56:171

Operations Research

Fall 2000

Quizzes

Dept of Mechanical & Industrial Engineering
University of Iowa

Name ______________________________

56:171 Operations Research
Quiz #1 – August 30, 2000

For each statement, indicate "+"=**true** or "o"=**false**.

_____ 1. When you enter an LP formulation into LINDO, you must first convert all inequalities to equations.

_____ 2. When you enter an LP formulation into LINDO, you must include any nonnegativity constraints.

_____ 3. When you enter an LP formulation into LINDO, you must manipulate your constraints so that all variables appear on the left, and all constants on the right.

_____ 4. LINDO would interpret the constraint "X1 + 2X2 > 10" as "X1 + 2X2 ≥ 10".

<><><><><><><><><><><><><><><>

Walnut Orchard has two farms that grow wheat and corn. Because of differing soil conditions, there are differences in the yields and costs of growing crops on the two farms. The yields and costs are shown in the table below. Farm #1 has 100 acres available for cultivation, while Farm #2 has 150 acres. The farm has contracted to grow 11,000 bushels of corn and 6000 bushels of wheat. Determine a planting plan that will minimize the cost of meeting these contracts.

	Farm #1	Farm #2
Corn yield/acre	100 bushels	120 bushels
Cost/acre of corn	$90	$115
Wheat yield/acre	40 bushels	35 bushels
Cost/acre of wheat	$90	$80

<u>Decision variables:</u>

C1 = # of acres of Farm 1 planted in corn
W1 = # of acres of Farm 1 planted in wheat
C2 = # of acres of Farm 2 planted in corn
W2 = # of acres of Farm 2 planted in wheat

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The model & LINDO output is below:

```
MIN     90 C1 + 115 C2 + 90 W1 + 80 W2
  SUBJECT TO
         2)   C1 + W1 <=   100
         3)   C2 + W2 <=   _______
         4)   100 C1 + 120 C2 >=   11000
         5)   40 W1 + 35 W2 >=   ________
  END
```

5. Complete the right-hand-sides of rows 3 & 5 above.

```
        OBJECTIVE FUNCTION VALUE
        1)      24096.15

  VARIABLE        VALUE          REDUCED COST
        C1         3.846154          0.000000
        C2        88.461540          0.000000
        W1        96.153847          0.000000
        W2        61.538460          0.000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         17.692308
        3)         0.000000         14.230769
        4)         0.000000         -1.076923
        5)         0.000000         -2.692308
```

6. The optimal solution is to plant ______ acres of Farm#1 in corn and _____ acres in wheat.

7. A total of _________ acres will be planted in corn.

8. The total cost of satisfying the grain contracts is $_________ .

Multiple choice:

___ 9. The additional restriction that the planted acres of Farm #1 cannot be more than 75% wheat could be stated as the linear inequality:

a. $W1 \leq 75$
b. $25W1 - 75C1 \leq 0$
c. $75W1 - 25C1 \geq 0$
d. $C1 \geq 25$
e. $25W1 - 75C1 \geq 0$
f. $75W1 - 25C1 \leq 0$

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Quiz #2 – September 13, 2000

Below are several simplex tableaus. Assume that the objective in each case is to be **minimized**. Classify each tableau by writing to the right of the tableau a letter **A** through **G**, according to the descriptions below. *Also answer the question accompanying each classification, if any.*

(**A**) Nonoptimal, nondegenerate tableau with bounded solution. *Circle a pivot element which would improve the objective.*

(**B**) Nonoptimal, degenerate tableau with bounded solution. *Circle an appropriate pivot element.*

(**C**) Unique optimum.

(**D**) Optimal tableau, with alternate optimum. *Circle a pivot element which would lead to another optimal basic solution.*

(**E**) Objective unbounded (below). *Specify a variable which, when going to infinity, will make the objective arbitrarily low.*

(F) Tableau with infeasible primal

Warning: *Some of these classifications might be used for more than one tableau, while others might not be used at all*

(1)

-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	-1	3	0	0	2	2	-36	
0	3	0	4	0	0	1	3	0	9	________
0	-1	1	2	-5	0	0	-2	1	0	
0	6	0	3	-2	1	0	-4	3	5	

(2)

-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	1	3	0	0	2	-2	-36	
0	3	0	4	0	0	1	3	0	9	________
0	-1	1	-2	-5	0	0	-2	1	4	
0	6	0	3	-2	1	0	-4	3	5	

(3)

-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	-1	3	0	0	2	2	-36	
0	3	0	4	0	0	1	3	0	9	________
0	-1	1	-2	-5	0	0	-2	1	0	
0	6	0	3	-2	1	0	-4	3	5	

(4)

-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	-1	3	0	0	2	2	-36	
0	3	0	-4	0	0	1	3	0	9	________
0	-1	1	-2	-5	0	0	-2	1	4	
0	6	0	0	-2	1	0	-4	3	5	

(5)

-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS	
1	3	0	-1	3	0	0	2	2	-36	
0	3	0	4	1	0	1	3	0	9	________
0	-1	1	-2	-5	0	0	-2	1	-4	
0	6	0	3	2	1	0	-4	3	5	

(6)

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	0	0	1	3	0	0	2	2	-36	
0	3	0	4	0	0	1	3	0	9	________
0	-1	1	-2	-5	0	0	-2	1	4	
0	6	0	3	-2	1	0	-4	3	5	

(7)

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	3	0	0	2	2	-36	
0	3	0	4	0	0	1	3	0	9	________
0	-1	1	-2	-5	0	0	-2	1	4	
0	6	0	3	-2	1	0	-4	3	5	

True (+) or False (o)?

_____ 8. If, when solving an LP by the simplex method, you make a mistake in choosing a pivot column, the resulting tableau is infeasible.

_____ 9. An LP with 5 variables and 2 equality constraints can have as many as (but no more than) ten basic solutions.

_____ 10. If, when solving an LP by the simplex method, you make a mistake in choosing a pivot row, the resulting tableau is infeasible.

_____ 11. In the simplex method, every variable of the LP is either basic or nonbasic..

_____ 12. In the simplex tableau, the objective row is written in the form of an equation.

_____ 13. When you enter an LP formulation into LINDO, you must first convert all inequalities to equations.

_____ 14. It may happen that an LP problem has (exactly) two optimal solutions.

_____ 15. The restriction that X1 be nonnegative should be entered into LINDO as the constraint X1 >= 0.

_____ 16. A "pivot" in a nonbasic column of a tableau will make it a basic column.

_____ 17. The "minimum ratio test" is used to select the pivot row in the simplex method for linear programming.

_____ 18. In the simplex method (as described in the lectures, not the textbook), the quantity $-Z$ serves as a basic variable, where Z is the value of the objective function.

_____ 19. Every optimal solution of an LP is a basic solution.

_____ 20. Basic feasible solutions of an LP with constraints $Ax \le b,\ x \ge 0$ correspond to "corner" points of the feasible region.

