56:171

Operations Research

Fall 1997

Quizzes & Solutions

Dept of Mechanical & Industrial Engineering
University of Iowa

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For each statement, indicate "+"=**true** or "o"=**false**.

Part I: LINDO

____ a. The **LOOK** command is used to display the LP model after you have entered the formulation.
____ b. After the LP model has been entered, the SOLVE command finds the optimal solution.
____ c. After the LP model has been entered, the **SAVE** command causes the LP model to be written to a file which can be used as input in another session.
____ d. To begin your entry of the LP model into LINDO, you should use the command **ENTER**.
____ e. The number of *rows* in the LP model is always greater than the number of constraints which you have entered.
____ f. LINDO does not accept "strict inequalities" such as X + 2Y < 10, so that you must enter X + 2Y <= 10 instead.
____ g. The command **DIVERT** will cause subsequent LINDO output to be sent to a text file.
____ h. The command **RVRT** will reverse the direction of an inequality which you previously have entered.
____ i. When entering your LP model, the last constraint which you enter should be followed by **END**.
____ j. The **PRINT** command may be used to print any LINDO output which has previously appeared on the monitor.
____ k. LINDO assumes that all variables are restricted to be nonnegative, so that you need not explicitly enter these constraints.
____ l. After you have entered the objective function, you must enter **SUBJECT TO** before entering the first constraint.
____ m. The command **DIVERT** will send all output which has previously appeared on the monitor display to a text file.

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Part II: Investment problem

Consider the following investment problem: You now have $100 available for investment (beginning of year #1). Your objective is to maximize the value of this initial investment after four years, i.e., the end of year #4 or equivalently, the beginning of year #5. The available investments are:

- Investment **A** is available <u>only at the beginning of years 1 and 2</u>; each $1 invested in A will be returned in two equal payments of $0.70 at the beginning of each of the following 2 years. (For example, if you invest $1 now, at the beginning of year 1, then you receive $0.70 at the beginning of year 2 <u>and</u> another $0.70 at the beginning of year 3.)
- Investment **B** is available only once, <u>at the beginning of year 2</u>; each $1 invested in B at the beginning of year 2 returns $2 after 3 years, i.e., the beginning of year 5.
- A Money Market fund (**R**) is <u>available every year</u>; each $1 invested in this way will return $1.10 after 1 year.

The following table displays these cash flows. For example, -1 indicates $1 put into the investment, and +0.70 indicates $0.70 received from the investment.

begin year #	A1	A2	B	R1	R2	R3	R4

1	-1			-1			
2	+0.7	-1	-1	+1.1	-1		
3	+0.7	+0.7			+1.1	-1	
4		+0.7				+1.1	-1
5			+2				+1.1

n. Complete the equation: 0.7A1 + 0.7A2 + 1.1R2 - R3 = _______________

o. The objective should be to maximize *(select one):*

___ 1.4A2 + 2B + 1.1R4
___ 2B + 1.1R4
___ 1.4A1 + 1.4A2 + 2B + 1.1R1 + 1.1R2 + 1.1R3 + 1.1R4
___ 0.4A2 + 1B + 0.1R4
___ none of the above

<><><><><><><><> Quiz #1 Solutions <><><><><><><><>

Part I: LINDO

__+__ a. The **LOOK** command is used to display the LP model after you have entered the formulation.

__o__ b. After the LP model has been entered, the SOLVE command finds the optimal solution. *Note: The command **GO** performs this function.*

__+__ c. After the LP model has been entered, the **SAVE** command causes the LP model to be written to a file which can be used as input in another session.

__o__ d. To begin your entry of the LP model into LINDO, you should use the command **ENTER**. *Note: The command **MAX** (or **MIN** if the LP is a minimization problem) performs this function.*

__+__ e. The number of *rows* in the LP model is always greater than the number of constraints which you have entered.

__o__ f. LINDO does not accept "strict inequalities" such as X + 2Y < 10, so that you must enter X + 2Y <= 10 instead. *Note: while it is true that LINDO does not handle strict inequalities, it treats "<" as equivalent to "<=", i.e., "≤".*

__+__ g. The command **DIVERT** will cause subsequent LINDO output to be sent to a text file.

__o__ h. The command **RVRT** will reverse the direction of an inequality which you previously have entered. *Note: this command (an acronym for REVERT) terminates the diversion of output to the print file, which was previously commanded by **DIVERT**.*

__+__ i. When entering your LP model, the last constraint which you enter should be followed by **END**.

__o__ j. The **PRINT** command may be used to print any LINDO output which has previously appeared on the monitor. *Note: LINDO does not have a print command. To print the LP output, you must DIVERT the output to a text file, and then use a wordprocesser or equivalent to print it.*

__+__ k. LINDO assumes that all variables are restricted to be nonnegative, so that you need not explicitly enter these constraints.

__o__ l. After you have entered the objective function, you must enter **SUBJECT TO** before entering the first constraint. *Note: you must enter **ST** rather than the complete spelling **SUBJECT TO**.*

__o__ m. The command **DIVERT** will send all output which has previously appeared on the monitor display to a text file. *Note: this command sends all output from following commands to the text file, until you issue the command **RVRT** (revert).*

Part II: Investment problem

Consider the following investment problem: You now have $100 available for investment (beginning of year #1). Your objective is to maximize the value of this initial investment after four years, i.e., the end of year #4 or equivalently, the beginning of year #5. The available investments are:

- Investment **A** is available only at the beginning of years 1 and 2; each $1 invested in A will be returned in two equal payments of $0.70 at the beginning of each of the following 2 years. (For example, if you invest $1 now, at the beginning of year 1, then you receive $0.70 at the beginning of year 2 and another $0.70 at the beginning of year 3.)
- Investment **B** is available only once, at the beginning of year 2; each $1 invested in B at the beginning of year 2 returns $2 after 3 years, i.e., the beginning of year 5.
- A Money Market fund (**R**) is available every year; each $1 invested in this way will return $1.10 after 1 year.

The following table displays these cash flows. For example, -1 indicates $1 put into the investment, and +0.70 indicates $0.70 received from the investment.

begin year #	A1	A2	B	R1	R2	R3	R4
1	-1			-1			
2	+0.7	-1	-1	+1.1	-1		
3	+0.7	+0.7			+1.1	-1	
4		+0.7				+1.1	-1
5			+2				+1.1

n. Complete the equation: 0.7A1 + 0.7A2 + 1.1R2 - R3 = ___0___

o. The objective should be to maximize (select one):

___ 1.4A2 + 2B + 1.1R4
_X 2B + 1.1R4
___ 1.4A1 + 1.4A2 + 2B + 1.1R1 + 1.1R2 + 1.1R3 + 1.1R4
___ 0.4A2 + 1B + 0.1R4
___ none of the above

<><><><><><><><><><> Quiz #2 <><><><><><><><><><>

For each statement, indicate "+"=**true** or "o"=**false**.

___ a. When you enter an LP formulation into LINDO, you must first convert all inequalities to equations.
___b. When you enter an LP formulation into LINDO, you must manipulate your constraints so that all variables appear on the left, and all constants on the right.
___c. A "pivot" in a nonbasic column of a tableau will make it a basic column.
___d. It may happen that an LP problem has (exactly) two optimal solutions.
___e. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *cannot* pivot in row i.
___f. A "pivot" in row i of the column for variable X_j will increase the number of basic variables.
___g. If a slack variable S_i for row i is basic in the optimal solution, then variable X_i cannot be basic.
___h. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *must* pivot in row i.
___i. A "pivot" in the simplex method corresponds to a move from one corner point of the feasible region to another.
___j. A basic solution of the problem "minimize cx subject to $Ax \leq b$, $x \geq 0$" corresponds to a corner of the feasible region.
___k. The feasible region is the set of all points that satisfy *at least* one constraint.
___l. Adding constraints to an LP may improve the optimal objective function value.
___m. The number of basic variables in an LP is equal to the number of rows, *including* the objective function row.
___n. In the simplex method, every variable of the LP is either basic or nonbasic.
___o. In a basic LP solution, the nonbasic variables equal zero.
___p. The restriction that X1 be nonnegative should be entered into LINDO as the constraint X1 >= 0.
___q. The "minimum ratio test" is used to select the pivot row in the simplex method for linear programming.
___r. The value in the objective row of the simplex tableau is referred to as "reduced cost" or "relative profit", depending upon whether you are minimizing or maximizing, respectively.
___s. In the simplex method (as described in the lectures, not the textbook), the quantity $-Z$ serves as a basic variable, where Z is the value of the objective function.
___t. Every optimal solution of an LP is a basic solution.
___u. Basic solutions of an LP with constraints $Ax \leq b$, $x \geq 0$ correspond to "corner" points of the feasible region.
___v. In the simplex tableau, the objective row is written in the form of an equation.
___w LINDO would interpret the constraint "X1 + 2X2 > 10" as "X1 + 2X2 ≥ 10"

Multiple-Choice:

____ x. If you make a mistake in choosing the pivot column in the simplex method, the solution in the next tableau
(a) will be nonbasic (c) will have a worse objective value
(b) will be infeasible (d) *None of the above*

____ y. If you make a mistake in choosing the pivot row in the simplex method, the solution in the next tableau
(a) will be nonbasic (c) will have a worse objective value
(b) will be infeasible (d) *None of the above*

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For each statement, indicate "+"=**true** or "o"=**false**.

_o_ a. When you enter an LP formulation into LINDO, you must first convert all inequalities to equations. *Note: LINDO accepts inequality constraints, and then adds any necessary slack/surplus variable to convert to equations, as required by the SIMPLEX method.*

_+_b. When you enter an LP formulation into LINDO, you must manipulate your constraints so that all variables appear on the left, and all constants on the right.

_+_c. A "pivot" in a nonbasic column of a tableau will make it a basic column.

_o_d. It may happen that an LP problem has (exactly) two optimal solutions. *Note: While an LP may have exactly two basic optimal solutions, corresponding to "corners" of the feasible region, all solutions on the "edge" joining those "corners" must also be optimal (but nonbasic).*

_o_e. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *cannot* pivot in row i. *Note: only a zero or negative value in a row of the pivot column can disqualify that row from use in the pivot.*

_o_f. A "pivot" in row i of the column for variable X_j will increase the number of basic variables. *Note: Every basic solution of a system of equations has the same number of basic variables (one per equation), so that the number of basic variables remains constant throughout the simplex method.*

_o_g. If a slack variable S_i for row i is basic in the optimal solution, then variable X_i cannot be basic.

_o_h. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next iteration you *must* pivot in row i. *Note: if there is only a single zero on the right-hand-side and if the corresponding number in the pivot column is positive, then this must be the pivot row. If, however, there is a negative substitution rate in the corresponding row of the pivot column, then this would disqualify the use of this row for the pivot.*

_+_i. A "pivot" in the simplex method corresponds to a move from one corner point of the feasible region to another.

_+_ j. A basic solution of the problem "minimize cx subject to $Ax \le b$, $x \ge 0$" corresponds to a corner of the feasible region.

_o_ k. The feasible region is the set of all points that satisfy *at least* one constraint. *Note: points in the feasible region must satisfy all of the constraints!*

_o_ l. Adding constraints to an LP may improve the optimal objective function value. *Note: the addition of a constraint to an LP might possible make the previous optimal solution infeasible, in which case the objective function might be worsened, but it cannot add a new improved optimal solution.*

_+_ m. The number of basic variables in an LP is equal to the number of rows, *including* the objective function row.

_+_ n. In the simplex method, every variable of the LP is either basic or nonbasic.

_+_ o. In a basic LP solution, the nonbasic variables equal zero.

_o_ p. The restriction that X1 be nonnegative should be entered into LINDO as the constraint X1 >= 0. *Note: LINDO assumes (requires) that all variables have nonnegativity constraints, so one should not include these constraints explicitly. Some other LP software packages assume by default that variables are nonnegative, but allow this default to be overridden.*

_+_ q. The "minimum ratio test" is used to select the pivot row in the simplex method for linear programming.

_+_ r. The value in the objective row of the simplex tableau is referred to as "reduced cost" or "relative profit", depending upon whether you are minimizing or maximizing, respectively.

_+_ s. In the simplex method (as described in the lectures, not the textbook), the quantity $-Z$ serves as a basic variable, where Z is the value of the objective function.

_o_ t. Every optimal solution of an LP is a basic solution. *Note: see the comment in (d) above.*

_+_ u. Basic solutions of an LP with constraints Ax≤b, x≥0 correspond to "corner" points of the feasible region. ***Note:*** *Actually, the statement might be considered false, since basic solutions of an LP might include infeasible as well as feasible solutions, and of course the infeasible basic solutions would not correspond to "corner" points of the feasible region!*

_+_ v. In the simplex tableau, the objective row is written in the form of an equation.

_+_ w LINDO would interpret the constraint "X1 + 2X2 > 10" as "X1 + 2X2 ≥ 10"

Multiple-Choice:

_c_ x. If you make a mistake in choosing the pivot column in the simplex method, the solution in the next tableau

(a) will be nonbasic	(c) will have a worse objective value
(b) will be infeasible	(d) *None of the above*

_b_ y. If you make a mistake in choosing the pivot row in the simplex method, the solution in the next tableau

(a) will be nonbasic	(c) will have a worse objective value
(b) will be infeasible	(d) *None of the above*

<><><><><><><><><><><> Quiz #3 <><><><><><><><><><><>

1. *Simplex Method.* Classify each simplex tableau below, using the following classifications, and write the appropriate letter on the right of the tableau. If class B, D, or E, indicate, by circling,the additional information requested.

A. **UNIQUE** OPTIMUM.
B. **OPTIMAL**, but with **ALTERNATE** optimal solutions. *Indicate (by circling) a ny pivot element which would yield an alternate basic optimal solution.*
C. **INFEASIBLE**
D. FEASIBLE but **NOT OPTIMAL**. *Indicate (by circling) any pivot element which would yield an improved solution.*
E. FEASIBLE but **UNBOUNDED**. *Indicate any variable which, by increasing without limits, will improve the objective without limit.*

Take careful note of whether the LP is being **min**imized or **max**imized! Note also that (-z), rather than z, appears in the first column (i.e., corresponding to the approach used in my notes instead of that in the text by Winston).

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MAX	1	2	0	-3	4	-1	0	1	0	-10	
	0	0	0	2	-4	0	0	-1	1	3	___E
	0	-3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	1	0	2	

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MAX	1	0	0	-4	-2	-3	0	-1	0	-10	
	0	0	0	2	1	0	0	-1	1	3	___A
	0	-3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	1	0	2	

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MAX	1	-2	0	4	-2	3	0	1	0	-10	
	0	0	0	2	1	0	0	-1	1	3	___C
	0	-3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	1	0	-2	

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MAX	1	0	0	-4	-2	-3	0	-1	0	-10	
	0	0	0	2	1	0	0	-1	1	3	____B
	0	3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	1	0	2	

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MAX	1	-2	0	-4	-2	-3	0	1	0	-10	
	0	0	0	2	1	0	0	-1	1	3	____D
	0	-3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	1	0	2	

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2. For each statement, indicate "+"=**true** or "o"=**false**.

____ a. If there is a tie in the "minimum-ratio test" of the simplex method, the next basic solution will be degenerate, i.e., one of the basic variables will be zero.

____ b. In the two-phase simplex method, an artificial variable is defined for each constraint row lacking a slack variable (assuming the right-hand-side of the LP tableau is nonnegative).

____ c. The Revised Simplex Method, for most LP problems, requires fewer computations per iteration than the ordinary simplex method.

