaining in planning period.) We define

S_n = stock on hand at stage n.

$f_n(S_n)$ = minimum total expected cost for the last n days if at the beginning of stage n the stock on hand is S_n.

Thus, we seek the value of $f_6(1)$, i.e., the minimum expected cost for six days, beginning with one unit in inventory.

(a.) What is the value of $f_6(1)$? _$61.77

(b.) What is the total expected cost for the six days, if there is one unit of stock on hand initially? $61.77

(c.) What should be the production quantity for Monday? __4__

(d.) If, on Monday, the demand happens to be 2 units, how many should be produced on Tuesday? __0___

(e.) Three values have been blanked out in the computer output, What are they?

- the cost associated with the decision to produce 1 unit on Friday when the inventory is 1 at the end of Thursday. **(A)**_ 33.09__ *(Note: this may or may not be the optimal decision!)*
- the optimal value $f_5(1)$, i.e., the minimum total cost of the last 5 days (Tuesday through Saturday) if there is one unit of stock on hand Tuesday morning. **(B) $**_51.21____
- the corresponding optimal decision $X_5^*(1)$ **(C)**____**4**______

Name ______________________________

---Stage 1--- (Saturday)

s \ x:	0	1	2	3	4
0	15.00	16.40	13.90	13.50	14.50
1	7.40	10.90	10.50	11.50	12.50
2	1.90	7.50	8.50	9.50	11.10
3	¯1.50	5.50	6.50	8.10	10.60
4	¯3.50	3.50	5.10	7.60	11.00
5	¯5.50	2.10	4.60	8.00	12.00

---Stage 2--- (Friday)

s \ x:	0	1	2	3	4
0	28.50	29.28	25.35	23.19	22.90
1	20.28	22.35	20.19	19.90	20.78
2	13.35	17.19	16.90	17.78	19.90
3	8.19	13.90	14.78	16.90	19.90
4	4.90	11.78	13.90	16.90	20.50
5	2.78	10.90	13.90	17.50	21.50

---Stage 3--- (Thursday)

s \ x:	0	1	2	3	4
0	37.90	39.30	36.09	34.19	33.42
1	30.30	A)33.09	31.19	30.42	30.15
2	24.09	28.19	27.42	27.15	28.50
3	19.19	24.42	24.15	25.50	28.20
4	15.42	21.15	22.50	25.20	28.78
5	12.15	19.50	22.20	25.78	29.78

---Stage 4--- (Wednesday)

s \ x:	0	1	2	3	4
0	48.42	49.77	46.57	44.80	44.10
1	40.77	43.57	41.80	41.10	40.63
2	34.57	38.80	38.10	37.63	38.54
3	29.80	35.10	34.63	35.54	37.81
4	26.10	31.63	32.54	34.81	38.15
5	22.63	29.54	31.81	35.15	39.15

---Stage 5--- (Tuesday)

s \ x:	0	1	2	3	4
0	59.10	60.41	57.15	55.34	54.66
1	51.41	54.15	52.34	51.66	51.21
2	45.15	49.34	48.66	48.21	49.11
3	40.34	45.66	45.21	46.11	48.33
4	36.66	42.21	43.11	45.33	48.63
5	33.21	40.11	42.33	45.63	49.63

---Stage 6--- (Monday)

s \ x:	0	1	2	3	4
0	69.66	70.97	67.72	65.91	65.22
1	61.97	64.72	62.91	62.22	61.77
2	55.72	59.91	59.22	58.77	59.67
3	50.91	56.22	55.77	56.67	58.90
4	47.22	52.77	53.67	55.90	59.21
5	43.77	50.67	52.90	56.21	60.21

❍ ❍

Stage 6 (Monday)

	State	Optimal Values	Optimal Decision
0	Empty	65.22	4 Prod 4
1	Stock1	61.77	4 Prod 4
2	Stock2	55.72	0 Idle
3	Stock3	50.91	0 Idle
4	Stock4	47.22	0 Idle
5	Stock5	43.77	0 Idle

Stage 5 (Tuesday)