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Quiz #3 – September 20, 2000

Consider the LP problem:

$$\text{Min } w = 4Y_1 + 2Y_2 - Y_3$$
$$\text{s.t.} \quad Y_1 + 2Y_2 \geq 10$$
$$Y_1 - Y_2 + 2Y_3 = 8$$
$$Y_1 \geq 0,\ Y_2 \leq 0 \ \ (Y_3 \text{ is unrestricted in sign})$$

The dual of the above problem is

Max _____ X_1 + ______ X_2

s.t _____ X_1 + ______ X_2 ☐ _____

_____ X_1 + ______ X_2 ☐ _____

_____ X_1 + ______ X_2 ☐ _____

sign restrictions: X_1 ☐ 0, X_2 ☐ 0

For each statement, indicate "+"=**true** or "o"=**false**.

_____ 1. If you increase the right-hand-side of a "≥" constraint in a <u>min</u>imization LP, the optimal objective value will either increase or stay the same.

_____ 2. If the increase in the cost of a nonbasic variable remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.

_____ 3. The "reduced cost" in an LP solution provides an estimate of the change in the objective value when a basic variable increases.

_____ 4. In the two-phase simplex method, Phase One computes the optimal dual variables, followed by Phase Two in which the optimal primal variables are computed.

_____ 5. At the completion of the revised simplex method applied to an LP, the simplex multipliers give the optimal solution to the dual of the LP.

_____ 6. If a minimization LP problem is feasible and unbounded below, then its dual problem has an objective (to be maximized) which must be unbounded above.

_____ 7. If a minimization LP problem has a cost which is infeasible, then its dual problem cannot be feasible.

_____ 8. If the increase in the right-hand-side of a "tight" constraint remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.

_____ 9. According to the Complementary Slackness Theorem, if constraint #1 of the primal problem is slack, then variable #1 of the dual problem must be zero.

_____ 10. According to the Complementary Slackness Theorem, if variable #1 of the primal problem is zero, then constraint #1 of the dual problem must be tight.

FYI:

Maximize	Minimize
Type of constraint i: ≤ = ≥	Sign of variable i: nonnegative unrestricted in sign nonpositive
Sign of variable j: nonnegative unrestricted in sign nonpositive	Type of constraint i: ≥ = ≤

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Quiz #4 – September 27, 2000

Consider again the LP model of PAR, Inc., which manufactures standard and deluxe golf bags:

X1 = number of **STANDARD** golf bags manufactured per quarter

X2 = number of **DELUXE** golf bags manufactured per quarter

Four operations are required, with the time per golf bag as follows:

	STANDARD	DELUXE	Available
Cut-&-Dye	0.7 hr	1 hr	630 hrs.
Sew	0.5 hr	0.8666 hr	600 hrs.
Finish	1 hr	0.6666 hr	708 hrs.
Inspect-&-Pack	0.1 hr	0.25 hr	135 hrs.
Profit ($/bag)	$10	$9	

LINDO provides the following output:

```
MAX     10 X1 + 9 X2
 SUBJECT TO
        2)   0.7 X1 + X2 <=   630
        3)   0.5 X1 + 0.86666 X2 <=   600
        4)   X1 + 0.66666 X2 <=   708
        5)   0.1 X1 + 0.25 X2 <=   135
 END
        OBJECTIVE FUNCTION VALUE
        1)    7668.01200

  VARIABLE        VALUE          REDUCED COST
        X1       540.003110           .000000
        X2       251.997800           .000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)          .000000          4.375086
        3)       111.602000           .000000
        4)          .000000          6.937440
        5)        18.000232           .000000

RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE          CURRENT        ALLOWABLE        ALLOWABLE
                    COEF          INCREASE         DECREASE
       X1        10.000000         3.500135         3.700000
       X2         9.000000         5.285715         2.333400

                           RIGHTHAND SIDE RANGES

      ROW          CURRENT        ALLOWABLE        ALLOWABLE
                    RHS           INCREASE         DECREASE
        2       630.000000        52.364582       134.400000
        3       600.000000         INFINITY       111.602000
        4       708.000000       192.000010       128.002800
        5       135.000000         INFINITY        18.000232
```

Name______________________________

THE TABLEAU

```
 ROW (BASIS) X1    X2    SLK 2  SLK 3  SLK 4 SLK 5
  1 ART      .00   .00   4.375   .00   6.937   .00     7668.012
  2 X2       .00  1.00   1.875   .00  -1.312   .00      251.998
  3 SLK 3    .00   .00  -1.000  1.00    .200   .00      111.602
  4 X1      1.00   .00  -1.250   .00   1.875   .00      540.003
  5 SLK 5    .00   .00   -.344   .00    .141  1.00       18.000
```

Enter the correct answer into each blank or check the correct alternative answer, as appropriate. If not sufficient information, write "NSI" in the blank:

a. If the profit on STANDARD bags were to decrease from $10 each to $7 each, the number of STANDARD bags to be produced would

|__| increase |__| decrease |__| remain the same |__| not sufficient info.

b. If the profit on DELUXE bags were to increase from $9 each to $15 each, the number of DELUXE bags to be produced would

|__| increase |__| decrease |__| remain the same |__| not sufficient info.

c. The LP problem above has

|__| exactly one optimal solution |__| exactly two optimal solutions

|__| an infinite number of optimal solutions

d. If an additional 10 hours were available in the cut-&-dye department, PAR would be able to obtain an additional $________ in profits.

e. If an additional 10 hours were available in the inspect-&-pack department, PAR would be able to obtain an additional $________ in profits.

f. If the variable "SLK 2" were increased, this would be equivalent to

___ increasing the hours used in the cut-&-dye department

___ decreasing the hours used in the cut-&-dye department

___ none of the above

g. If the variable "SLK 2" were increased by 10, X1 would |__| increase |__| decrease by ________ STANDARD golf bags/quarter.

h. If the variable "SLK 2" were increased by 10, X2 would |__| increase |__| decrease by ________ DELUXE golf bags/quarter.

i. If a pivot were to be performed to enter the variable SLK2 into the basis, then according to the "minimum ratio test", the value of SLK2 in the resulting basic solution would be approximately

__1.875/252 __1/111.6 __1.25/540 __0.344/18

__252/1.875 __111.6 __540/1.25 __18/0.344

__ not sufficient information

j. If the variable SLK2 were to enter the basis, then the variable __________ will leave the basis.

FYI:

Maximize	Minimize
Type of constraint i: ≤ = ≥	Sign of variable i: nonnegative unrestricted in sign nonpositive
Sign of variable j: nonnegative unrestricted in sign nonpositive	Type of constraint i: ≥ = ≤

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Quiz #5 – October 4, 2000

PART ONE: Data Envelopment Analysis (Note: *DMU* = "decision-making-unit")

___ 1. In the *maximization* problem of the primal-dual pair of LP models, the decision variables are:
   a. The amount of each input and output to be used by the DMU
   b. The multipliers for the DMUs which are used to form a linear combination of DMUs in the reference set.
   c. The "prices" assigned to the inputs and outputs.
   c. None of the above

___ 2. The "prices" or weights assigned to the input & output variables in the maximization problem must
   a. be nonnegative
   b. sum to 1.0
   c. Both a & b
   d. Neither a nor b.