____ d. When maximizing in the simplex method, the value of the objective function increases at every iteration, except possibly in the case of a degenerate tableau (0 on the right-hand-side).

____ e. If you make a mistake in choosing the pivot column in the simplex method, the next basic solution will be infeasible.

____ f. A basic solution of an LP is always feasible, but not all feasible solutions are basic.

____ g. The coefficients of a nonbasic variable in the current simplex tableau are the *negatives* of the so-called "substitution rates".

____ h. In Phase One of the 2-Phase method, one should never pivot in the column of an artificial variable.

____ i. If the nonbasic variable X_j has a negative reduced cost in a simplex iteration for a minimization LP, then increasing X_j will *worsen* the objective function.

____ j. In the revised simplex method, the simplex multiplier vector π will have an element for each of the nonbasic variables in the LP.

____ k. For a minimization LP with constraints $Ax \geq b$, where b is a positive right-hand-side vector, one must introduce artificial variables in the simplex algorithm.

____ l. The simplex multiplier vector π is used in computing the substitution rates of a pivot column.

____ m. The Revised Simplex Method, for most LP problems, requires fewer iterations than the ordinary simplex method.

____ n. If you make a mistake in choosing the pivot row in the simplex method, the next basic solution will be infeasible.

____ o. If a column of a nonbasic variable in the current tableau contains only zero and negative elements, then the LP must be unbounded.

____ p. The simplex multiplier vector at each iteration is computed by $\pi = \left(A^B\right)^{-1} c_B$, where B is the ordered set of current basis indices.

____ q. In the ordinary simplex method, the current basis inverse matrix may always be found somewhere in the current tableau, without further computation.

<><><><><><><><> Quiz #3 Solutions <><><><><><><><>

1. *Simplex Method.* Classify each simplex tableau below, using the following classifications, and write the appropriate letter on the right of the tableau. If class B, D, or E, indicate, by circling,the additional information requested.

A. **UNIQUE** OPTIMUM.
B. **OPTIMAL**, but with **ALTERNATE** optimal solutions. *Indicate (by circling) a ny pivot element which would yield an alternate basic optimal solution.*
C. **INFEASIBLE**
D. FEASIBLE but **NOT OPTIMAL**. *Indicate (by circling) any pivot element which would yield an improved solution.*
E. FEASIBLE but **UNBOUNDED**. *Indicate any variable which, by increasing without limits, will improve the objective without limit.*

Take careful note of whether the LP is being **min**imized or **max**imized! Note also that (-z), rather than z, appears in the first column (i.e., corresponding to the approach used in my notes instead of that in the text by Winston).

pivot

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MAX	1	2	0	-3	4	-1	0	1	0	-10	
	0	0	0	2	-4	0	0	-1	1	3	__E_
	0	-3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	1	0	2	

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MAX	1	0	0	-4	-2	-3	0	-1	0	-10	
	0	0	0	2	1	0	0	-1	1	3	__A__
	0	-3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	1	0	2	

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MAX	1	-2	0	4	-2	3	0	1	0	-10	
	0	0	0	2	1	0	0	-1	1	3	__C_
	0	-3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	1	0	-2	

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MAX	1	0	0	-4	-2	-3	0	-1	0	-10	
	0	0	0	2	1	0	0	-1	1	3	__B_
	0	3	1	0	-1	2	0	2	0	6	
	0	$\boxed{2}$	0	3	0	5	1	1	0	2	

	-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
MAX	1	-2	0	-4	-2	-3	0	1	0	-10	
	0	0	0	2	1	0	0	-1	1	3	__D__
	0	-3	1	0	-1	2	0	2	0	6	
	0	2	0	3	0	5	1	$\boxed{1}$	0	2	

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2. For each statement, indicate "+"=**true** or "o"=**false**.

__+_ a. If there is a tie in the "minimum-ratio test" of the simplex method, the next basic solution will be degenerate, i.e., one of the basic variables will be zero.

__+_ b. In the two-phase simplex method, an artificial variable is defined for each constraint row lacking a slack variable (assuming the right-hand-side of the LP tableau is nonnegative).

__+_ c. The Revised Simplex Method, for most LP problems, requires fewer computations per iteration than the ordinary simplex method.

__+_ d. When maximizing in the simplex method, the value of the objective function increases at every iteration, except possibly in the case of a degenerate tableau (0 on the right-hand-side).

__o_ e. If you make a mistake in choosing the pivot column in the simplex method, the next basic solution will be infeasible. *Note: the solution remains feasible, but the objective value worsens!*

__o_ f. A basic solution of an LP is always feasible, but not all feasible solutions are basic. *Note: This might be judged false for two reasons! Firstly, if "basis solutions of the LP" means the basic solutions of the system of equations represented by the tableau, then it might be classified as either* *feasible or infeasible, depending upon its satisfaction of the nonnegativity restrictions.* *Secondly, the entire line segment between two basic feasible solutions consists of points which are not basic, but which are feasible (as are any solutions represented by points within the interior of the feasible region.*

__o_ g. The coefficients of a nonbasic variable in the current simplex tableau are the *negatives* of the so-called "substitution rates". *Note: the substitution rates are exactly as they appear in the left side of the tableau.*

__+_ h. In Phase One of the 2-Phase method, one should never pivot in the column of an artificial variable.

__o_ i. If the nonbasic variable X_j has a negative reduced cost in a simplex iteration for a minimization LP, then increasing X_j will *worsen* the objective function. *Note: a negative reduced cost indicates that the objective will go down as* X_j *is increased, which is an improvement, not a worsening of the objective function value.*

__o_ j. In the revised simplex method, the simplex multiplier vector π will have an element for each of the nonbasic variables in the LP. *Note: The number of elements in* π *is identical to the number of constraints, which (if -Z is ignored) is the number of basic variables.*

__+_ k. For a minimization LP with constraints $Ax \geq b$, where b is a positive right-hand-side vector, one must introduce artificial variables in the simplex algorithm.

__o_ l. The simplex multiplier vector π is used in computing the substitution rates of a pivot column. *Note: The simplex multiplier vector* π *is used to compute the reduced costs (or relative profits), in order to select the next pivot column. That is, the reduced cost of Xj is* $\bar{c}_j = c_j - \pi A^j$

__o_ m. The Revised Simplex Method, for most LP problems, requires fewer iterations than the ordinary simplex method. *Note: If the same rule is used to choose the pivot column (e.g., column with the largest relative profit), then the number of iterations is identical.*

__+_ n. If you make a mistake in choosing the pivot row in the simplex method, the next basic solution will be infeasible.

__o_ o. If a column of a nonbasic variable X_j in the current tableau contains only zero and negative elements, then the LP optimal solution must be unbounded. Note: The truth of this statement depends upon the sign of the reduced cost/relative profit in the objective row. If it is a maximation and the relative profit in this column is negative, then increasing X_j will worsen the objective function value rather than improving it.

__o_ p. The simplex multiplier vector at each iteration is computed by $\pi = \left(A^B\right)^{-1} c_B$, where B is the ordered set of current basis indices. *Note: the correct formula is* $\pi = c_B \left(A^B\right)^{-1}$, *which gives a different result than the formula above.*

____ q. In the ordinary simplex method, the current basis inverse matrix may always be found somewhere in the current tableau, without further computation. *Note: If every constraint, when converted to an equation, includes a slack variable (i.e., it was a "≤" constraint), then an identity matrix appears initially in the slack columns, and in the current tableau will be found* $\left(A^B\right)^{-1} I = \left(A^B\right)^{-1}$. *Likewise, if all constraints were of type "≥" then surplus variables would be subtraced from the left side to transform to equations, and the matrix -I would appear in the surplus columns of the original tableau, and* $\left(A^B\right)^{-1}\left(-I\right) = -\left(A^B\right)^{-1}$ *(i.e., the negative of the basis inverse matrix) will appear this that location of the current tableau. If, on the other hand, all constraints were originally equations, with no slack or surplus variables, then the basis inverse matrix would be nowhere to be found.*

<><><><><><><><> Quiz #4 <><><><><><><><>

1. Sensitivity Analysis. Consider the LP problem (lumber processing) below:

Brady Corporation produces wooden cabinets. Each week, they require 90,000 cu ft of processed lumber. They may obtain processed lumber in two ways. First, they may purchase lumber from an outside supplier and then dry it at their kiln. Second, they may chop down logs on their land, cut them into lumber at their sawmill, and finally dry the lumber at their kiln.

Brady can purchase grade 1 or grade 2 lumber. Grade 1 lumber costs \$3 per cu ft and when dried yields 0.7 cu ft of useful lumber. Grade 2 lumber costs \$7 per cu ft and when dried yields 0.9 cu ft of useful lumber. It costs the company \$3 per cu ft to chop down a log. After being cut and dried, one cubic foot of log yields 0.8 cu ft of lumber. Brady incurs costs of \$4 per cu ft of lumber dried.

It costs \$2.50 per cu ft of logs sent through their sawmill. Each week, the sawmill can process up to 35,000 cu ft of lumber. Each week, up to 40,000 cu ft of grade 1 lumber and up to 60,000 cu ft of grade 2 lumber can be purchased.

Each week, 40 hours of time are available for drying lumber. The time it takes to dry 1 cu ft of grade 1 lumber, grade 2 lumber, or logs is as follows:

grade 1:	2 seconds
grade 2:	0.8 seconds
log:	1.3 seconds

Brady has formulated an LP to minimize the weekly cost of meeting the demand for processed lumber:

Define the **decision variables**

G1 = the # of cu ft /week of grade 1 lumber purchased and used,
G2 = the # of cu ft /week of grade 2 lumber purchased and used,
LOG = the # of cu ft /week of the corporation's own lumber used.

LP Model :

Min	$3G1+7G2$	(purchase cost)
	$+4(G1+G2+LOG)$	(dry cost for lumber)
	$+3LOG$	(cost for chopping)
	$+2.5LOG$	(cost for sawmill)

s.t.

$0.7G1+0.9G2+0.8LOG\geq 90000$	(constraint for demand)
$2G1+0.8G2+1.3LOG\leq 144000$	(available hours for drying)
$G1\leq 40000$	(available cu ft of grade 1 per week)
$G2\leq 60000$	(available cu ft of grade 2 per week)
$LOG\leq 35000$	(available cu ft of own lumber per week)
$G1,G2,LOG\geq 0$	

LINDO output : *(Note that the upper bounds on G1, G2, and LOG were entered into LINDO using the SUB (simple upper bound) command.)*

```
MIN     7 G1 + 11 G2 + 9.5 LOG
  SUBJECT TO
         2)   0.7 G1 + 0.9 G2 + 0.8 LOG >=   90000
         3)   2 G1 + 0.8 G2 + 1.3 LOG <=   144000
  END
  SUB       G1    40000.00000
  SUB       G2    60000.00000
  SUB      LOG    35000.00000
```

```
       LP OPTIMUM FOUND AT STEP      3

              OBJECTIVE FUNCTION VALUE

              1)      1033585.

        VARIABLE        VALUE          REDUCED COST
              G1     40000.000000         -.905659
              G2     55471.703000          .000000
             LOG     15094.340000          .000000


             ROW   SLACK OR SURPLUS     DUAL PRICES
              2)         -.000000       -12.641510
              3)          .000000          .471699

       NO. ITERATIONS=       3

       RANGES IN WHICH THE BASIS IS UNCHANGED:

                                 OBJ COEFFICIENT RANGES
       VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                         COEF          INCREASE         DECREASE
             G1        7.000000          .905659         INFINITY
             G2       11.000000          .695651          .312500
            LOG        9.500000          .277778          .387096

                                 RIGHTHAND SIDE RANGES
            ROW         CURRENT        ALLOWABLE        ALLOWABLE
                          RHS          INCREASE         DECREASE
              2   90000.000000      1846.151400     13187.500000
              3  144000.000000     11722.222000      2999.996000

THE TABLEAU
     ROW (BASIS)      G1       G2       LOG     SLK 2   SLK 3
     1 ART           .91    0.69E-06 0.72E-06  13.      .47         -0.10E+07
     2 LOG         -2.340   0.000    1.000      1.509   1.698   15094.34
      3 G2  1.302 1.000 .000  -2.453      -1.509      55471.70
```

a. Suppose that the company can increase the amount of time available for drying each week, through use of overtime, which will cost $0.50/second, including additional labor and energy costs. Should they schedule the overtime? Circle: YES or NO

b. If so, how many seconds should they schedule? ______

c. Whether or not you answered "yes" in part (a), suppose that 1000 additional seconds are available on the dryer. Using the substitution rates, compute the modifications to the optimal values of the variables which would result from the use of this additional time.
G1 ___________ cubic feet ___ increase? ___decrease? ___ no change?
G2 ___________ cubic feet ___ increase? ___decrease? ___ no change?
LOG___________ cubic feet ___ increase? ___decrease? ___ no change?

d. If the cost of cutting and processing logs from its own land were to increase to $10/cu ft, will the optimal solution change? ____________

e. What additional cost would result if the demanded amount of lumber were to increase by 1000 cubic feet (i.e., from 90,000 to 91,000)? $___________

<><><><><><><>

2. For each statement, indicate "+"=**true** or "o"=**false**.

_____ a. The optimal basic solution to an LP with *m* constraints (excluding non-negativity constraints) can have at most *m* positive decision variables.

_____ b. If the primal LP has an equality constraint, the corresponding dual variable must be zero.

_____ c. The dual of an LP problem is always a MAXIMIZE problem with "≤" constraints.

_____ d. If an LP problem has 3 constraints (not including non-negativity) and 5 variables, then its dual problem has 5 constraints (not including non-negativity) and 3 variables.

_____ e. If you increase the right-hand-side of a "≥" constraint in a minimization LP, the optimal objective value will either increase or stay the same.

_____ f. The "reduced cost" in LP provides an estimate of the change in the objective value when the right-hand-side of a constraint changes.

_____ g. In the two-phase simplex method, Phase One computes the optimal dual variables, followed by Phase Two in which the optimal primal variables are computed.

_____ h. At the completion of the revised simplex method applied to an LP, the simplex multipliers give the optimal solution to the dual of the LP.

_____ i. At the end of the first phase of the two-phase simplex method, the phase-one objective function must be zero if the LP is feasible.

_____ j. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next simplex iteration you *cannot* pivot in row i.

_____ k. If a minimization LP problem has a cost which is unbounded below, then its dual problem has an objective (to be maximized) which is unbounded above.

_____ l. If you increase the right-hand-side of a "≤" constraint in a maximization LP, the optimal objective value will either increase or stay the same.

_____ m. If the increase in the cost of a nonbasic variable remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.

_____ n. If the increase in the right-hand-side of a "tight" constraint remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.

<><><><><><><><><><> Quiz #4 Solutions <><><><><><><><><><>

1. Sensitivity Analysis.

a. Suppose that the company can increase the amount of time available for drying each week, through use of overtime, which will cost $0.50/second, including additional labor and energy costs. Should they schedule the overtime? Circle: YES or [✓ NO]
Solution: The DUAL PRICE indicates that each unit (second) increase in the right-hand-side of row 3, which imposes the restriction on the time available for drying, will improve ti.e. lower) the total cost by $0.471699, less than the cost of increasing the right-hand-side.

b. If so, how many seconds should they schedule? __none_

c. Whether or not you answered "yes" in part (a), suppose that 1000 additional seconds are available on the dryer. Using the substitution rates, compute the modifications to the optimal values of the variables which would result from the use of this additional time.