	State	Optimal Values	Optimal Decision
0	Empty	54.66	4 Prod 4
1	Stock1	B)51.21	C)4 Prod 4
2	Stock2	45.15	0 Idle
3	Stock3	40.34	0 Idle
4	Stock4	36.66	0 Idle
5	Stock5	33.21	0 Idle

Stage 4 (Wednesday)

	State	Optimal Values	Optimal Decision
0	Empty	44.10	4 Prod 4
1	Stock1	40.63	4 Prod 4
2	Stock2	34.57	0 Idle
3	Stock3	29.80	0 Idle
4	Stock4	26.10	0 Idle
5	Stock5	22.63	0 Idle

Stage 3 (Thursday)

	State	Optimal Values	Optimal Decision
0	Empty	33.42	4 Prod 4
1	Stock1	30.15	4 Prod 4
2	Stock2	24.09	0 Idle
3	Stock3	19.19	0 Idle
4	Stock4	15.42	0 Idle
5	Stock5	12.15	0 Idle

Stage 2 (Friday)

	State	Optimal Values	Optimal Decision
0	Empty	22.90	4 Prod 4
1	Stock1	19.90	3 Prod 3
2	Stock2	13.35	0 Idle
3	Stock3	8.19	0 Idle
4	Stock4	4.90	0 Idle
5	Stock5	2.78	0 Idle

Stage 1 (Saturday)

	State	Optimal Values	Optimal Decision
0	Empty	13.50	3 Prod 3
1	Stock1	7.40	0 Idle
2	Stock2	1.90	0 Idle
3	Stock3	¯1.50	0 Idle
4	Stock4	¯3.50	0 Idle
5	Stock5	¯5.50	0 Idle

56:171 Operations Research
Quiz #12 Solution (Version B) – Fall 2000

Stochastic Production Planning with Backordering

We wish to plan next week's production (Monday through Saturday) of an expensive, low-demand item.

- **the cost of *production* is $7 for setup, plus $3 per unit produced,** up to a maximum of 4 units.
- the *storage* cost for inventory is **$1** per unit, based upon the level at the beginning of the day.
- a maximum of 5 units may be kept in inventory at the end of each day; any excess inventory is simply discarded.
- the demand D is random, with the same probability distribution each day:

demand d	0	1	2	3
P{D=d}	0.2	0.3	0.3	0.2

- there is a *penalty* of **$10** per unit for any demand which cannot be satisfied. Any customer whose demand cannot be met takes his business elsewhere.
- the initial inventory is **1**.
- a *salvage* value of **$3** per unit is received for any inventory remaining at the end of the last day (Saturday).

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 6= Monday, stage 5= Tuesday, etc.** (i.e., n = # days remaining in planning period.) We define

S_n = stock on hand at stage n.

$f_n(S_n)$ = minimum total expected cost for the last n days if at the beginning of stage n the stock on hand is S_n.

Thus, we seek the value of $f_6(1)$, i.e., the minimum expected cost for six days, beginning with one unit in inventory.

(a.) What is the value of $f_6(1)$? $ 55.31__

(b.) What is the total expected cost for the six days, if there is one unit of stock on hand initially? $ 55.31__

(c.) What should be the production quantity for Monday? __4__

(d.) If, on Monday, the demand happens to be 2 units, how many should be produced on Tuesday? __0__

(e.) Three values have been blanked out in the computer output, What are they?

- the cost associated with the decision to produce 1 unit on Thursday when the inventory is 1 at the end of Wednesday. **(A)** $ 30.75__ *(Note: this may or may not be the optimal decision!)*
- the optimal value $f_5(1)$, i.e., the minimum total cost of the last 5 days (Tuesday through Saturday) if there is one unit of stock on hand Tuesday morning. **(B) $_45.84_**
- the corresponding optimal decision $X_5^*(1)$ **(C)___4_______**

---Stage 1--- (Saturday)

s \ x:	0	1	2	3	4
0	15.00	16.40	12.90	11.50	11.50
1	7.40	10.90	9.50	9.50	9.50
2	1.90	7.50	7.50	7.50	8.10
3	¯1.50	5.50	5.50	6.10	7.60
4	¯3.50	3.50	4.10	5.60	8.00
5	¯5.50	2.10	3.60	6.00	9.00