True (+) or false (o)?

___ 3. To perform a complete DEA analysis, an LP must be solved for *every* DMU.
___ 4. In the maximization LP form of the problem, all constraints have non-zero right-hand-sides.
___ 5. There is a constraint for *every* DMU (in the maximization LP form of the problem).
___ 6. The optimal value of the LP cannot exceed 1.0.
___ 7. The number of input and output variables must be equal.

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Assignment Problem. **Three machines are to be assigned to three jobs (one machine per job),** so that total machine hours used is minimized. The hours required by each machine for the jobs is given in the table below.

	Job 1	Job 2	Job 3
Machine A	4	2	9
Machine B	2	1	5
Machine C	5	2	10

a. Perform the row reduction step of the Hungarian method so that every row contains at least one zero. (Write the updated matrix below.)

	Job 1	Job 2	Job 3
Machine A			
Machine B			
Machine C			

b. Perform the column reduction step so that every column contains at least one zero, and write the updated matrix below:

	Job 1	Job 2	Job 3
Machine A			
Machine B			
Machine C			

c. What is the smallest number of (horizontal & vertical) lines required to cover all the zeroes? ____

d. Are any further steps required? If so, perform the next step, and write the resulting matrix below:

	Job 1	Job 2	Job 3
Machine A			
Machine B			
Machine C			

e. What is now the smallest number of (horizontal & vertical) lines required to cover all the zeroes? ____

f. Find the optimal assignment: Machine A performs job ___. Machine B performs job ___. Machine C performs job ___. Total machine hours required is ____.

g. The assignment problem can be modeled as a transportation problem with ___ sources and ____ destinations, with the supplies available at the sources equal to _____ and the demands at the destinations equal to _______. The number of basic variables will be ____ , while the number of positive variables in a basic solution will be ____. Every basic solution is therefore classified as "d__________".

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<table><tr><td align="center">56:171 Operations Research
Quiz #6 Solution -- Fall 2000</td></tr></table>

Consider the following payoff table in which you must choose among three possible alternatives, after which two different states of nature may occur. (Nothing is known about the probability distribution of the state of nature.)

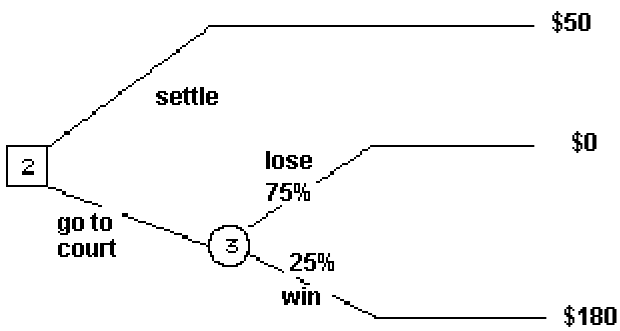
	State of Nature	
Decision	1	2
1	6	3
2	3	8
3	4	2

1. What is the optimal decision if the maximin criterion is used? _____
2. What is the optimal decision if the maximax criterion is used? _____
3. Create the regret table:

	State of Nature	
Decision	1	2
1	__	__
2	__	__
3	__	__

4. What is the optimal decision if the minimax regret is used? _____

General Custard Corporation is being sued by Sue Smith. Sue must decide whether to accept an offer of $50,000 by the corporation to settle out of court, or she can go to court. If she goes to court, there is a 25% chance that she will win the case *(event W)* and a 75% chance she will lose *(event L)*. If she wins, she will receive $180,000, but if she loses, she will net $0. A decision tree representing her situation appears below, where payoffs are in thousands of dollars:



5. What is the decision which maximizes the expected value? ___ a. settle ___b. go to court

For $20,000, Sue can hire a consultant who will predict the outcome of the trial, i.e., either he predicts a loss of the suit *(event PL)*, or he predicts a win *(event PW)*. The consultant is correct 80% of the time, e.g., if the suit will win, the probability that the consultant predicts the win is 80%.

Bayes' Rule states that if S_i is one of the *n* states of nature and O_j is the outcome of an experiment,

$$P\{S_i|O_j\} = \frac{P\{O_j|S_i\}P\{S_i\}}{P\{O_j\}}, \text{ where } P\{O_j\} = \sum_k P\{O_j|S_k\}P\{S_k\}$$

_____ 6. The probability that the consultant will predict a loss, i.e. P{PL} is *(choose nearest value)*

a. ≤ 35% b. 40% c. 45% d. 50%
e. 55% f. 60% g. 65% h. ≥ 70%

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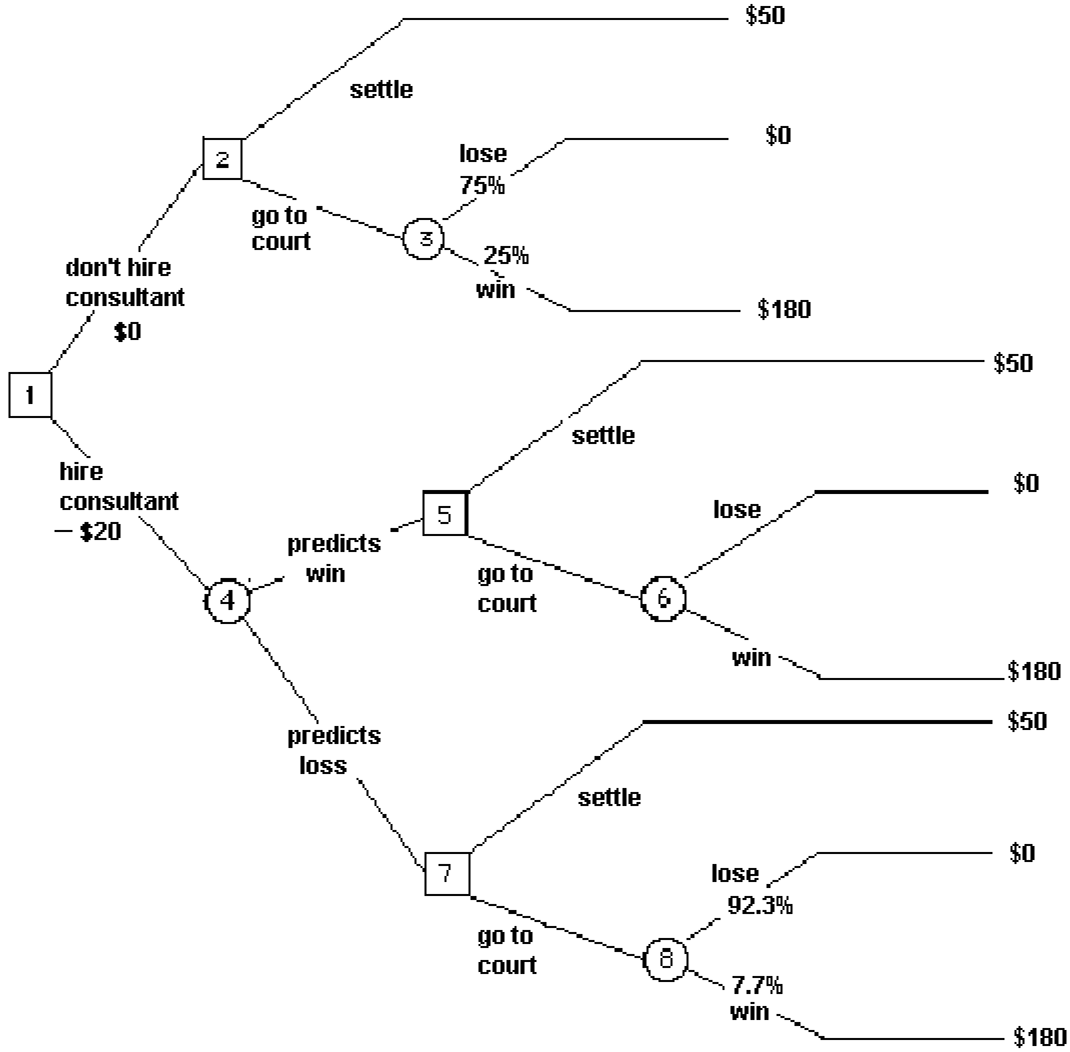
_____ 7. According to Bayes' theorem, the probability that Sue *will* win, given that the consultant predicts a win, i.e. $P\{W|PW\}$, is *(choose nearest value)*