G1 ______0_____ cubic feet ___ increase? ___decrease? _X_ no change?
G2 ___1509___ cubic feet ___ increase? _X_decrease? ___ no change?

LOG___1698___ cubic feet _X_ increase? ___decrease? ___ no change?

Solution: In order to increase the number of seconds used in the drying process by 1000 in the equation

```
3)   2 G1 + 0.8 G2 + 1.3 LOG + SLK3 =   144000,
```

the slack variable SLK3 must decrease from its current value of zero by 1000. Consulting the substitution rates of SLK3 in the tableau, we find that the substitution rate for LOG is +1.698 and that for G2 is -1.509. A positive substitution rate indicates that the basic variable (LOG) will change in the direction opposite to that of the nonbasic variable SLK3, i.e., it will increase. Conversely, a negative substitution rate indicates that the basic variable (G2) will change in the same direction as that of the nonbasic variable SLK3, i.e., it will decrease. The variable G1, on the other hand, remains unchanged (at its upper bound).

d. If the cost of harvesting logs (now $9.50/cu ft) were to increase to $10.00/cu ft, will the optimal solution change? Circle: [✓ YES] or NO

Solution: The ALLOWABLE INCREASE for the objective coefficient of LOG is 0.277778, i.e., $0.277778/cubic foot. Therefore, an increase of $0.50 is outside of the range, meaning that the optimal basis will change, and therefore, the basic solution as well. (The optimal value of LOG will undoubtedly decrease, although not necessarily to zero.)

e. What additional cost would result if the demanded amount of lumber were to increase by 1000 cubic feet (i.e., from 90,000 to 91,000)? $_12641.51__

Solution: The ALLOWABLE INCREASE for the right-hand-side of row 2 is 1846.1514 cubic feet. This means that an increase of only 1000 cubic feet is within the allowable range, and the optimal basis will not change and the DUAL PRICE, which gives the rate of increase in cost, remains valid for all 1000 cubic feet, i.e., the increase in cost will be $12641.51.

<><><><><><><><>

2. For each statement, indicate "+"=**true** or "o"=**false**.

__+__ a. The optimal basic solution to an LP with *m* constraints (excluding non-negativity constraints) can have at most *m* positive decision variables.

__o__ b. If the primal LP has an equality constraint, the corresponding dual variable must be zero.
In this case, the corresponding dual variable will be unrestricted in sign.

__o__ c. The dual of an LP problem is always a MAXIMIZE problem with "≤" constraints.
This is true only if the primal is a MINIMIZE problem with nonnegativity constraints on all of the variables.

__+__ d. If an LP problem has 3 constraints (not including non-negativity) and 5 variables, then its dual problem has 5 constraints (not including non-negativity) and 3 variables.

__+__ e. If you increase the right-hand-side of a "≥" constraint in a minimization LP, the optimal objective value will either increase or stay the same.

__o__ f. The "reduced cost" in LP provides an estimate of the change in the objective value when the right-hand-side of a constraint changes.
The reduced cost of a nonbasic variable is the rate of change in the objective value when that nonbasic variable is increased!

__o__ g. In the two-phase simplex method, Phase One computes the optimal dual variables, followed by Phase Two in which the optimal primal variables are computed.
Phase One has the task of eliminating the artificial variables (forming the initial basic solution) from the basis, in order to obtain a truly feasible solution.

__+__ h. At the completion of the revised simplex method applied to an LP, the simplex multipliers give the optimal solution to the dual of the LP.

__+__ i. At the end of the first phase of the two-phase simplex method, the phase-one objective function must be zero if the LP is feasible.

__o__ j. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next simplex iteration you *cannot* pivot in row i.
On the contrary, if there is a positive substitution rate in that row of the pivot column, then the minimum ratio test will force a pivot in that row (unless another right-hand-side is also zero).

__o__ k. If a minimization LP problem has a cost which is unbounded below, then its dual problem has an objective (to be maximized) which is unbounded above.
If the objective of the minimization problem is unbounded below, then its dual cannot be feasible, since in this case any feasible dual solution would provide a lower bound on the primal optimal value.

__+__ l. If you increase the right-hand-side of a "≤" constraint in a maximization LP, the optimal objective value will either increase or stay the same.

__+__ m. If the increase in the cost of a nonbasic variable remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.

__o__ n. If the increase in the right-hand-side of a "tight" constraint remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.
Although in this case the basis will remain unchanged, the basic variables, given by the formula $x_B = \left(A^B\right)^{-1} b$, *will change if the right-hand-side vector b changes.*

<><><><><><><><> Quiz #5 <><><><><><><><>

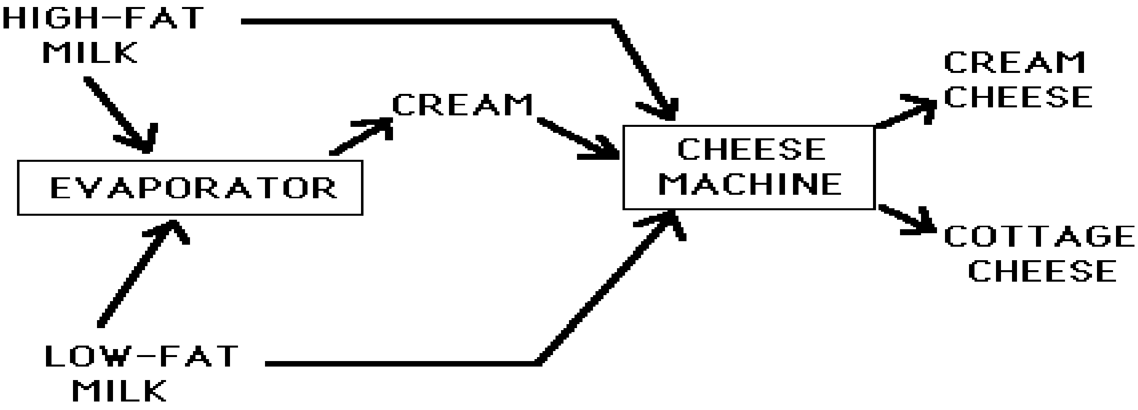
1. Sensitivity Analysis. Consider the LP problem:
Lizzie's Dairy produces cream cheese and cottage cheese. Milk and cream are blended together to produce these two products. Both high-fat and low-fat milk can be used to produce cream cheese and cottage cheese. High-fat milk is 60% fat; low-fat milk is 30% fat. The milk used to produce the cream cheese must average at least 50% fat, and the milk used to produce cottage cheese must average at least 35% fat. At least 40% (by weight) of the inputs to cream cheese must be cream. At least 20% (by weight) of the input to cottage cheese must be cream.

Both cottage cheese and cream cheese are produced by putting milk and cream through the cheese machine. It costs 40¢ to process 1 lb of inputs into a pound of cream cheese. It costs 40¢ also to produce 1 lb of cottage cheese, but every pound of input for cottage cheese yields 0.9 lb of cottage cheese and 0.1 lb of waste. Every day, up to 3000 lb of input may be sent through the cheese machine.

Cream can be produced by evaporating high-fat and low-fat milk. It costs 40¢ to evaporate 1 lb of high-fat milk. Each pound of high-fat milk that is evaporated yields 0.6 lb of cream. It costs 40¢ to evaporate 1 lb of low-fat milk. Each pound of low-fat milk that is evaporated yields 0.3 lb o cream. The evaporator can process at most 2000 lb of milk daily.

Every day, at least 1000 lb of cottage cheese and at least 1000 lb of cream cheese must be produced. Up to 1500 lb of cream cheese and up to 2000 lb of cottage cheese can be sold each day.

Cottage cheese is sold for $1.20/lb and cream cheese for $1.50/lb. High-fat milk is purchased for 80¢/lb and low-fat milk for 40¢/lb.



```
MAX      1.1 P1 + 0.8 P2 - 0.4 HFE - 0.4 LFE - 0.4 LF - 0.8 HF
  SUBJECT TO
         2)   P1 - HF1 - LF1 - C1 =    0
         3)   P2 - 0.9 HF2 - 0.9 LF2 - 0.9 C2 =    0
         4)   P1 >=   1000
         5)   P2 >=   1000
         6)   HFE + LFE <=   2000
         7)   HF1 + LF1 + C1 + HF2 + LF2 + C2 <=   3000
         8) - HFE + HF - HF1 - HF2 =    0
         9) - LFE + LF - LF1 - LF2 =    0
        10) - 0.6 HFE - 0.3 LFE + C1 + C2 =    0
        11)   0.1 HF1 - 0.2 LF1 >=   0
        12)   0.25 HF2 - 0.05 LF2 >=   0
        13) - 0.4 HF1 - 0.4 LF1 + 0.6 C1 >=   0
```

```
       14) - 0.2 HF2 - 0.2 LF2 + 0.8 C2 >=   0
 END
 SUB       P1     1500.00000
 SUB       P2     2000.00000

 LP OPTIMUM FOUND AT STEP     11

       OBJECTIVE FUNCTION VALUE

       1)     -159.2592

 VARIABLE        VALUE          REDUCED COST
       P1      1000.000000           .000000
       P2      1000.000000           .000000
      HFE      1037.037000           .000000
      LFE          .000000           .200000
       LF       940.740800           .000000
       HF      1585.185100           .000000
      HF1       400.000000           .000000
      LF1       200.000000           .000000
       C1       400.000000           .000000
      HF2       148.148140           .000000
      LF2       740.740800           .000000
       C2       222.222220           .000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)          .000000         1.200000
       3)          .000000          .859259
       4)         -.000000         -.100000
       5)         -.000000         -.059259
       6)       962.963010          .000000
       7)       888.888900          .000000
       8)          .000000         -.800000
       9)          .000000         -.400000
      10)          .000000         2.000000
      11)         -.000000        -1.333333
      12)         -.000000        -1.333333
      13)         -.000000        -1.333333
      14)         -.000000        -1.533333

 NO. ITERATIONS=      11

 RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT          ALLOWABLE        ALLOWABLE
                   COEF            INCREASE         DECREASE
      P1        1.100000           .100000         INFINITY
      P2         .800000           .059259         INFINITY
     HFE        -.400000           .150000          .400000
     LFE        -.400000           .200000         INFINITY
      LF        -.400000           .080000          .400000
      HF        -.800000           .093750          .400000
     HF1        -.000000           .250000         2.000000
     LF1        -.000000           .500000          .400000
      C1        -.000000           .250000         INFINITY
     HF2        -.000000           .400000         9.200000
     LF2        -.000000           .080000          .400000
      C2        -.000000           .266667         INFINITY

                           RIGHTHAND SIDE RANGES
     ROW          CURRENT          ALLOWABLE        ALLOWABLE
                    RHS            INCREASE         DECREASE
```

```
       2          .000000      1000.000000       888.888900
       3          .000000      1000.000000       799.999930
       4      1000.000000       500.000000      1000.000000
       5      1000.000000       799.999930      1000.000000
       6      2000.000000         INFINITY       962.963010
       7      3000.000000         INFINITY       888.888900
       8          .000000         INFINITY      1585.185100
       9          .000000         INFINITY       940.740800
      10          .000000       622.222220       577.777830
      11          .000000        60.000000       120.000000
      12          .000000       222.222240        44.444450
      13          .000000       577.777900       400.000000
      14          .000000       577.777830       222.222220


 THE TABLEAU
     ROW  (BASIS)          P1         P2        HFE        LFE         LF         HF
       1 ART            0.000      0.000       .000       .200       .000       .000
       2        P1      1.000       .000       .000       .000       .000       .000
       3        P2       .000      1.000       .000       .000       .000       .000
       4        LF       .000       .000       .000     -1.000      1.000       .000
       5       LF2       .000       .000       .000       .000       .000       .000
       6 SLK     6       .000       .000       .000       .500       .000       .000
       7 SLK     7       .000       .000       .000       .000       .000       .000
       8       HF1       .000       .000       .000       .000       .000       .000
       9       LF1       .000       .000       .000       .000       .000       .000
      10        C1       .000       .000       .000       .000       .000       .000
      11        HF       .000       .000       .000       .500       .000      1.000
      12       HF2       .000       .000       .000       .000       .000       .000
      13       HFE       .000       .000      1.000       .500       .000       .000
      14        C2       .000       .000       .000       .000       .000       .000


     ROW        HF1        LF1         C1        HF2        LF2         C2  SLK    4
       1      0.000       .000      0.000      0.000      0.000       .000       .100
       2       .000       .000       .000       .000       .000       .000     -1.000
       3       .000       .000       .000       .000       .000       .000       .000
       4      0.000       .000       .000      0.000      0.000       .000      -.200
       5       .000       .000       .000      0.000      1.000       .000       .000
       6       .000       .000      0.000      0.000      0.000      0.000       .667
       7       .000       .000       .000      0.000      0.000      0.000      1.000
       8      1.000      0.000       .000       .000       .000       .000      -.400
       9      0.000      1.000       .000       .000       .000       .000      -.200
      10       .000       .000      1.000       .000       .000       .000      -.400
      11       .000      0.000      0.000      0.000      0.000      0.000     -1.067
      12       .000       .000       .000      1.000      0.000      0.000       .000
      13       .000       .000      0.000      0.000      0.000      0.000      -.667
      14       .000       .000       .000       .000       .000      1.000       .000


     ROW   SLK 5     SLK 6    SLK 7     SLK 11     SLK 12   SLK 13  SLK 14   RHS
      1      .059      .000     .000     1.333      1.333     1.333   1.533   -159.259
      2      .000      .000     .000      .000       .000      .000    .000   1000.000
      3    -1.000      .000     .000      .000       .000      .000    .000   1000.000
      4     -.741      .000     .000     3.333      3.333      .333    .833    940.741
      5     -.741      .000     .000      .000      3.333      .000    .833    740.741
      6      .370     1.000     .000     0.000      0.000     1.667   1.667    962.963
      7     1.111      .000    1.000      .000      0.000      .000   0.000    888.889
      8      .000      .000     .000    -3.333       .000      .667    .000    400.000
      9      .000      .000     .000     3.333       .000      .333    .000    200.000
     10      .000      .000     .000      .000       .000    -1.000    .000    400.000
     11     -.519      .000     .000    -3.333     -3.333    -1.000  -1.500   1585.185
     12     -.148      .000     .000      .000     -3.333      .000    .167    148.148
     13     -.370      .000     .000     0.000      0.000    -1.667  -1.667   1037.037
```

```
     14     -.222     .000     .000      .000      .000      .000  -1.000    222.222
```

a. Suppose that the evaporator malfunctions during the day, and is able to process only 1500 lb. of milk, instead of the original 2000 lb. capacity. If possible, determine the resulting loss of profit: $_________

b. Suppose that there is a 10% **in**crease in the minimum requirement for cottage cheese (**P2**). If possible, determine the resulting change in profit: $_______ (increase or decrease?)

c. In the situation of (b) above, determine (if possible) the **change**, if any, of the optimal quantity of

pounds of high-fat milk to be purchased (**HF**): _______ (increase or decrease?)
pounds of low-fat milk to be purchased (**LF**): ________ (increase or decrease?)

d. Suppose that, due to a misunderstanding, 100 pounds of low-fat milk was put through the evaporator. Determine, if possible,

- the resulting **loss** in profit, if any: $______
- the **change** in the optimal quantity
  - of high-fat milk to be put through the evaporator: ______ lb
  - of high-fat milk to be purchased: ______pounds
  - of low-fat milk to be purchased: ______ pounds

<><><><><><><><>

2. For each statement, indicate "+"=**true** or "o"=**false**.

_____ a. If the primal LP has an equality constraint, the corresponding dual variable must be zero.
_____ b. The dual of an LP problem is always a MAXIMIZE problem with "≤" constraints.
_____ c. The optimal basic solution to an LP with *m* constraints (excluding non-negativity constraints) can have at most *m* positive decision variables.
_____ d. If you increase the right-hand-side of a "≤" constraint in a maximization LP, the optimal objective value will either increase or stay the same.
_____ e. If the increase in the cost of a nonbasic variable remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.
_____ f. If the increase in the right-hand-side of a "tight" constraint remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.
_____ g. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next simplex iteration you *cannot* pivot in row i.
_____ h. If an LP problem has 3 constraints (not including non-negativity) and 5 variables, then its dual problem has 5 constraints (not including non-negativity) and 3 variables.
_____ i. If you increase the right-hand-side of a "≥" constraint in a minimization LP, the optimal objective value will either increase or stay the same.
_____ j. At the end of the first phase of the two-phase simplex method, the phase-one objective function must be zero if the LP is feasible.
_____ k. The "reduced cost" in LP provides an estimate of the change in the objective value when the right-hand-side of a constraint changes.
_____ l. In the two-phase simplex method, Phase One computes the optimal dual variables, followed by Phase Two in which the optimal primal variables are computed.
_____ m. At the completion of the revised simplex method applied to an LP, the simplex multipliers give the optimal solution to the dual of the LP.