---Stage 2--- (Friday)

s \ x:	0	1	2	3	4
0	26.50	27.68	23.35	20.79	19.90
1	18.68	21.35	18.79	17.90	17.78
2	12.35	16.79	15.90	15.78	16.90
3	7.79	13.90	13.78	14.90	16.90
4	4.90	11.78	12.90	14.90	17.50
5	2.78	10.90	12.90	15.50	18.50

---Stage 3--- (Thursday)

s \ x:	0	1	2	3	4
0	34.90	36.48	32.75	30.58	29.58
1	27.48	**A**)30.75	28.58	27.58	26.83
2	21.75	26.58	25.58	24.83	25.42
3	17.58	23.58	22.83	23.42	25.20
4	14.58	20.83	21.42	23.20	25.78
5	11.83	19.42	21.20	23.78	26.78

---Stage 4--- (Wednesday)

s \ x:	0	1	2	3	4
0	44.58	46.03	42.19	40.01	39.08
1	37.03	40.19	38.01	37.08	36.36
2	31.19	36.01	35.08	34.36	34.81
3	27.01	33.08	32.36	32.81	34.38
4	24.08	30.36	30.81	32.38	34.83
5	21.36	28.81	30.38	32.83	35.83

---Stage 5--- (Tuesday)

s \ x:	0	1	2	3	4
0	54.08	55.54	51.69	49.48	48.55
1	46.54	49.69	47.48	46.55	45.84
2	40.69	45.48	44.55	43.84	44.31
3	36.48	42.55	41.84	42.31	43.91
4	33.55	39.84	40.31	41.91	44.36
5	30.84	38.31	39.91	42.36	45.36

---Stage 6--- (Monday)

s \ x:	0	1	2	3	4
0	63.55	65.01	61.16	58.96	58.03
1	56.01	59.16	56.96	56.03	55.31
2	50.16	54.96	54.03	53.31	53.78
3	45.96	52.03	51.31	51.78	53.38
4	43.03	49.31	49.78	51.38	53.84
5	40.31	47.78	49.38	51.84	54.84

Stage 6 (Monday)

State	Optimal Values	Optimal Decision
0 Empty	58.03	4 Prod 4
1 Stock1	55.31	4 Prod 4
2 Stock2	50.16	0 Idle
3 Stock3	45.96	0 Idle
4 Stock4	43.03	0 Idle
5 Stock5	40.31	0 Idle

Stage 5 (Tuesday)

State	Optimal Values	Optimal Decision
0 Empty	48.55	4 Prod 4
1 Stock1	**B**)45.84	**C**)4 Prod 4
2 Stock2	40.69	0 Idle
3 Stock3	36.48	0 Idle
4 Stock4	33.55	0 Idle
5 Stock5	30.84	0 Idle

Stage 4 (Wednesday)

State	Optimal Values	Optimal Decision
0 Empty	39.08	4 Prod 4
1 Stock1	36.36	4 Prod 4
2 Stock2	31.19	0 Idle
3 Stock3	27.01	0 Idle
4 Stock4	24.08	0 Idle
5 Stock5	21.36	0 Idle

Stage 3 (Thursday)

State	Optimal Values	Optimal Decision
0 Empty	29.58	4 Prod 4
1 Stock1	26.83	4 Prod 4
2 Stock2	21.75	0 Idle
3 Stock3	17.58	0 Idle
4 Stock4	14.58	0 Idle
5 Stock5	11.83	0 Idle

Stage 2 (Friday)

State	Optimal Values	Optimal Decision
0 Empty	19.90	4 Prod 4
1 Stock1	17.78	4 Prod 4
2 Stock2	12.35	0 Idle
3 Stock3	7.79	0 Idle
4 Stock4	4.90	0 Idle
5 Stock5	2.78	0 Idle

Stage 1 (Saturday)

State	Optimal Values	Optimal Decision
0 Empty	11.50	3 Prod 3
1 Stock1	7.40	0 Idle
2 Stock2	1.90	0 Idle
3 Stock3	¯1.50	0 Idle
4 Stock4	¯3.50	0 Idle
5 Stock5	¯5.50	0 Idle