a. ≤ 35% b. 40% c. 45% d. 50%
e. 55% f. 60% g. 65% h. ≥ 70%

The decision tree below includes Sue's decision as to whether or not to hire the consultant. Insert the probability that you computed in (7) above, together with its complement $P\{L|PW\}$.



Note that in this tree, the cost of the consultant is not included in the "payoffs" at the ends of the tree, but is included only when comparing the payoffs at nodes 2 & 4.

8. "Fold back" nodes 2 through 8, and write the value of each node below:

Node	Value	Node	Value	Node	Value
8	______	5	102.85	2	50
7	50	4	______	1	______
6	102.85	3	45		

9. Should Sue hire the consultant? *Circle:* Yes No

____ 10. The expected value of the consultant's opinion is (in thousands of $) *(Choose nearest value)*:

a. ≤ 17 b. 18 c. 19 d. 20
e. 21 f. 22 g. 22 h. ≥ 23

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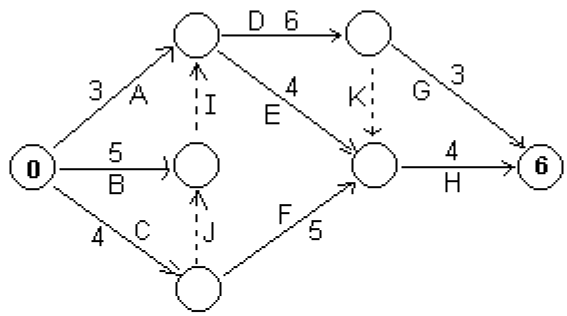
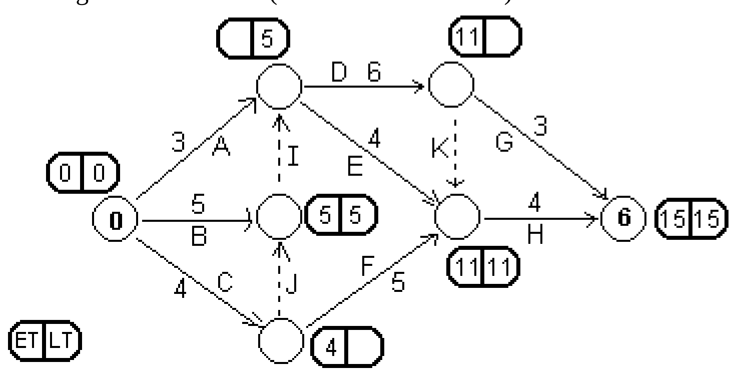
Quiz #7 -- Fall 2000



a. Complete the labeling of the nodes on the A-O-A project network above (so that if arrow goes from node **i** to node **j**, then **i**<**j**). *Note that I, J, & K are "dummy" activities.*

b. The activity durations are given below on the arrows. Finish computing the Early Times (ET) and Late Times (LT) for each node, writing them in the box (with rounded corners) beside each node.



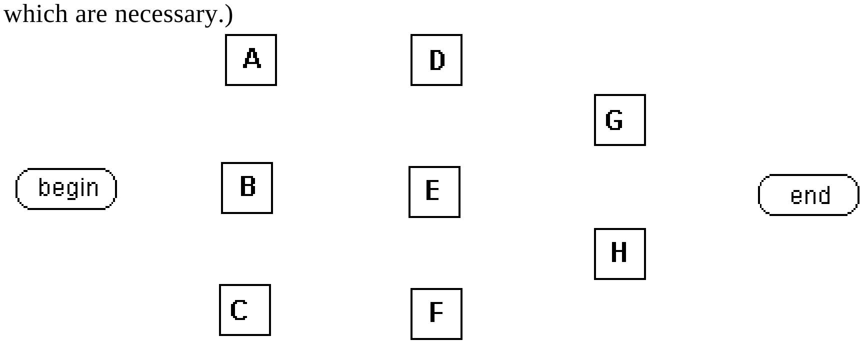
c. Complete the table:: *(You may omit "Late Start" if you don't use it for computing slack.)*

Activity	Duration	Early Start	Early Finish	*Late Start*	Late Finish	Total Slack
A	3					
B	5					
C	4					
D	6					
E	4					
F	5					
G	3					
H	4					

d. Which activities are critical? (circle: A B C D E F G H I J K)

e. Suppose that the durations are random, with the expected value as given, but with standard deviations all equal to 1.00. What is the standard deviation of the project completion time? ________

f. Complete the A-O-N (activity-on-node) network below for this same project. (Add any "dummy" activities which are necessary.)



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Quiz #8 -- Fall 2000

1. The Cubs are trying to determine which of the following free agent pitchers should be signed: Rick Sutcliffe (RS), Bruce Sutter (BS), Dennis Eckersley (DE), Steve Trout (ST), Tim Stoddard (TS).

Define five binary variables, one for each pitcher: for example: **RS** = 1 if Rick Sutcliffe is signed, 0 otherwise
Find the appropriate constraint corresponding to each restriction. If none apply, write "z"

___ If DE and ST are signed, then BS cannot be signed.
___ The Cubs cannot sign both ST and TS.
___ If TS is signed, then DE must also be signed.
___ If DE is signed, then TS cannot be signed.
___ If ST is not signed, then DE must be signed.
___ The Cubs must sign either BS or RS (or both)

a. $ST + TS = 1$	b. $ST + TS \leq 1$	c. $ST \leq TS$	d. $TS \leq ST$
e. $DE + ST \geq 1$	f. $DE + ST = 1$	g. $DE + ST \leq 1$	h. $DE \leq ST$
i. $TS \leq DE$	j. $TS + DE = 1$	k. $TS + DE \geq 1$	l. $ST \leq DE$
m. $DE + ST + BS \leq 2$	n. $DE + ST + BS = 2$	o. $DE + ST + BS \geq 2$	p. $BS \leq DE + ST$
q. $DE + ST - 1 \geq BS$	r. $ST + DE \geq 1$	s. $TS \geq DE$	t. $TS \leq DE$
u. $DE + ST - 1 \leq BS$	v. $RS + BS \geq 1$	w. $RS + BS \leq 1$	x. $RS + BS = 1$

z. *None of the above*

2. Four trucks are available to deliver milk to five grocery stores. Capacities and daily operating costs vary among the trucks. Trucks are to make a single trip per day. Each grocery may be supplied by only one truck, but a truck may deliver to more than one grocery in one trip. No more than $100 may be spent on the trucks.