_____ n. If a minimization LP problem has a cost which is unbounded below, then its dual problem has an objective (to be maximized) which is unbounded above.

<><><><><><><><> Quiz #5 Solutions <><><><><><><><>

1. Sensitivity Analysis.

a. Suppose that the evaporator malfunctions during the day, and is able to process only 1500 lb. of milk, instead of the original 2000 lb. capacity. If possible, determine the resulting loss of profit: $___none_
Solution: The 500 decrease in the right-hand-side of row (6) is in the given range for which the current basis remains optimal (ALLOWABLE DECREASE = 962.963) and the DUAL PRICE of row (6) is zero. Therefore, there will be no change in the objective function.

b. Suppose that there is a 10% **in**crease in the minimum requirement for cottage cheese (**P2**). If possible, determine the resulting change in profit: $_decrease $5.93_
Solution: Row (5) enforces the minimum requirement for P2 (cottage cheese):

(5) P2 >= 1000

The DUAL PRICE for row (5) is - 0.059259, the rate of improvement in the optimal objective value as the right-hand-side of row (5) increases. Therefore, because of the negative DUAL PRICE, the profit will decrease by the amount ($0.0593/lb)(100 lb) = $5.93.

c. In the situation of (b) above, determine (if possible) the **change**, if any, of the optimal quantity of

pounds of high-fat milk to be purchased (**HF**): __Increase 100_
pounds of low-fat milk to be purchased (**LF**): __Increase 74.1_

Solution: In order to determine the effect of an increase in P2, we consider row (5) after it has been converted to an equation:

(5) P2 - SLK5 = 1000

If P2 increases from 1000 to 1100 lb., then SLK5 must increase from 0 to 100 lb in order to balance the equation. We next look at the substitution rates of SLK5 in the tableau, and find

Row #	Basic variable	Substitution rate
4	LF	-0.741
11	HF	-0.519

Whereas a positive substitution rate for a basic variable X would mean that increaseing SLK5 would replace (i.e., reduce) the basic variable X, a negative substitution rate means that the basic variable will increase. Therefore, as SLK5 increases by 100 lb, LF will increase by 100(0.741) = 74.1 lb, and HF will *increase* by 100(0.519) = 51.9 lb.

d. Suppose that, due to a misunderstanding, 100 pounds of low-fat milk was put through the evaporator. Determine, if possible,

- the resulting **loss** in profit, if any: $__20.00_
- the **change** in the optimal quantity
  - of high-fat milk to be put through the evaporator: __50 lb decrease
  - of high-fat milk to be purchased: __50 lb decrease
  - of low-fat milk to be purchased: __100 lb. increase

Solution: LFE (# lb of low-fat milk input to evaporator) is nonbasic (zero) in the optimal solution, with a REDUCED COST of $0.20/lb. That is, the objective function will deteriorate at the rate of $0.20/lb as LFE increases. If LFE were to be increased by 100 lb, the resulting deterioration in the proit would be ($0.20/lb)(100 lb) = $20. *Note that this assumes that no basic variable decreases below its lower bound (namely, zero) as LFE is increased by 100. One* *could determine the amount of LFE required for a basis change by using the minimum ratio test. If this is done, it would be determined that the minimum ratio is 962/0.5 = 1924 lb, much larger then the increase of 100 which we are considering.*

Next, look at the substitution rates of LFE in the tableau:

Row #	Basic variable	Substitution rate
4	LF	-1.000
11	HF	0.500
13	HFE	0.500

This means that if LFE increases by 100, LF will also increase by 100, while HF and HFE will both decrease by 0.500(100) = 50.

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2. For each statement, indicate "+"=**true** or "o"=**false**.

__o__ a. If the primal LP has an equality constraint, the corresponding dual variable must be zero.
Note: the corresponding dual variable is unrestricted in sign, being either positive, negative, or zero.

__o__ b. The dual of an LP problem is always a MAXIMIZE problem with "≤" constraints.
Note: This statement is not true in general-- it is true only if the primal is a MINIMIZE problem with nonnegative variables.

__+__ c. The optimal basic solution to an LP with *m* constraints (excluding non-negativity constraints) can have at most *m* positive decision variables.

__+__ d. If you increase the right-hand-side of a "≤" constraint in a maximization LP, the optimal objective value will either increase or stay the same.

__+__ e. If the increase in the cost of a nonbasic variable remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged.

__o__ f. If the increase in the right-hand-side of a "tight" constraint remains less than the "ALLOWABLE INCREASE" reported by LINDO, then the optimal values of all variables will be unchanged. Note: the optimal basis will be unchanged if the increase

__o__ g. If a zero appears on the right-hand-side of row i of an LP tableau, then at the next simplex iteration you *cannot* pivot in row i.

__+__ h. If an LP problem has 3 constraints (not including non-negativity) and 5 variables, then its dual problem has 5 constraints (not including non-negativity) and 3 variables.

__+__ i. If you increase the right-hand-side of a "≥" constraint in a minimization LP, the optimal objective value will either increase or stay the same.

__+__ j. At the end of the first phase of the two-phase simplex method, the phase-one objective function must be zero if the LP is feasible.

__o__ k. The "reduced cost" in LP provides an estimate of the change in the objective value when the right-hand-side of a constraint changes.

__o__ l. In the two-phase simplex method, Phase One computes the optimal dual variables, followed by Phase Two in which the optimal primal variables are computed.

__+__ m. At the completion of the revised simplex method applied to an LP, the simplex multipliers give the optimal solution to the dual of the LP.

__o__ n. If a minimization LP problem has a cost which is unbounded below, then its dual problem has an objective (to be maximized) which is unbounded above.

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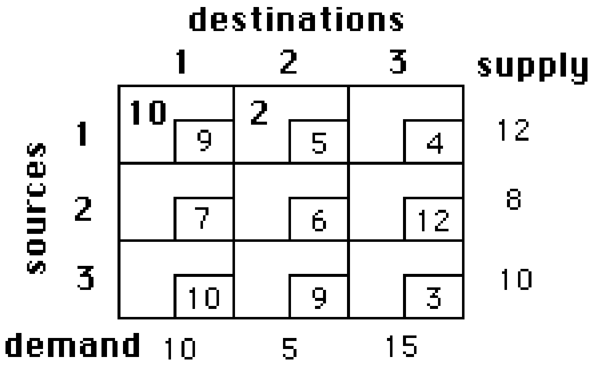
1. **Transportation Problem:** Consider the transportation problem with the tableau below:



		destinations			
		1	2	3	supply
sources	1	10 9	2 5	4	12
	2	7	6	12	8
	3	10	9	3	10
demand		10	5	15	

a. If the LP formulation of this transportation problem were to be written, how many constraint rows (excluding nonnegativity) will it have? _____

How many variables will it have? ________

b. Is this a "balanced" transportation problem? ____ (Yes/No)

c. How many *basic* variables (shipments) must this problem have (excluding the negative of the cost, -z) ? __________

d. An initial basic feasible solution is to be found using the "Northwest Corner Method"; complete the computation of this solution and write the values of the variables in the tableau above.

e. Is the basic feasible solution found by the NW-corner method *degenerate* ? ____ (yes/no)

f. If, for the basic solution found by the NW-corner method, U_1 (the dual variable for the first source) is set equal to 0, what must be the value of V_2 (the dual variable for the second destination)? ________

g. For the basic solution found by the NW-corner method, what will be the reduced cost of the variable X_{21}? ___________

h. According to your answer to (g), will increasing X_{21} improve the objective function? ___________(Yes/No)

i. Regardless of whether the answer to (h) is "yes" or "no", what basic variable must leave the basis if X_{21} enters the basis?_________

j. What will be the value of X_{21} if it is entered into the solution as in (i)? ___________

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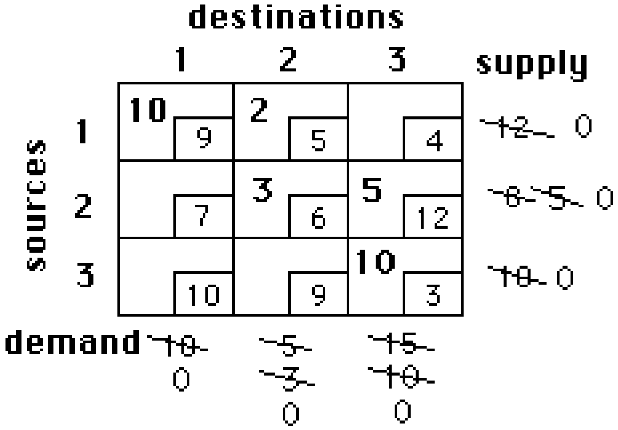
a. If the LP formulation of this transportation problem were to be written, how many constraint rows (excluding nonnegativity) will it have? __6__ (3 supply constraints and 3 demand constraints)

How many variables will it have? ___9___ (= 3x3, one for every source-destination pair)

b. Is this a "balanced" transportation problem? __Yes_ (Total supply = 12+8+10=30 and total demand = 10+5+15=30)

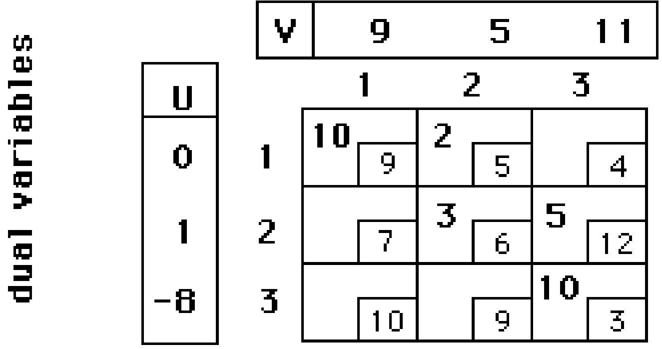
c. How many *basic* variables (shipments) must this problem have (excluding the negative of the cost, -z) ? __5__ (m+n-1, where m=3 and n=3)

d. An initial basic feasible solution is to be found using the "Northwest Corner Method"; complete the computation of this solution and write the values of the variables in the tableau:



e. Is the basic feasible solution found by the NW-corner method *degenerate* ? _No_ (There are five positive shipments, which must be basic, and since there must be five basic variables, no basic variable can have the value zero!)

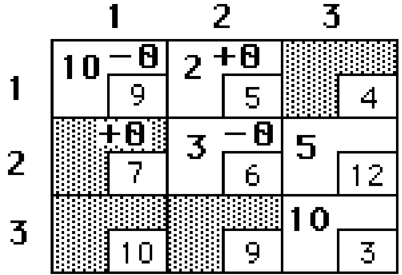
f. If, for the basic solution found by the NW-corner method, U_1 (the dual variable for the first source) is set equal to 0, what must be the value of V_2 (the dual variable for the second destination)? ___5____
Solution: According to complementary slackness conditions, is X_{12} is >0 (as in the NW-corner solution above), the corresponding dual constraint ($U_1 + V_2 \leq C_{12}$) must be tight, i.e., $U_1 + V_2 = C_{12}$. If $U_1=0$, and since $C_{12}=5$, $0+V_2=5$ which implies that $V_2=5$.



g. For the basic solution found by the NW-corner method, what will be the reduced cost of the variable X_{21}? ___-3___ **Solution:** The reduced cost is $\underline{C}_{21} = C_{21} - (U_2 + V_1) = 7 - (1+9) = -3$

h. According to your answer to (g), will increasing X_{21} improve the objective function? __Yes____(The value -3 indicates that each unit shipped from source 2 to destination 1 will (because of the negative sign!) reduce our total cost by $3.)

i. Regardless of whether the answer to (h) is "yes" or "no", what basic variable must leave the basis if X_{21} enters the basis?___X_{22}___
Solution: Entering a shipment in cell (2,1) will create a "cycle":



	1	2	3
1	10 $-\theta$ 9	2 $+\theta$ 5	4
2	$+\theta$ 7	3 $-\theta$ 6	5 12
3	10	9	10 3

Note that an increase of 3 in cell (2,1) will cause decreases of 3 in cells (1,1) and (2,2). Any further increase in cell (2,1) would force the shipment in cell (2,2) to become negative (infeasible).

j. What will be the value of X_{21} if it is entered into the solution as in (i)? ___3___ (see discussion in (i) above.)

<><><><><><><><> Quiz #7 <><><><><><><><>

Consider the following payoff table in which you must choose among three possible alternatives, after which two different states of nature may occur. (Nothing is known about the probability distribution of the state of nature.)

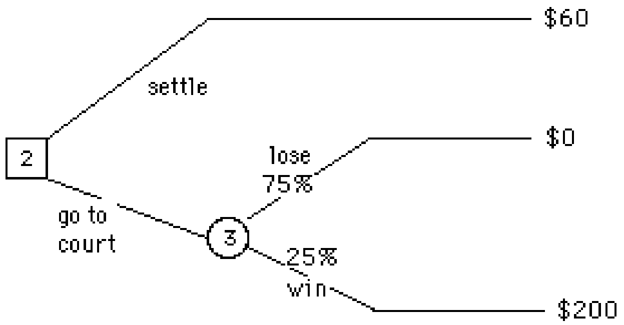
	State of Nature	
Decision	1	2
1	7	1
2	5	6
3	2	8

1. What is the optimal decision if the maximin criterion is used? _____
2. What is the optimal decision if the maximax criterion is used? _____
3. Create the regret table:

	State of Nature	
Decision	1	2
1	__	__
2	__	__
3	__	__

4. What is the optimal decision if the minimax regret is used? _____

General Custard Corporation is being sued by Sue Smith. Sue must decide whether to accept an offer of $60,000 by the corporation to settle out of court, or she can go to court. If she goes to court, there is a 25% chance that she will win the case *(event W)* and a 75% chance she will lose *(event L)*. If she wins, she will receive $200,000, but if she loses, she will net $0. A decision tree representing her situation appears below, where payoffs are in thousands of dollars:



5. What is the decision which maximizes the expected value? ___ a. settle ___b. go to court

For $20,000, Sue can hire a consultant who will predict the outcome of the trial, i.e., either he predicts a loss of the suit *(event PL)*, or he predicts a win *(event PW)*. The consultant is correct 80% of the time.