Truck #	Capacity (gallons)	Daily operating cost ($)
1	400	45
2	500	50
3	600	55
4	1100	60

Grocery #	Daily Demand (gallons)
1	100
2	200
3	300
4	500
5	800

Define binary variables
$Y_i = 1$ if truck i is used, 0 otherwise
$X_{ij} = 1$ if truck i delivers to grocery j, 0 otherwise

Put an "X" beside each of the constraints below which would be valid in the integer LP model.

__ $X_{13} + X_{23} + X_{33} + X_{43} = 1$
__ $45Y_1 + 50Y_2 + 55Y_3 + 60Y_4 \leq 100$
__ $X_{41} + X_{42} + X_{43} + X_{44} + X_{45} = 1$
__ $X_{31} + X_{32} + X_{33} + X_{34} + X_{35} \leq 600Y_3$
__ $100X_{31} + 200X_{32} + 300X_{33} + 500X_{34} + 800X_{35} \leq 600Y_3$
__ $X_{41} + X_{42} + X_{43} + X_{44} + X_{45} \leq Y_4$
__ $400X_{14} + 500X_{24} + 600X_{34} + 1100X_{44} \leq 500Y_4$
__ $300X_{43} \leq 1100Y_4$
__ $X_{43} \leq Y_4$
__ $45Y_1 + 50Y_2 + 55Y_3 + 60Y_4 = 100$
__ $Y_4 \leq X_{43}$
__ $X_{13} + X_{23} + X_{33} + X_{43} \leq 300Y_3$
__ $X_{13} + X_{23} + X_{33} + X_{43} \leq 4Y_3$
__ $X_{41} + X_{42} + X_{43} + X_{44} + X_{45} \leq 5Y_4$
__ $X_{41} + X_{42} + X_{43} + X_{44} + X_{45} \leq 1100$
__ $300X_{43} \geq 1100Y_4$

__

Name_______________________________

56:171 Operations Research
Quiz #9 – Fall 2000

Consider an **(s,S) inventory system** in which the number of items on the shelf is checked at the end of each day. The demand distribution is as follows:

n=	0	1	2
P{D=n}	0.2	0.5	0.3

To avoid shortages, the current policy is to restock the shelf at the end of each day (after spare parts have been removed) so that the shelf is again filled to its limit (i.e., 4) **if** there are fewer than 2 parts on the shelf. *(That is, it is an (s,S) inventory system, with s=2 and S=4.)*

The inventory system has been modeled as a Markov chain, with the state of the system defined as the end-of-day inventory level (before restocking). Refer to the computer output which follows to answer the following questions:

P =

\	0	1	2	3	4
0	0	0	0.3	0.5	0.2
1	0	0	0.3	0.5	0.2
2	0.3	0.5	0.2	0	0
3	0	0.3	0.5	0.2	0
4	0	0	0.3	0.5	0.2

P^2 =

\	0	1	2	3	4
0	0.09	0.3	0.37	0.2	0.04
1	0.09	0.3	0.37	0.2	0.04
2	0.06	0.1	0.28	0.4	0.16
3	0.15	0.31	0.29	0.19	0.06
4	0.09	0.3	0.37	0.2	0.04

P^3 =

\	0	1	2	3	4
0	0.111	0.245	0.303	0.255	0.086
1	0.111	0.245	0.303	0.255	0.086
2	0.084	0.26	0.352	0.24	0.064
3	0.087	0.202	0.309	0.298	0.104
4	0.111	0.245	0.303	0.255	0.086

P^4 =

\	0	1	2	3	4
0	0.0909	0.228	0.3207	0.272	0.0884
1	0.0909	0.228	0.3207	0.272	0.0884
2	0.1056	0.248	0.3128	0.252	0.0816
3	0.0927	0.2439	0.3287	0.2561	0.0786
4	0.0909	0.228	0.3207	0.272	0.0884

P^5 =

\	0	1	2	3	4
0	0.0962	0.2419	0.3223	0.2580	0.0814
1	0.0962	0.2419	0.3223	0.2580	0.0814
2	0.0938	0.2320	0.3191	0.2680	0.0870
3	0.0986	0.2412	0.3183	0.2588	0.0830
4	0.0962	0.2419	0.3223	0.2580	0.0814

$\sum_{n=1}^{5} P^n =$

\	0	1	2	3	4
0	0.388	1.015	1.616	1.485	0.495
1	0.388	1.015	1.616	1.485	0.495
2	0.643	1.340	1.463	1.160	0.392
3	0.428	1.297	1.746	1.202	0.325
4	0.388	1.015	1.616	1.485	0.495

First Passage Probabilities

n	$f_{4,0}^{(n)}$
1	0.0
2	0.09
3	0.111
4	0.0828
5	0.07623

Mean First Passage Times

\	0	1	2	3	4
0	10.408	4.192	2.653	2.75	11.953
1	10.408	4.192	2.653	2.75	11.953
2	7.755	2.822	3.122	4	13.203
3	10	3.014	2.245	3.83	13.984
4	10.408	4.192	2.653	2.75	11.953

Steady State Distribution

i	name	πi
0	SOH 0	0.09607
1	SOH 1	0.23856
2	SOH 2	0.32026
3	SOH 3	0.26144
4	SOH 4	0.08366

Name ______________________________

____ 1. the value $P_{4,2}$ is
- a. P{demand=0} b. P{demand=1} c. P{demand=2}
- d. P{demand≤1} e. P{demand≥1} f. *none of the above*

____ 2. the value $P_{0,3}$ is
- a. P{demand=0} b. P{demand=1} c. P{demand=2}
- d. P{demand≤1} e. P{demand≥1} f. *none of the above*

____ 3. the value $P_{2,0}$ is
- a. P{demand=0} b. P{demand=1} c. P{demand=2}
- d. P{demand≤1} e. P{demand≥1} f. *none of the above*

____ 4. If the shelf is full Monday morning, the expected number of days until a stockout occurs is *(select nearest value):*
- a. 2.5 b. 5 c. 7.5 d. 10 e. 12.5
- f. 15 g. 17.5 h. 20 i. more than 20

____ 5. If the shelf is full Monday morning, the probability that the shelf is full Thursday night (i.e., after 4 days of sales) is *(select nearest value):*
- a. 5% b. 6% c. 7% d. 8% e. 9%
- f. 10% g. 11% h. 12% i. 13% j. ≥14%