Bayes' Rule states that if S_i are the states of nature and O_j are the outcomes of an experiment,

$$\boxed{\begin{array}{c} P\left\{S_i \mid O_j\right\} = \dfrac{P\left\{O_j \mid S_i\right\} P\left\{S_i\right\}}{P\left\{O_j\right\}} \\ \text{where } P\left\{O_j\right\} = \sum_i P\left\{O_j \mid S_i\right\} P\left\{S_i\right\} \end{array}}$$

_____ 6. The probability that the consultant will predict a win, i.e. P{PW} is *(choose nearest value)*

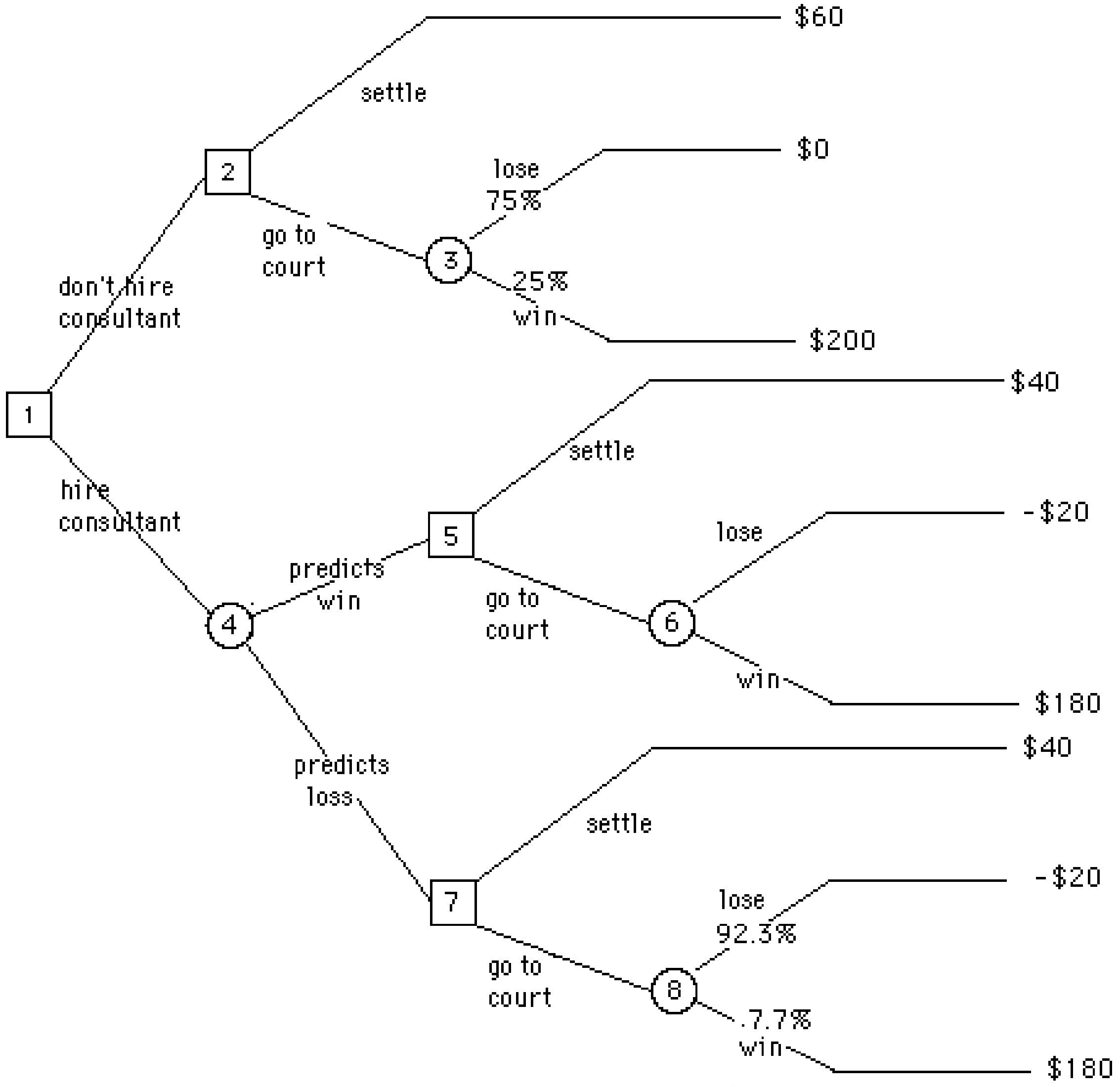
a. ≤25% b. 30% c. 35%

d. 40% e. 45% f. ≥ 50%

_____ 7. According to Bayes' theorem, the probability that Sue *will* win, given that the consultant predicts a win, i.e. P{W | PW}, is *(choose nearest value)*

a. ≤25% b. 30% c. 35%
d. 40% e. 45% f. ≥ 50%

The decision tree below includes Sue's decision as to whether or not to hire the consultant. Insert the probability that you computed in (7) above, together with its complement P{L êPW}.



8. "Fold back" nodes 2 through 8, and write the value of each node below:

Node	Value	Node	Value	Node	Value
8	_____	5	94.286	2	60
7	40	4	______	1	______
6	94.286	3	50		

9. Should Sue hire the consultant? *Circle:* Yes No

____ 10. The expected value of the consultant's opinion is (in thousands of $) *(Choose nearest value)*:

a. ≤16 b. 17 c. 18 d. 19
e. 20 f. 21 g. 22 h. ≥23

<><><><><><><><> Quiz #7 Solutions <><><><><><><><><>

1. What is the optimal decision if the maximin criterion is used? __2__
2. What is the optimal decision if the maximax criterion is used? __3__
3. Create the regret table:

	State of Nature	
Decision	1	2
1	_0	_7
2	_2	_2
3	_5	_0

4. What is the optimal decision if the minimax regret is used? __2__

5. What is the decision which maximizes the expected value? _X_ a. settle ___b. go to court

Solution: *Expected value of node 3 is (0.75)(0) + (0.25)(200) = 50 < 60!)*

For $20,000, Sue can hire a consultant who will predict the outcome of the trial, i.e., either he predicts a loss of the suit *(event PL)*, or he predicts a win *(event PW)*. The consultant is correct 80% of the time.

Bayes' Rule states that if S_i are the states of nature and O_j are the outcomes of an experiment,

$$P\left\{S_i \mid O_j\right\} = \frac{P\left\{O_j \mid S_i\right\} P\left\{S_i\right\}}{P\left\{O_j\right\}}$$

$$\text{where } P\left\{O_j\right\} = \sum_i P\left\{O_j \mid S_i\right\} P\left\{S_i\right\}$$

__c__ 6. The probability that the consultant will predict a win, i.e. P{PW} is *(choose nearest value)*

a. ≤25% b. 30% c. 35%
d. 40% e. 45% f. ≥ 50%

Solution:

$$P\left\{O_j\right\} = \sum_i P\left\{O_j \mid S_i\right\} P\left\{S_i\right\}$$

$$\begin{aligned} P\left\{PW\right\} &= P\left\{PW \mid W\right\} P\left\{W\right\} + P\left\{PW \mid L\right\} P\left\{L\right\} \\ &= (0.8)(0.25) + (0.2)(0.75) \\ &= 0.35 \end{aligned}$$

__f__ 7. According to Bayes' theorem, the probability that Sue *will* win, given that the consultant predicts a win, i.e. $P\left\{W \mid PW\right\}$, is *(choose nearest value)*

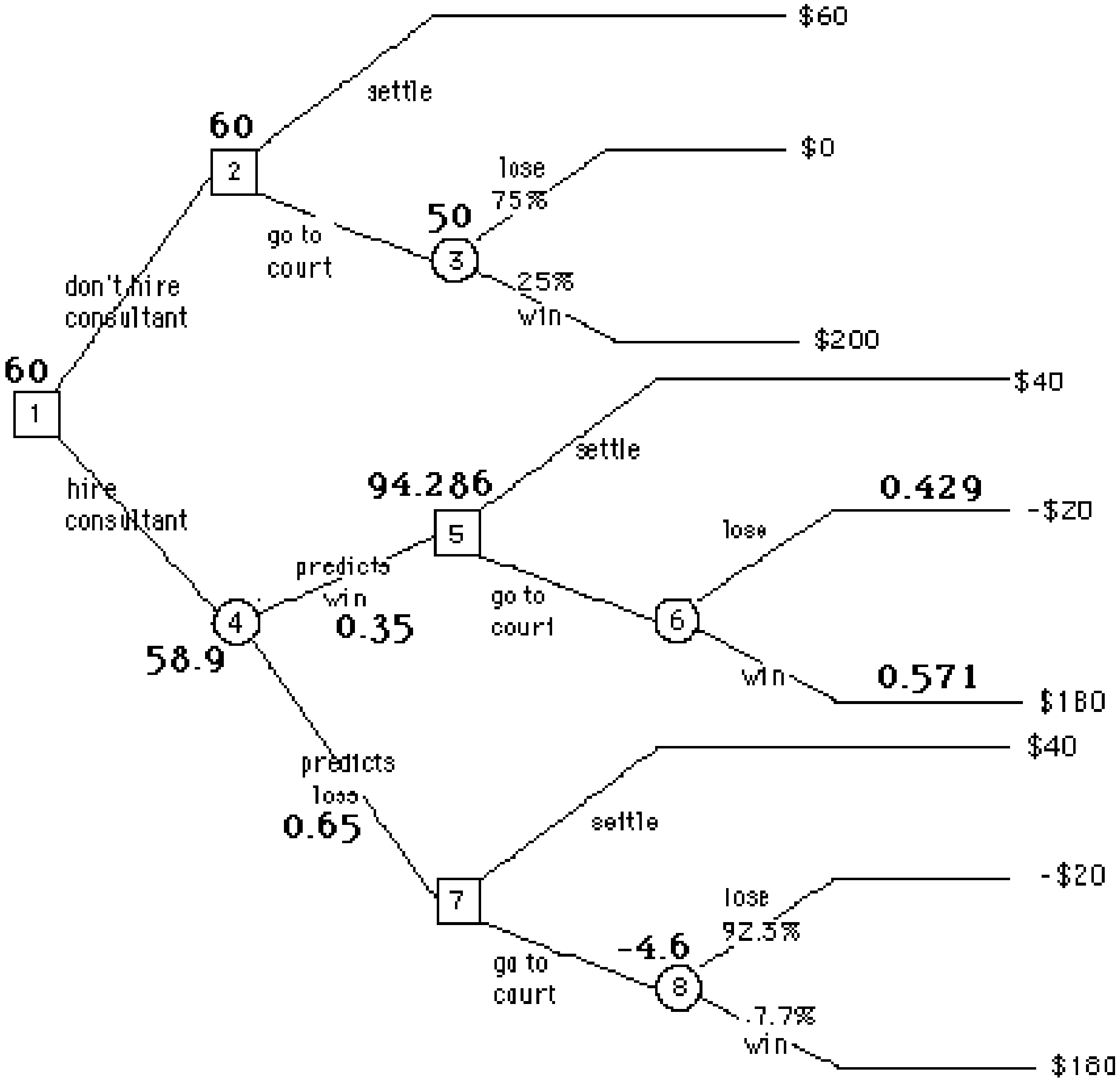
a. ≤25% b. 30% c. 35%
d. 40% e. 45% f. ≥ 50%

Solution:

$$P\left\{S_i \mid O_j\right\} = \frac{P\left\{O_j \mid S_i\right\} P\left\{S_i\right\}}{P\left\{O_j\right\}}$$

$$\begin{aligned} P\left\{W \mid PW\right\} &= \frac{P\left\{PW \mid W\right\} P\left\{W\right\}}{P\left\{PW\right\}} \\ &= \frac{(0.8))(0.25)}{(0.35)} \\ &= 0.571 \end{aligned}$$

The decision tree below includes Sue's decision as to whether or not to hire the consultant. Insert the probability that you computed in (7) above, together with its complement P{L êPW}.



8. "Fold back" nodes 2 through 8, and write the value of each node below:

Node	Value	Node	Value	Node	Value
8	**-4.6**	5	94.286	2	60
7	40	4	**58.9**	1	**60**
6	94.286	3	50		

Solution:

Expected value of node 8 is (0.923)(-20) + (0.077)(180) = - 4.6
Expected value of node 4 is (0.35)(94.286) + (0.65)(40) = 58.9 (Note that from questions (6), the probability that the consultant predicts a win is 35%.)
Expected value at node 1 is Max{60, 58.9} = 60.

9. Should Sue hire the consultant? *Circle:* Yes [✓ NO]

Solution: The optimal decision at node 1 is not to hire the consultant!

__d_ 10. The expected value of the consultant's opinion is (in thousands of $) *(Choose nearest value)*:

a. ≤16 b. 17 c. 18 d. 19
e. 20 f. 21 g. 22 h. ≥23

Solution: The expected payoff at node 4, before subtracting the cost of the consultant (20), is 78.9. Compare this to the expected payoff at node 2. Thus EVSI = 78.9 - 60 = 18.9.

<><><><><><><><> Quiz #8 <><><><><><><><>

Part I: Integer Programming Model Formulation Part I. You have been assigned to arrange the songs on the cassette version of Madonna's latest album. A cassette tape has two sides (#1 and #2). The length and type of each song are given in the table below:

Song	Type	Length (minutes)
1	Ballad	4
2	Hit	5
3	Ballad	3
4	Hit	2
5	Ballad	4
6	Hit	3
7	neither ballad nor hit	5
8	Ballad & hit	4

Define the variables

Y_i = 1 if song #i is on side 1;
0 otherwise (i.e., if on side 2)

Thus, $1-Y_i$ = 1 if song #i is on side 2;
0 otherwise (i.e., if on side 1)

For each restriction, choose a linear constraint from the list (a) through (ee) below.

___ 1. Side #1 must have exactly 3 ballads
___ 2. Side #1 must have at least 2 hit songs
___ 3. If song #2 is on side 1, then song #3 must be on side 2
___ 4. The number of hit songs on side 2 should be no more than 2
___ 5. If both songs 1 & 2 are on side 1, then song 3 must be on side 2.
___ 6. Side #1 cannot contain more than 8 minutes of ballads

Part II: Integer Programming Model Formulation. Comquat owns four production plants at which personal computers are produced. In order to use a plant to produce computers, a fixed cost must be paid to set up the production line in that plant.

Define the variables:

Y_i = 1 if the production line has been set up at plant #i
0 otherwise
X_i = # of computers produced at plant #i

For each restriction, choose a linear constraint from the list (a) through (ee) below.

___ 7. Computers are to be produced at no more than 3 plants.
___ 8. If the production line at plant 2 is set up, then that plant can produce up to 8000 computers; otherwise, none can be produced at that plant.
___ 9. The production lines at plants 2 and 3 cannot both be set up.
___ 10. The total production must be at least 20,000 computers.
___ 11. If the production line at plant 2 is set up, that plant must produce at least 2000 computers.
___ 12. If the production line at plant 2 is not set up, then the production line at plant 3 cannot be set up.

Constraints:

a. $Y_1 + Y_2 + Y_3 + Y_4 \le 3$
b. $Y_2 \le Y_3$
c. $Y_3 \le Y_2$
d. $Y_1 + Y_2 + Y_3 + Y_4 \ge 20000$
e. $Y_3Y_2 \ge 1$
f. $X_2 \ge 2000Y_2$
g. $X_1 + X_2 + X_3 + X_4 \ge 20000$
h. $X_2 \le 8000Y_2$
i. $X_2 \le 2000Y_2$
j. $Y_1 + Y_2 + Y_3 + Y_4 \ge 3$
k. $X_2 \ge 8000Y_2$
l. $8000X_2 \le Y_2$
m. $Y_1 + Y_3 + Y_5 + Y_8 = 3$
n. $2000X_2 \le Y_2$
o. $Y_2 + Y_3 \le 1$
p. $Y_1 + Y_3 + Y_5 + Y_8 \le 3$
q. $Y_2 + Y_3 \ge 1$
r. $Y_2 - Y_3 = 1$
s. $Y_2 + Y_4 + Y_6 + Y_8 \le 2$
t. $Y_3Y_2 = 1$
u. $Y_3Y_2 \le 1$
v. $Y_2 + Y_4 + Y_6 + Y_8 \ge 2$
w. $X_2Y_2 \ge 2000$
x. $X_2Y_2 \le 8000$
y. $Y_2 + Y_4 + Y_6 + Y_8 \ge 3$
z. $Y_2 \le Y_3$
aa. $Y_3 \le Y_2$
bb. $4Y_1 + 3Y_3 + 4Y_5 + 4Y_8 \le 8$
cc. $4Y_1 + 3Y_3 + 4Y_5 + 4Y_8 \ge 8$
dd. $X_2 \le Y_2$
ee. ***None of the above***

<><><><><><><><><><> Quiz #8 Solutions <><><><><><><><><><>

For each restriction, choose a linear constraint from the list (a) through (ee).

_m_ 1. Side #1 must have exactly 3 ballads

Solution: $Y_1 + Y_3 + Y_5 + Y_8 = 3$

_v_ 2. Side #1 must have at least 2 hit songs

Solution: Since songs #2,4,6, &8 are classified as "hits", $Y_2 + Y_4 + Y_6 + Y_8 \ge 2$

_o_ 3. If song #2 is on side 1, then song #3 must be on side 2

Solution: One approach to deriving this constraint is to first write $Y_2 \le 1-Y_3$, i.e., if Y_2 is 1 then $(1-Y_3)$ is forced also to be 1, i.e., song #3 is forced to be on side 2. By re-arranging terms, we obtain $Y_2 + Y_3 \le 1$. Alternately, we can deduce that the restriction is equivalent to saying that songs #2 and #3 cannot both be on side 1.

_v_ 4. The number of hit songs on side 2 should be no more than 2

Solution: Since songs #2,4,6, &8 are classified as "hits", $Y_2 + Y_4 + Y_6 + Y_8 \ge 2$

_ee_ 5. If both songs 1 & 2 are on side 1, then song 3 must be on side 2.

Solution: The correct constraint is $\boxed{1 - Y_3 \ge Y_1 + Y_2 - 1}$ $\boxed{Y_1 + Y_2 + Y_3 \le 2}$, which does not appear in the list below.

_bb_ 6. Side #1 cannot contain more than 8 minutes of ballads

Solution: Since songs #1, 3, 5, & 8 (with playing times 4, 3, 4, & 4, respectively) are classified as "ballads", the constraint is $4Y_1 + 3Y_3 + 4Y_5 + 4Y_8 \le 8$

Part II: Integer Programming Model Formulation.

_a_ 7. Computers are to be produced at no more than 3 plants.

Solution: $Y_1 + Y_2 + Y_3 + Y_4 \le 3$

_h_ 8. If the production line at plant 2 is set up, then that plant can produce up to 8000 computers; otherwise, none can be produced at that plant.

Solution: The correct constraint is $X_2 \le 8000Y_2$, which restricts X_2 to be 0 if Y_2 is 0, while if $Y_2=1$ then X_2 has an upper bound of 8000 computers.

_o_ 9. The production lines at plants 2 and 3 cannot both be set up.

Solution: $Y_2 + Y_3 \le 1$

_g_ 10. The total production must be at least 20,000 computers.

Solution: $X_1 + X_2 + X_3 + X_4 \ge 20000$

_f_ 11. If the production line at plant 2 is set up, that plant must produce at least 2000 computers.

Solution: The constraint is $X_2 \geq 2000Y_2$, which, if $Y_2 = 1$, sets a lower bound of 2000 on X_2.

c, aa 12. If the production line at plant 2 is not set up, then the production line at plant 3 cannot be set up.

Solution: The required constraint is $Y_3 \leq Y_2$, which, if $Y_2 = 0$, restricts Y_3 to be 0.

<><><><><><><><> Quiz #9 <><><><><><><><>

Consider an **(s,S) inventory system** in which the number of items on the shelf is checked at the end of each day. The demand distribution is as follows:

n=	0	1	2
P{n}=	0.3	0.5	0.2

To avoid shortages, the current policy is to restock the shelf at the end of each day (after spare parts have been removed) so that the shelf is again filled to its limit (i.e., 4) **if** there are fewer than 2 parts on the shelf. *(That is, it is an (s,S) inventory system, with s=2 and S=4.)*

The inventory system has been modeled as a Markov chain, with the state of the system defined as the end-of-day inventory level (before restocking). Refer to the computer output which follows to answer the following questions: **Note that in the computer output, state #1 is inventory level 0, state #2 is inventory level 1, etc.**

$$P=\begin{bmatrix} 0 & 0 & 0.2 & 0.5 & 0.3 \\ 0 & 0 & 0.2 & 0.5 & 0.3 \\ 0.2 & 0.5 & \boxed{A} & 0 & 0 \\ 0 & 0.2 & 0.5 & 0.3 & 0 \\ 0 & 0 & 0.2 & 0.5 & 0.3 \end{bmatrix},\ P^2=\begin{bmatrix} 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \\ 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \\ 0.06 & 0.15 & 0.23 & 0.35 & 0.21 \\ 0.1 & 0.31 & 0.34 & 0.19 & 0.06 \\ 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \end{bmatrix}$$

$$P^3=\begin{bmatrix} 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \\ 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \\ 0.046 & 0.185 & 0.328 & 0.315 & 0.126 \\ 0.068 & 0.208 & 0.291 & 0.292 & 0.141 \\ 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \end{bmatrix},\ P^4=\begin{bmatrix} 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \\ 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \\ 0.065 & 0.227 & 0.327 & 0.273 & 0.107 \\ 0.058 & 0.203 & 0.316 & 0.296 & 0.125 \\ 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \end{bmatrix}$$

$$\sum_{n=1}^{4} P^n=\begin{bmatrix} 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \\ 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \\ 0.3716 & 1.062 & 1.1853 & 0.938 & 0.4431 \\ 0.2262 & 0.9219 & 1.4477 & 1.0781 & 0.3261 \\ 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \end{bmatrix}$$

$$M=\begin{bmatrix} 15.769 & 4.641 & 3.076 & 2.571 & 8.367 \\ 15.769 & 4.641 & 3.076 & 2.571 & 8.367 \\ 12.692 & 2.754 & 3.153 & 4 & 9.795 \\ 15 & 3.396 & 2.307 & 3.514 & 10.816 \\ 15.769 & 4.641 & 3.076 & 2.571 & \boxed{B} \end{bmatrix}$$

i	π_i
1	0.063
2	0.215
3	0.317
4	0.284
5	0.119

____ 1. the value $P_{5,3}$ is
- a. P{demand=0}
- b. P{demand=1}
- c. P{demand=2}
- d. P{demand≤1}
- e. P{demand≥1}
- f. none of the above

____ 2. the value $P_{1,4}$ is
- a. P{demand=0}
- b. P{demand=1}
- c. P{demand=2}
- d. P{demand≤1}
- e. P{demand≥1}
- f. none of the above

____ 3. the value $P_{3,1}$ is
- a. P{demand=0}
- b. P{demand=1}
- c. P{demand=2}
- d. P{demand≤1}
- e. P{demand≥1}
- f. none of the above

____ 4. the numerical value **A** in the matrix above is *(select nearest value)*

a. 0 b. 0.1 c. 0.2 d. 0.3
e. 0.4 f. 0.5

____ 5. the numerical value **B** in the mean-first-passage time matrix (M) above is *(select nearest value)*
a. 1 b. 2 c. 4
d. 6 e. 8 f. 10

____ 6. If the shelf is full Monday morning, the expected number of days until a stockout occurs is *(select nearest value):*
a. 2 b. 5 c. 10
d. 15 e. 20 f. more than 20

____ 7. If the shelf is full Monday morning, the probability that the shelf is full Thursday night (i.e., after 4 days of sales) is *(select nearest value):*
a. 7% b. 8% c. 9%
d. 10% e. 11% f. more than 12%

____ 8. If the shelf is full Monday morning, the probability that the shelf is restocked Thursday night is (select nearest value):
a. 10% b. 15% c. 20%
d. 25% e. 30% f. more than 30%

____ 9. If the shelf is full Monday morning, the expected number of nights that the shelf is restocked before Friday morning is (select nearest value):
a. 0.6 b. 0.7 c. 0.8 d. 0.9
e. more than once but less than twice f. more than 2

____ 10. The number of transient states in this Markov chain model is
a. zero b. 1
c. 2 d. 5 e. none of the above

____ 11. The number of recurrent states in this Markov chain model is
a. zero b. 1
c. 2 d. 5 e. none of the above

____ 12. The number of absorbing states in this Markov chain model is
a. zero b. 1
c. 2 d. 5 e. none of the above

____ 13. Which (one or more) of the following equations are among those solved to compute the steady state probability distribution?
a. $\pi_1 = 0.2\pi_3$
b. $\pi_1 = 0.2\pi_3 + 0.5\pi_4 + 0.3\pi_5$
c. $\pi_3 = 0.2\pi_1 + 0.2\pi_2 + 0.3\pi_3 + 0.5\pi_4 + 0.2\pi_5$
d. $\pi_4 = 0.2\pi_2 + 0.5\pi_3 + 0.3\pi_4$
e. $\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1$

<><><><><><><><> Quiz #9 Solutions <><><><><><><><>

__c_ 1. the value $P_{5,3}$ is
a. P{demand=0} b. P{demand=1}
c. P{demand=2} d. P{demand≤1}
e. P{demand≥1} f. none of the above

__b_ 2. the value $P_{1,4}$ is
a. P{demand=0} b. P{demand=1}
c. P{demand=2} d. P{demand≤1}
e. P{demand≥1} f. none of the above

__c_ 3. the value $P_{3,1}$ is
a. P{demand=0} b. P{demand=1}
c. P{demand=2} d. P{demand≤1}
e. P{demand≥1} f. none of the above

__d_ 4. the numerical value **A** in the matrix above is *(select nearest value)*

a. 0 b. 0.1 c. 0.2 d. 0.3
e. 0.4 f. 0.5

__e_ 5. the numerical value **B** in the mean-first-passage time matrix (M) above is *(select nearest value)*
a. 1 b. 2 c. 4
d. 6 e. 8 ($^1/\pi_5 = 8.4$) f. 10

__d_ 6. If the shelf is full Monday morning, the expected number of days until a stockout occurs is *(select nearest value):*
a. 2 b. 5 c. 10
d. 15 (m_{51}=15.769) e. 20 f. more than 20

__f_ 7. If the shelf is full Monday morning, the probability that the shelf is full Thursday night (i.e., after 4 days of sales) is *(select nearest value):*
a. 7% b. 8% c. 9%
d. 10% e. 11% f. more than 12% $\left(p_{55}^{(4)} = 0.125\right)$

__e_ 8. If the shelf is full Monday morning, the probability that the shelf is restocked Thursday night is (select nearest value): $\left(p_{51}^{(4)}+p_{52}^{(4)} = 0.065 + 0.214 = 0.279\right)$
a. 10% b. 15% c. 20%
d. 25% e. 30% f. more than 30%

__c_ 9. If the shelf is full Monday morning, the expected number of nights that the shelf is restocked before Friday morning is (select nearest value):
$\left(\sum_{k=1}^{4}\left(p_{51}^{(k)}+p_{52}^{(k)}\right) = 0.1794 + 0.6595 = 0.8389\right)$ a. 0.6 b. 0.7
c. 0.8 d. 0.9
e. more than once but less than twice f. more than 2

__a_ 10. The number of transient states in this Markov chain model is
a. zero b. 1
c. 2 d. 5 e. none of the above

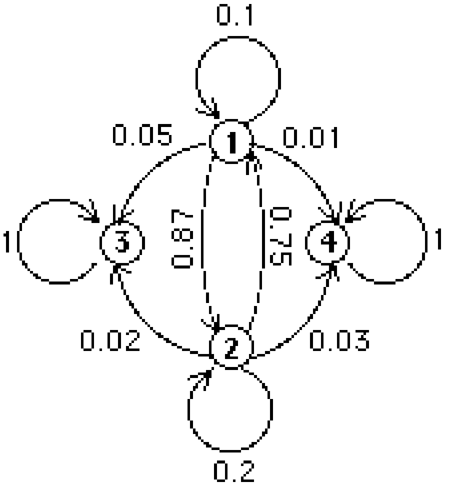
__d_ 11. The number of recurrent states in this Markov chain model is
a. zero b. 1
c. 2 d. 5 e. none of the above

__a_ 12. The number of absorbing states in this Markov chain model is
a. zero b. 1
c. 2 d. 5 e. none of the above

a,c,e 13. Which (one or more) of the following equations are among those solved to compute the steady state probability distribution?
a. $\pi_1 = 0.2\pi_3$
b. $\pi_1 = 0.2\pi_3 + 0.5\pi_4 + 0.3\pi_5$
c. $\pi_3 = 0.2\pi_1 + 0.2\pi_2 + 0.3\pi_3 + 0.5\pi_4 + 0.2\pi_5$
d. $\pi_4 = 0.2\pi_2 + 0.5\pi_3 + 0.3\pi_4$
e. $\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1$

<><><><><><><> Quiz #10 <><><><><><><>

Part I: Consider the discrete-time Markov chain diagrammed below:



Transition Probability Matrix

		1	2	3	4
from	1	0.1	0.84	0.05	0.01
	2	0.75	0.2	0.02	0.03
	3	0	0	1	0
	4	0	0	0	1

The following matrices were computed, based upon the above Markov chain data:

A

		3	4
from	1	0.6311	0.3688
	2	0.6166	0.3833

E

		1	2
from	1	8.888	9.333
	2	8.333	10

$$P^2 = \begin{bmatrix} 0.64 & 0.252 & 0.0718 & 0.0362 \\ 0.225 & 0.67 & 0.0615 & 0.0435 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 0.253 & 0.588 & 0.108 & 0.050 \\ 0.525 & 0.323 & 0.086 & 0.065 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad P^4 = \begin{bmatrix} 0.4663 & 0.33012 & 0.13325 & 0.07033 \\ 0.29475 & 0.5056 & 0.11886 & 0.08079 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

___ 1. Which is the matrix Q (used in computation of E)?

a. $\begin{bmatrix} 0.05 & 0.01 \\ 0.02 & 0.03 \end{bmatrix}$ b. $\begin{bmatrix} 0.1 & 0.01 \\ 0.75 & 0.03 \end{bmatrix}$ c. $\begin{bmatrix} 8.888 & 9.333 \\ 8.333 & 10 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0.1 & 0.84 \\ 0.75 & 0.2 \end{bmatrix}$ f. None of the above