____ 6. If the shelf is full Monday morning, the probability that the shelf is restocked Thursday night is *(select nearest value):*
- a. 5% b. 10% c. 15% d. 20% e. 25%
- f. 30% g. 35% h. 40% i. 45% j. ≥50%

____ 7. If the shelf is full Monday morning, the *expected* number of nights that the shelf is restocked during the next five nights is *(select nearest value):*
- a. 0.25 b. 0.5 c. 0.75 d. 1 e. 1.25
- f. 1.5 g. 1.75 h. 2 i. 2.25 j. ≥2.5

____ 8. How frequently will the shelf be restocked? *(select nearest value): once every ______ days*
- a. 0.5 days b. 1 days c. 1.5 days d. 2 days e. 2.5 days
- f. 3 days g. 3.5 days h. 4 days i. 4.5 days j. ≥5 days

____ 9. What is the probability of a stockout Thursday night? *(select nearest value):*
- a. 5% b. 6% c. 7% d. 8% e. 9%
- f. 10% g. 11% h. 12% i. 13% j. ≥14%

10. Circle (one or more) of the following equations which are among those solved to compute the steady state probability distribution:

a. $p_0 = 0.3p_2$

b. $p_3 = 0.3p_0 + 0.5p_1 + 0.2p_2$

c. $p_2 = 0.3p_0 + 0.3p_1 + 0.2p_2 + 0.5p_3 + 0.3p_4$

d. $p_4 = 0.3p_2 + 0.5p_3 + 0.2p_4$

e. $p_4 = 0.2p_2 + 0.5p_3 + 0.3p_4$

f. $p_0 + p_1 + p_2 + p_3 + p_4 = 1$

Name ____________________________

56:171 Operations Research
Quiz #10 – Fall 2000

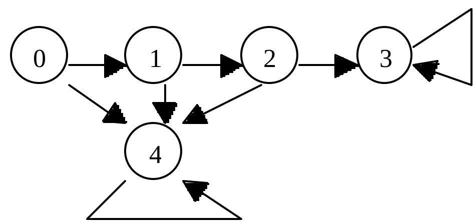
Coldspot manufactures refrigerators. The company has issued a warranty on all refrigerators that requires free replacement of any refrigerator that fails before it is three years old.

- 3% of all new refrigerators fail during their first year of operation.
- 5% of all 1-year-old refrigerators fail during their second year of operation.
- 7% of all 2-year-old refrigerators fail during their third year of operation.

Replacement refrigerators are not covered by the warranty.

Define a discrete-time Markov chain, with states

(0.) new refrigerators
(1.) 1-year-old refrigerators
(2.) 2-year-old refrigerators
(3.) refrigerators that have passed their third anniversary
(4.) replacement refrigerators



Define the stages to be years, with the refrigerator observed on the anniversaries of its purchase.

P=

	0	1	2	3	4
0	0	0.97	0	0	0.03
1	0	0	0.95	0	0.05
2	0	0	0	0.93	0.07
3	0	0	0	1	0
4	0	0	0	0	1

Q=

\	0	1	2
0	0	0.97	0
1	0	0	0.95
2	0	0	0

R=

\	3	4
0	0	0.03
1	0	0.05
2	0.93	0.07

E =

\	0	1	2
0	1	0.97	0.9215
1	0	1	0.95
2	0	0	1

A=

\	3	4
0	0.857	0.14301
1	0.8835	0.1165
2	0.93	0.07

___ 1. Which states are transient, and which are absorbing?

a. All are transient & none are absorbing
b. All are absorbing & none are transient
c. States {0, 1, 2} are transient & {3, 4} are absorbing
d. States {0, 1, 2} are absorbing & {3, 4} are transient
e. None of the above

___ 2. What fraction of the refrigerators will Coldspot expect to replace? *(Choose nearest value!)*

a. 6% b. 8% c. 10% d. 12% e. 14% f. 16% g. 18% h. 20%

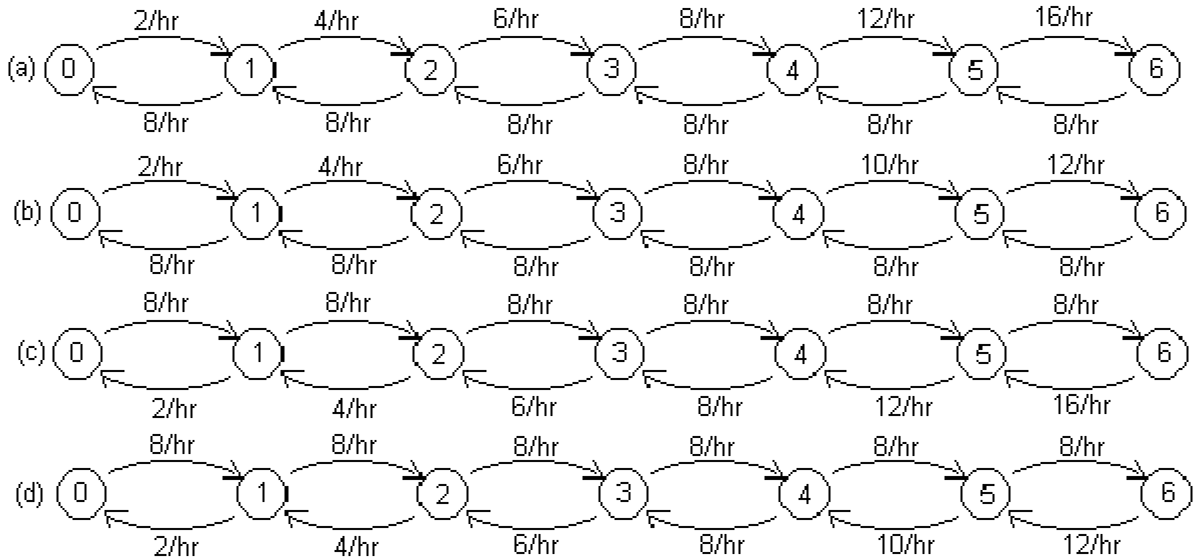
___ 3. What fraction of one-year-old refrigerators are expected to survive past the warranty period? *(Choose nearest value!)*

a. 88% b. 89% c. 90% d. 91% e. 92% f. 93% g. 94% h. 95%

Birth-death model of queue. A parking lot consists of four spaces. Cars making use of these spaces arrive according to a Poisson process at the rate of *eight cars per hour.* Parking time is exponentially distributed with mean of *30 minutes*. Cars who cannot find an empty space immediately on arrival may temporarily wait inside the lot until a parked car leaves, but may get impatient and leave before a parking space opens up. Assume that the time that a driver is willing to wait has exponential distribution with an average of *15 minutes*. The temporary space can hold only two cars. All other cars that cannot park or find a temporary waiting space must go elsewhere. Model this system as a birth-death process, with states 0, 1, … 6.