___ 2. Which is the matrix R (used in computation of A)?

a. $\begin{bmatrix} 0.05 & 0.01 \\ 0.02 & 0.03 \end{bmatrix}$ b. $\begin{bmatrix} 0.1 & 0.01 \\ 0.75 & 0.03 \end{bmatrix}$ c. $\begin{bmatrix} 8.888 & 9.333 \\ 8.333 & 10 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0.1 & 0.84 \\ 0.75 & 0.2 \end{bmatrix}$ f. None of the above

___ 3. If the system begins in state #1, what is the probability that it is absorbed into state #3? (Choose <u>nearest</u> value)

a. 10% *or less* b. 20% c. 30% d. 40%

e. 50% f. 60% g. 70% h. 80% *or more*

___ 4. If the system begins in state #1, what is the expected number of stages (including the initial stage) that the system exists before it is absorbed into one of the two absorbing states? (Choose nearest value)

a. 2 *or less* b. 4 c. 6 d. 8
e. 10 f. 12 g. 14 h. 16 *or more*

___ 5. If the system begins in state #2, what is the probability that the system survives for at least 3 stages before being absorbed into one of the two absorbing states ? (Choose nearest value)

a. 10% *or less* b. 20% c. 30% d. 40%
e. 50% f. 60% g. 70% h. 80% *or more*

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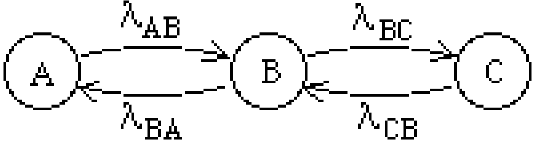
Part II: A repairman is responsible for maintaining two machines in working condition. When both are in good condition, they operate simultaneously. However, a machine operates for an average of only 1 hour, when it fails and repair begins. Repair of a machine requires an average of 30 minutes. (Only one machine at a time can be repaired.) Define a continuous-time Markov chain with states:

A. Both machines have failed, with repair in progress on one machine
B. One machine is operable, and the other is being repaired
C. Both machines are in operating condition



___ 1. In this model, the probability distribution of the time required to repair a machine is assumed to be:

a. Uniform b. Markov c. Poisson
d. Normal e. exponential f. None of the above

___ 2. The transition rate λ_{AB} is

a. 0.5/hour b. 1/hour c. 2/hour
d. $-\lambda_{BA}$ e. λ_{BA} f. None of the above

___ 3. The transition rate λ_{CB} is

a. 0.5/hour b. 1/hour c. 2/hour
d. $-\lambda_{CB}$ e. λ_{BC} f. None of the above

___ 4. The steady-state probability distribution must satisfy the equation(s) *(one or more):*

a. $\pi_A + \pi_B + \pi_C = 1$ b. $\lambda_{AB}\pi_A = \lambda_{BA}\pi_B$
c. $\lambda_{BA}\pi_A = \lambda_{AB}\pi_B$ d. $\pi_A = \lambda_{AB}\pi_A + (\lambda_{BA} + \lambda_{BC})\pi_B + \lambda_{CB}\pi_C$
e. $\lambda_{BC}\pi_B = \lambda_{CB}\pi_C$ f. $\pi_B = \lambda_{AB}\pi_A + (\lambda_{BA} + \lambda_{BC})\pi_B + \lambda_{CB}\pi_C$

___ 5. The average *utilization* of these machines in steady state (i.e., the fraction of maximum capacity at which they will operate), is:

a. $\pi_B+\pi_C$ b. $0.5(\pi_B+\pi_C)$ c. $\pi_B+2\pi_C$
d. $\pi_A+\pi_B+\pi_C$ e. $2(\pi_B+\pi_C)$ f. $0.5(\pi_B+2\pi_C)$

<><><><><><><><> Quiz #10 Solutions <><><><><><><><>

Part I: Discrete-time Markov Chain:

_e_ 1. Which is the matrix Q (used in computation of E)?

a. $\begin{bmatrix} 0.05 & 0.01 \\ 0.02 & 0.03 \end{bmatrix}$ b. $\begin{bmatrix} 0.1 & 0.01 \\ 0.75 & 0.03 \end{bmatrix}$ c. $\begin{bmatrix} 8.888 & 9.333 \\ 8.333 & 10 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0.1 & 0.84 \\ 0.75 & 0.2 \end{bmatrix}$ f. None of the above

Solution: Q is the submatrix of P whose elements are transition probabilities between transient states! $E = (I - Q)^{-1}$

_a_ 2. Which is the matrix R (used in computation of A)?

a. $\begin{bmatrix} 0.05 & 0.01 \\ 0.02 & 0.03 \end{bmatrix}$ b. $\begin{bmatrix} 0.1 & 0.01 \\ 0.75 & 0.03 \end{bmatrix}$ c. $\begin{bmatrix} 8.888 & 9.333 \\ 8.333 & 10 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 0.1 & 0.84 \\ 0.75 & 0.2 \end{bmatrix}$ f. None of the above

Solution: R is the submatrix of P whose elements are transition probabilities from transient states to absorbing states! A = E R.

_f_ 3. If the system begins in state #1, what is the probability that it is absorbed into state #3? (Choose nearest value)

a. 10% *or less*	b. 20%	c. 30%	d. 40%
e. 50%	f. 60%	g. 70%	h. 80% *or more*

Solution: $a_{13} = 0.6310$

_h_ 4. If the system begins in state #1, what is the expected number of stages (including the initial stage) that the system exists before it is absorbed into one of the two absorbing states? (Choose nearest value)

a. 2 *or less*	b. 4	c. 6	d. 8
e. 10	f. 12	g. 14	h. 16 *or more*

Solution: $e_{11} + e_{12} = 8.888 + 9.333 = 18.222$

_h_ 5. If the system begins in state #2, what is the probability that the system survives for at least 3 stages before being absorbed into one of the two absorbing states ? (Choose nearest value)

a. 10% *or less*	b. 20%	c. 30%	d. 40%
e. 50%	f. 60%	g. 70%	h. 80% *or more*

Solution: $p_{21}^{(3)} + p_{22}^{(3)} = 0.525 + 0.323 = 0.848$

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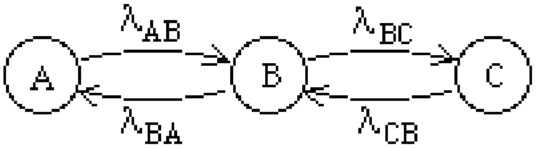
Part II: States:

A. Both machines have failed, with repair in progress on one machine
B. One machine is operable, and the other is being repaired
C. Both machines are in operating condition



_e_ 1. In this model, the probability distribution of the time required to repair a machine is assumed to be:

a. Uniform b. Markov c. Poisson
d. Normal e. exponential f. None of the above

Solution: In a Markov chain model, the probability distribution until the time that a certain transition occurs must have the exponential (i.e., memoryless) distribution.

_c_ 2. The transition rate λ_{AB} is

a. 0.5/hour b. 1/hour c. 2/hour
d. $-\lambda_{BA}$ e. λ_{BA} f. None of the above

Solution: A transition from state A to state B occurs when the current repair is completed. This happens once every 30 minutes when a repair is in progress, i.e., 2 per 60 minutes, or 2/hour.

_c_ 3. The transition rate λ_{CB} is

a. 0.5/hour b. 1/hour c. 2/hour
d. $-\lambda_{CB}$ e. λ_{BC} f. None of the above

Solution: A transition from state C to state B occurs when one of the two machines (both operating) fails and requires repair. Each machine fails at the rate of once per hour, and so the two machines combined fail at the rate of 2/hour.

a,b,e 4. The steady-state probability distribution must satisfy the equation(s) *(one or more):*

a. $\pi_A + \pi_B + \pi_C = 1$ b. $\lambda_{AB}\pi_A = \lambda_{BA}\pi_B$
c. $\lambda_{BA}\pi_A = \lambda_{AB}\pi_B$ d. $\pi_A = \lambda_{AB}\pi_A + (\lambda_{BA} + \lambda_{BC})\pi_B + \lambda_{CB}\pi_C$
e. $\lambda_{BC}\pi_B = \lambda_{CB}\pi_C$ f. $\pi_B = \lambda_{AB}\pi_A + (\lambda_{BA} + \lambda_{BC})\pi_B + \lambda_{CB}\pi_C$

Solution:

- Equation (a) must be satisfied in order to be a probability distribution.
- Equation (b) states that (in steady state) the rate at which the system makes transitions from state A (namely, the probability π_A that the system is in state A, times the rate λ_{AB} at which the system makes transitions into state B when it is in state A) must be the same as the rate at which the system makes transitions into state A (namely, $\lambda_{BA}\pi_B$ since there is only one way that the system can make a transition into state A). This equation is referred to as a *balance equation* state A.
- Equation (e) is the balance equation for state C.

Note that the balance equation for state B is $\left(\lambda_{BA} + \lambda_{BC}\right)\pi_B = \lambda_{AB}\pi_A + \lambda_{CB}\pi_C$ *, which does not appear in the list.*

_f_ 5. The average *utilization* of these machines in steady state (i.e., the fraction of maximum capacity at which they will operate), is:

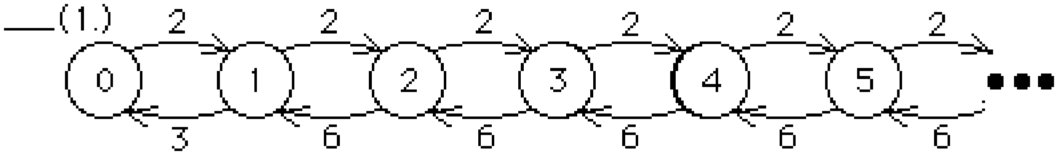
a. $\pi_B + \pi_C$ b. $0.5(\pi_B + \pi_C)$ c. $\pi_B + 2\pi_C$
d. $\pi_A + \pi_B + \pi_C$ e. $2(\pi_B + \pi_C)$ f. $0.5(\pi_B + 2\pi_C)$

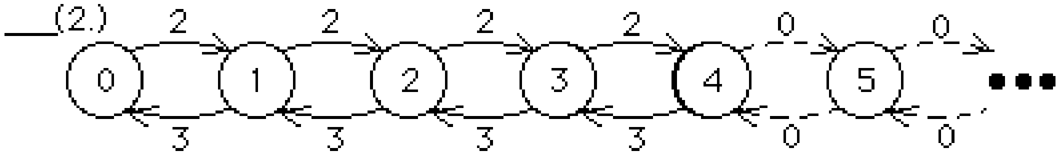
Solution: The expected (i.e., average) number of machines operating is $0\pi_A + 1\,\pi_B + 2\pi_C$ (since in state A, no machines are operating, one machine is operating in state B, and 2 in state C). Dividing this by the total number of machines gives us the average utilization of each machine.

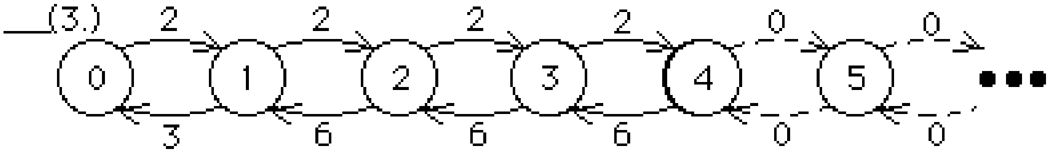
<><><><><><><><> Quiz #11 <><><><><><><><>

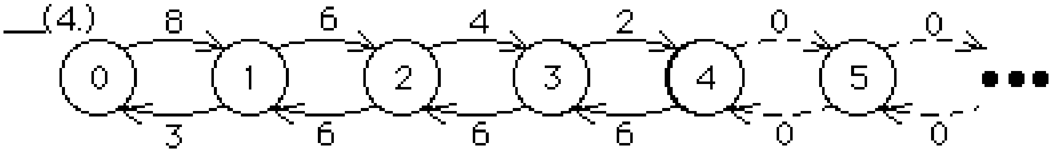
Part I: For each diagram of a Markov model of a queue in (1) through (5) below, indicate the correct Kendall's classification from among the following choices :

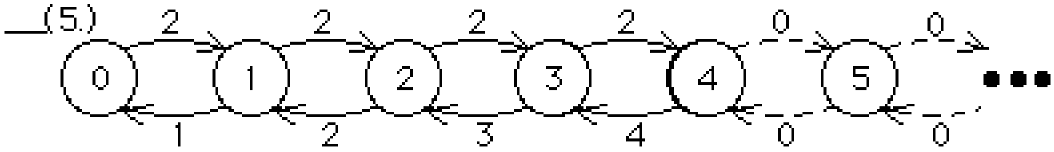
(a) M/M/1	(b) M/M/2	(c) M/M/1/4
(d) M/M/4	(e) M/M/2/4	(f) M/M/2/4/4
(g) M/M/1/2/4	(h) M/M/4/2	(i) M/M/4/4
	(j) none of the above	



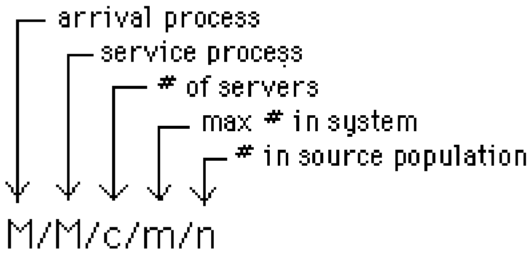








Note: Kendall's notation:



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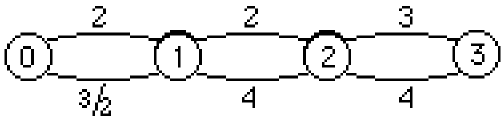
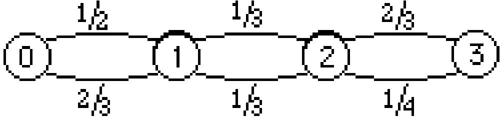
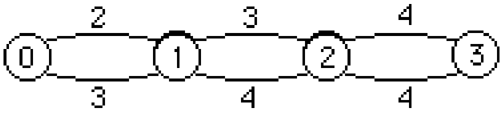
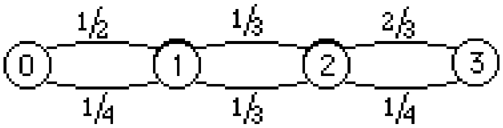
Part II: Two mechanics work in an auto repair shop, with a capacity of 3 cars. If there are 2 or more cars in the shop, each mechanic works individually, each completing the repair of a car in an average of **4** hours (the actual time being random with exponential distribution). If there is only one car in the shop, both mechanics work together on it, completing the repair in an average time of **3** hours (also exponentially distributed). Cars arrive randomly, according to a Poisson process, at the rate of one every two hours when there are less than two cars in the shop, but one every **3** hours when both mechanics are busy. If 3 cars are already in the shop, no cars arrive.

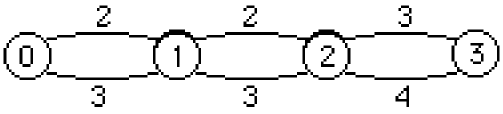
____ 6. Choose the transition diagram below corresponding to this system.

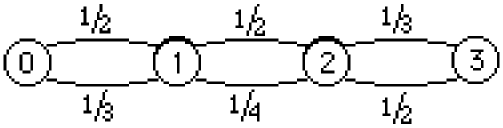
a 

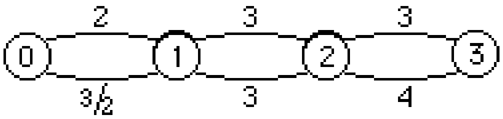
e 

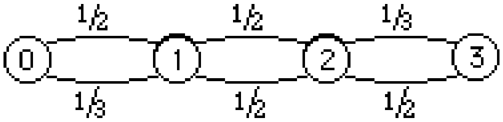
b 

f 

c 

g 

d 

h 

i. *None of the above*

For ease of computation, suppose that the steady-state probabilities for this system are *(not the actual values)*:

π_0=20%, π_1=30%, π_2=30%, & π_3=20%.