Name______________________________

___ 1. Which are the correct transition rates?



The steadystate probability distribution of the number of cars in the system is:

n	0	1	2	3	4	5	6
π_n	0.02	0.08	0.18	0.24	0.24	0.16	0.08

___ 2. What is the fraction of the time that there is at least one empty space? *(Choose nearest value!)*

a. 10% c. 30% e. 50% g. 70%
b. 20% d. 40% f. 60% h. 80%

___ 3. What is the average number of cars in the lot? *(Choose nearest value!)*

a. 1 c. 2 e. 3 g. 4
b. 1.5 d. 2.5 f. 3.5 h. 4.5

___ 4. What is the average number of cars waiting? *(Choose nearest value!)*

a. 0.1 c. 0.3 e. 0.5 g. 0.7
b. 0.2 d. 0.4 f. 0.6 h. 0.8

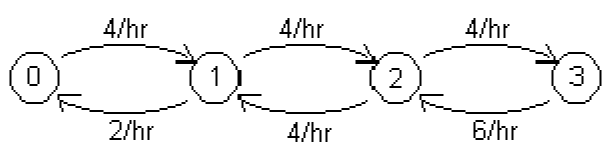
___ 5. What is the average arrival rate (keeping in mind that the arrival rate is zero when n=6)? *(Choose nearest value!)*

a. 5/hr c. 7/hr e. 9/hr g. 11/hr
b. 6/hr d. 8/hr f. 10/hr h. 12/hr

___ 6. According to Little's Law, what is the average time that a car waits for a parking space? *(Choose nearest value!)*

a. 0.025 hr c. 0.075 hr e. 0.25 hr g. 0.75 hr
b. 0.05hr d. 0.1 hr f. 0.5 hr h. 1 hr.

Consider the birth-death process:



___ 7. The probability of state 0 in steady-state is found by

a. $\frac{1}{p_0} = 1 + \frac{2}{4} + \frac{4}{4} + \frac{6}{4} = 4$ c. $\frac{1}{p_0} = \frac{4}{2} \times \frac{4}{4} \times \frac{4}{6} = \frac{4}{3}$

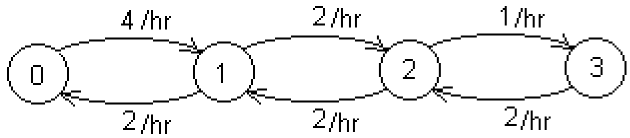
b. $\frac{1}{p_0} = 1 + \frac{4}{2} + \frac{4}{4} + \frac{4}{6} = \frac{14}{3}$ d. $\frac{1}{p_0} = 1 + \frac{4}{2} + \frac{4}{2} \times \frac{4}{4} + \frac{4}{2} \times \frac{4}{4} \times \frac{4}{6} = 6$

e. *None of the above*

Name______________________________

56:171 Operations Research
Quiz #11 – Fall 2000

Consider the single-server queue with the birth-death model shown below:



___ 1. The probability distribution of the time between arrivals is

a. Markov b. Exponential c. Poisson
d. Normal e. Binomial f. Bernouilli

___ 2. The steadystate probability that the queue is empty is π_0, where

a. $\frac{1}{p_0} = 1 + \frac{4}{2} + \frac{2}{2} + \frac{1}{2} = \frac{11}{2}$

b. $\frac{1}{p_0} = 1 + \frac{4}{2} + \frac{4}{2}\frac{2}{2} + \frac{4}{2}\frac{2}{2}\frac{1}{2} = 6$

c. $\frac{1}{p_0} = \frac{4}{2} + \frac{4}{2}\frac{2}{2} + \frac{4}{2}\frac{2}{2}\frac{1}{2} = 5$

d. $\frac{1}{p_0} = \frac{4}{2} + \frac{2}{2} + \frac{1}{2} = \frac{9}{2}$

e. $\frac{1}{p_0} = \frac{4 \times 2 \times 1}{2 \times 2 \times 2} = 1$

f. *None of the above*

___ 3. The steadystate probability π_1 that the server is busy with no customers waiting is equal to

a. p_0 b. $\frac{1}{2}p_0$ c. $\frac{1}{4}p_0$
d. $2p_0$ e. $4p_0$ f. *None of the above*

___ 4. The average number of customers in the system (including the one being served) is denoted by

a. λ b. μ c. M
d. N e. L f. *None of the above*

___ 5. If the average number of customers in the system is 1.5, and the average arrival rate is 5/3 per hour, then the average time spent by a customer in the system is (choose nearest value)

a. 0.5 hr b. 1 hr c. 1.5 hr
d. 2 hr e. 2.5 hr f. None of the above

Deterministic Dynamic Programming Model: Power Plant Capacity Planning:

This DP model schedules the construction of powerplants over a six-year period, given

R[t] = cumulative number of plants required at the end of year t (t=1,2,…6)
C[t] = cost per plant (in $millions) during year t, where

Year t	C_t	R_t
1	4	1
2	4	2
3	5	4
4	5	5
5	6	6
6	6	8

A total of eight plants are to be constructed during the six-year period, with a restriction that no more than three may be built during each one-year period. Any year in which a plant is built, a cost of $2 million is incurred (independent of number of plants built). Future costs will *not* be discounted, i.e., the time value of money is being ignored. As in the homework assignment, the stages are numbered in *increasing* order, i.e., t=1 is the first year and t=6 is the final year.

Consult the computer output to answer the questions below.

Name________________________________

1. The minimum total construction cost is $_______ million
2. Several values are missing in the tables-- compute them:
   A. _____ B. _____ C. _____
3. The optimal number of plants to be built in the first year is ____
4. The optimal number of plants to be built in the third year is ____

```
---Stage 6---
         s \x: 0    1    2
         6 | 999  999  14
         7 | 999    8  999
         8 |    0 999  999

---Stage 5---
         s \x: 0    1    2    3
         5 | 999   22   22   20
         6 |  14   16   14  999
         7 |   8    8  999  999
         8 |   0  999  999  999

---Stage 4---
         s \x: 0    1    2    3
         4 | 999   27   26   25
         5 |  20   21   20   17
         6 |  14   _A   12  999
         7 |   8    7  999  999
         8 |   0  999  999  999

---Stage 3---
         s \x: 0    1    2    3
         2 | 999  999   37   34
         3 | 999   32   29   29
         4 |  25   24   24   24
         5 |  17   19   19   17
         6 |  12   14   12  999

---Stage 2---
         s \x: 0    1    2    3
         1 | 999   40   39   38
         2 |  34   35   34   31
         3 |  29   30   27   26

---Stage 1---
         s \x: 0    1    2    3
         0 | 999   44   41   40
==============================================================================
Stage 6:
      Optimal  Optimal   Resulting
State  Values Decisions   State
6         14       2        8
7          8       1        8
8          0       0        8
---------------------------------
Stage 5:
      Optimal  Optimal   Resulting
State  Values Decisions   State
  5       20       3        8
  6       14       0        6
  7        8       0        7
  8        0       0        8
--------------------------------
Stage 4:
      Optimal  Optimal   Resulting
State  Values Decisions   State
  4       25       3        7
  5       _B       3       _C
  6       12       2        8
  7        7       1        8
  8        0       0        8

Stage 3:
      Optimal  Optimal   Resulting
State  Values Decisions   State
  2       34       3        5
  3       29       2        5
  4       24       1        5
  5       17       0        5
  6       12       0        6
----------------------------------
Stage 2:
      Optimal  Optimal   Resulting
State  Values Decisions   State
  1       38       3        4
  2       31       3        5
  3       26       3        6
----------------------------------
Stage 1:
      Optimal  Optimal   Resulting
State  Values Decisions   State
  0       40       3        3

==============================================================================
```

Name______________________________

56:171 Operations Research
Quiz #12 – Fall 2000

Stochastic Production Planning with Backordering

We wish to plan next week's production (Monday through Saturday) of an expensive, low-demand item.

- the cost of *production* is **$6** for setup, plus **$4** per unit produced, up to a maximum of 4 units.
- the *storage* cost for inventory is **$1** per unit, based upon the level at the beginning of the day.
- a maximum of 5 units may be kept in inventory at the end of each day; any excess inventory is simply discarded.
- the demand D is random, with the same probability distribution each day:

demand d	0	1	2	3
P{D=d}	0.2	0.3	0.3	0.2

- there is a *penalty* of **$10** per unit for any demand which cannot be satisfied. Any customer whose demand cannot be met takes his business elsewhere.
- the initial inventory is **1**.
- a *salvage* value of **$3** per unit is received for any inventory remaining at the end of the last day (Saturday).

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 6= Monday, stage 5= Tuesday, etc.** (i.e., n = # days remaining in planning period.) We define

S_n = stock on hand at stage n.

$f_n(S_n)$ = minimum total expected cost for the last n days if at the beginning of stage n the stock on hand is S_n.

Thus, we seek the value of $f_6(1)$, i.e., the minimum expected cost for six days, beginning with one unit in inventory.

(a.) What is the value of $f_6(1)$? $_______

(b.) What is the total expected cost for the six days, if there is one unit of stock on hand initially? $______

(c.) What should be the production quantity for Monday? ______

(d.) If, on Monday, the demand happens to be 2 units, how many should be produced on Tuesday? ______

(e.) Three values have been blanked out in the computer output, What are they?

- the cost associated with the decision to produce 1 unit on Thursday when the inventory is 1 at the end of Wednesday. **(A)**____________ *(Note: this may or may not be the optimal decision!)*
- the optimal value $f_5(1)$, i.e., the minimum total cost of the last 5 days (Tuesday through Saturday) if there is one unit of stock on hand Tuesday morning. **(B) $**___________
- the corresponding optimal decision $X_5^*(1)$ **(C)**___________

---Stage 1--- (Saturday)

s \ x:	0	1	2	3	4
0	15.00	16.40	13.90	13.50	14.50
1	7.40	10.90	10.50	11.50	12.50
2	1.90	7.50	8.50	9.50	11.10
3	¯1.50	5.50	6.50	8.10	10.60
4	¯3.50	3.50	5.10	7.60	11.00
5	¯5.50	2.10	4.60	8.00	12.00

---Stage 2--- (Friday)

s \ x:	0	1	2	3	4
0	28.50	29.28	25.35	23.19	22.90
1	20.28	22.35	20.19	19.90	20.78
2	13.35	17.19	16.90	17.78	19.90
3	8.19	13.90	14.78	16.90	19.90
4	4.90	11.78	13.90	16.90	20.50
5	2.78	10.90	13.90	17.50	21.50

---Stage 3--- (Thursday)

s \ x:	0	1	2	3	4
0	37.90	39.30	36.09	34.19	33.42
1	30.30	__**A**__	31.19	30.42	30.15
2	24.09	28.19	27.42	27.15	28.50
3	19.19	24.42	24.15	25.50	28.20
4	15.42	21.15	22.50	25.20	28.78
5	12.15	19.50	22.20	25.78	29.78

---Stage 4--- (Wednesday)

s \ x:	0	1	2	3	4
0	48.42	49.77	46.57	44.80	44.10
1	40.77	43.57	41.80	41.10	40.63
2	34.57	38.80	38.10	37.63	38.54
3	29.80	35.10	34.63	35.54	37.81
4	26.10	31.63	32.54	34.81	38.15
5	22.63	29.54	31.81	35.15	39.15

---Stage 5--- (Tuesday)

s \ x:	0	1	2	3	4
0	59.10	60.41	57.15	55.34	54.66
1	51.41	54.15	52.34	51.66	51.21
2	45.15	49.34	48.66	48.21	49.11
3	40.34	45.66	45.21	46.11	48.33
4	36.66	42.21	43.11	45.33	48.63
5	33.21	40.11	42.33	45.63	49.63

---Stage 6--- (Monday)

s \ x:	0	1	2	3	4
0	69.66	70.97	67.72	65.91	65.22
1	61.97	64.72	62.91	62.22	61.77
2	55.72	59.91	59.22	58.77	59.67
3	50.91	56.22	55.77	56.67	58.90
4	47.22	52.77	53.67	55.90	59.21
5	43.77	50.67	52.90	56.21	60.21

❍❍

Stage 6 (Monday)

State	Optimal Values	Optimal Decision
0 Empty	65.22	4 Prod 4
1 Stock1	61.77	4 Prod 4
2 Stock2	55.72	0 Idle
3 Stock3	50.91	0 Idle
4 Stock4	47.22	0 Idle
5 Stock5	43.77	0 Idle

Stage 5 (Tuesday)

State	Optimal Values	Optimal Decision
0 Empty	54.66	4 Prod 4
1 Stock1	__**B**__	__**C**__
2 Stock2	45.15	0 Idle
3 Stock3	40.34	0 Idle
4 Stock4	36.66	0 Idle
5 Stock5	33.21	0 Idle

Stage 4 (Wednesday)

State	Optimal Values	Optimal Decision
0 Empty	44.10	4 Prod 4
1 Stock1	40.63	4 Prod 4
2 Stock2	34.57	0 Idle
3 Stock3	29.80	0 Idle
4 Stock4	26.10	0 Idle
5 Stock5	22.63	0 Idle

Stage 3 (Thursday)

State	Optimal Values	Optimal Decision
0 Empty	33.42	4 Prod 4
1 Stock1	30.15	4 Prod 4
2 Stock2	24.09	0 Idle
3 Stock3	19.19	0 Idle
4 Stock4	15.42	0 Idle
5 Stock5	12.15	0 Idle

Stage 2 (Friday)

State	Optimal Values	Optimal Decision
0 Empty	22.90	4 Prod 4
1 Stock1	19.90	3 Prod 3
2 Stock2	13.35	0 Idle
3 Stock3	8.19	0 Idle
4 Stock4	4.90	0 Idle
5 Stock5	2.78	0 Idle

Stage 1 (Saturday)

State	Optimal Values	Optimal Decision
0 Empty	13.50	3 Prod 3
1 Stock1	7.40	0 Idle
2 Stock2	1.90	0 Idle
3 Stock3	¯1.50	0 Idle
4 Stock4	¯3.50	0 Idle
5 Stock5	¯5.50	0 Idle