____ 7. What fraction of the day will both mechanics be idle?
a. 10% c. 30% e. 50% g. 70%
b. 20% d. 40% f. 60% h. *NOTA*

____ 8. What fraction of the day will both mechanics be working on the same car?
a. 10% c. 30% e. 50% g. 70%
b. 20% d. 40% f. 60% h. *NOTA*

____ 9. What is the average number of cars in the shop? (Choose nearest answer.)
a. 0.5 c. 1.0 e. 1.5 g. 2.0
b. 0.75 d. 1.25 f. 1.75 h. 2.5

____ 10. What is the average number of cars waiting to be serviced? (Choose nearest answer.)
a. 0.5 c. 1.0 e. 1.5 g. 2.0
b. 0.75 d. 1.25 f. 1.75 h. 2.5

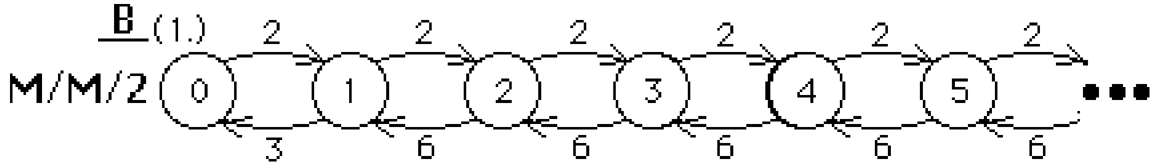
____ 11. The average arrival rate in steady state is approximately one every 2.85 hours, i.e., 0.35/hour. According to Little's Formula, the average total time spent by a car in the shop (including both waiting and repair time) is *(choose nearest value):*
a. 4 hours c. 5 hours e. 6 hours g. 7 hours
b. 8 hours d. 9 hours f. 10 hours h. 11 hours

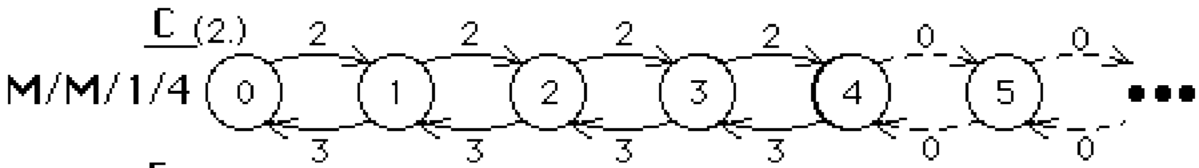
____ 12. What is the average time that a car spends in the shop waiting to be serviced? (Choose nearest answer.)
a. 0.5 c. 1.0 e. 1.5 g. 2.0
b. 0.75 d. 1.25 f. 1.75 h. 2.5

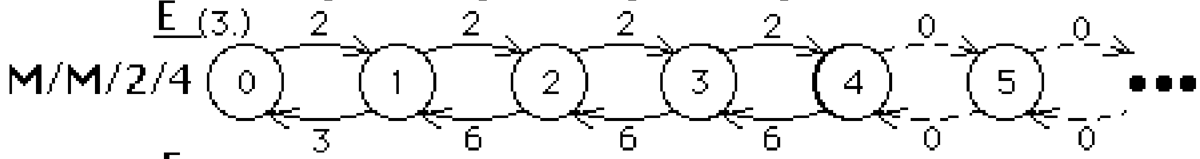
<><><><><><><><> Quiz #11 Solutions <><><><><><><><>

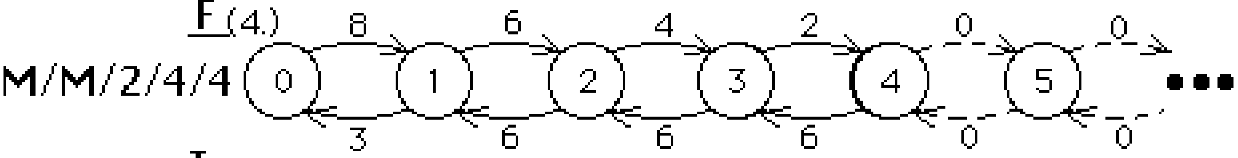
Part I:

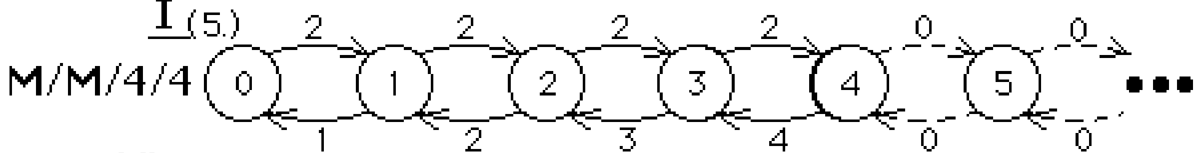
<u>B</u> (1.)



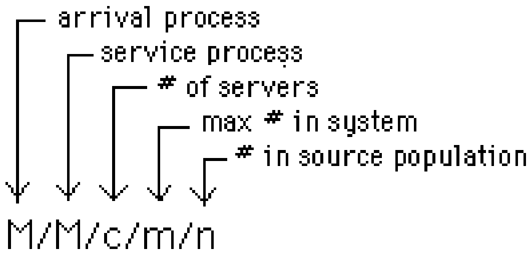






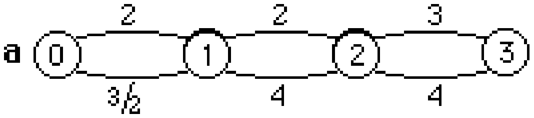


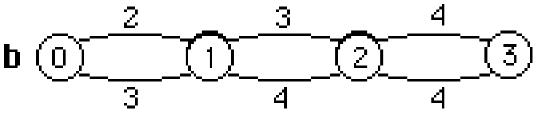
Note: Kendall's notation:

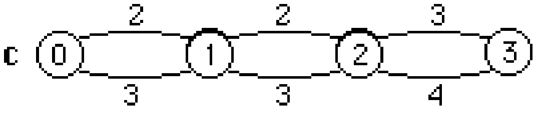


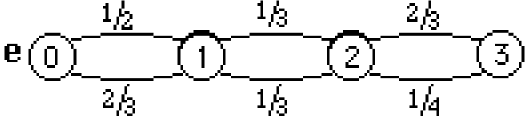
Part II: Two mechanics work in an auto repair shop, with a capacity of 3 cars. If there are 2 or more cars in the shop, each mechanic works individually, each completing the repair of a car in an average of **4** hours (the actual time being random with exponential distribution). If there is only one car in the shop, both mechanics work together on it, completing the repair in an average time of **3** hours (also exponentially distributed). Cars arrive randomly, according to a Poisson process, at the rate of one every two hours when there are less than two cars in the shop, but one every **3** hours when both mechanics are busy. If 3 cars are already in the shop, no cars arrive.

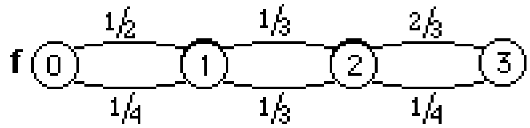
_**h**__ 6. Choose the transition diagram below corresponding to this system.

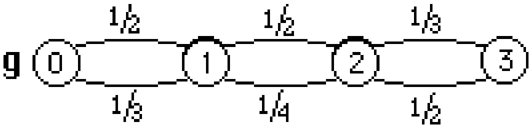


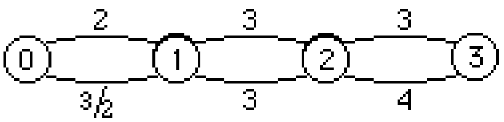










d


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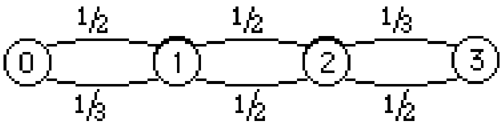


i. *None of the above*

For ease of computation, suppose that the steady-state probabilities for this system are *(not the actual values)*:

π_0=20%, π_1=30%, π_2=30%, & π_3=20%.

__**b**_ 7. What fraction of the day will both mechanics be idle?
a. 10% c. 30% e. 50% g. 70%
b. 20% = π_0 d. 40% f. 60% h. *NOTA*

__**c**_ 8. What fraction of the day will both mechanics be working on the same car?
a. 10% c. 30% = π_1 e. 50% g. 70%
b. 20% d. 40% f. 60% h. *NOTA*

__**e**_ 9. What is the average number of cars in the shop? (Choose nearest answer.)
a. 0.5 c. 1.0 e. 1.5 g. 2.0
b. 0.75 d. 1.25 f. 1.75 h. 2.5

Solution: **L =** $0\pi_0 + 1\pi_1 + 2\pi_2 + 3\pi_3 = 1.5$

__**a**_ 10. What is the average number of cars waiting to be serviced? (Choose nearest answer.)
a. 0.5 c. 1.0 e. 1.5 g. 2.0
b. 0.75 d. 1.25 f. 1.75 h. 2.5

Solution: $L_q = 0\pi_0 + 0\pi_1 + 0\pi_2 + 1\pi_3 = 0.2$

__**a**_ 11. The average arrival rate in steady state is approximately one every 2.85 hours, i.e., 0.35/hour. According to Little's Formula, the average total time spent by a car in the shop (including both waiting and repair time) is *(choose nearest value):*
a. 4 hours c. 5 hours e. 6 hours g. 7 hours
b. 8 hours d. 9 hours f. 10 hours h. 11 hours

Solution: According to *Little's Law*, $L=\underline{\lambda}W$, where L is the average number of customers in the system, $\underline{\lambda}$ is the average arrival rate, and W is the average time per customer in the system. Using L=1.5 from (9) and $\underline{\lambda}$ = 0.35/hr, we obtain 0.35W=1.5 from which we compute W = 4.275 hr.

__**a**_ 12. What is the average time that a car spends in the shop waiting to be serviced? (Choose nearest answer.)
a. 0.5 c. 1.0 e. 1.5 g. 2.0
b. 0.75 d. 1.25 f. 1.75 h. 2.5

Solution: Again by Little's Law, $L_q=\underline{\lambda}W_q$ where L_q is the average number in the queue (not including those being served) and W_q is the average time in the queue (not including service time). Using L_q=0.2 from (10) and $\underline{\lambda}$ = 0.35/hr, we obtain W_q = 0.2/0.35/hr = 0.4 hour.

<><><><><><><><> Quiz #12 <><><><><><><><>

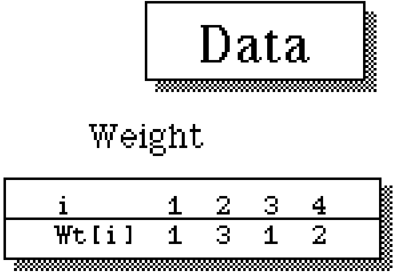
1. A system consists of 4 devices, each subject to possible failure, all of which must function in order for the system to function. In order to increase the reliability of the system, redundant units may be included, so that the system continues to function if at least one of the redundant units remains functional. The data are:

Device	Reliability (%)	Weight (kg.)
1	80	1
2	90	3
3	75	1
4	85	2

Suppose that the system may weigh no more than 12 kg. (Since at least one of each device *must* be included, a total of 7 kg, this leaves 5 kg available for redundant units.) Assume that no more than 3 units of any type need be considered. We wish to compute the number of units of each device type to be installed in order to maximize the system reliability, subject to the maximum weight restriction.
The dynamic programming model arbitrarily assumes that the devices are considered in the order: #4, #3, #2, and finally, #1. The optimal value function is defined to be:

$F_n(S)$ = maximum reliability which can be achieved for devices #n, n-1, ... 1, given that the weight used by these devices cannot exceed S *(the state variable)*

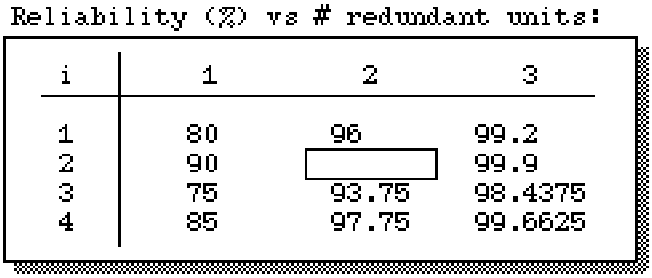
Note that the computation is done in the backward order, i.e., first the optimal value function $F_1(S)$ is computed for each value of the available weight S, then $F_2(S)$, etc., until finally $F_4(S)$ has been computed.



Data

Weight

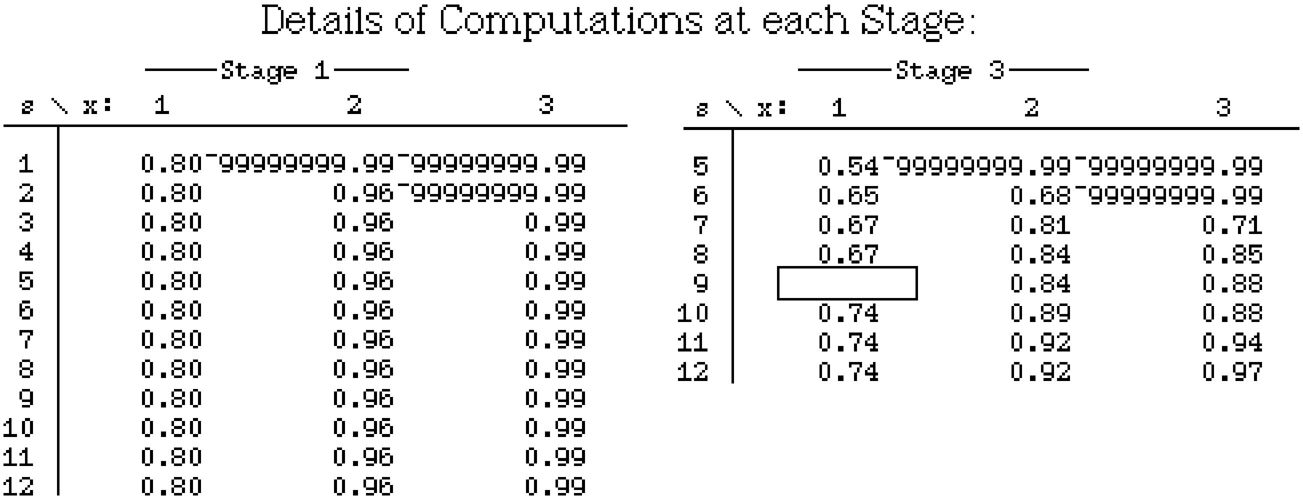
i	1	2	3	4
Wt[i]	1	3	1	2



Reliability (%) vs # redundant units:

i	1	2	3
1	80	96	99.2
2	90		99.9
3	75	93.75	98.4375
4	85	97.75	99.6625

a. Compute reliability for 2 units of device #2 (in blank above). _________%



Details of Computations at each Stage:

```
           ———Stage 1———                                ———Stage 3———
s \ x:  1           2            3             s \ x:  1           2            3

 1      0.80¯99999999.99¯99999999.99            5      0.54¯99999999.99¯99999999.99
 2      0.80        0.96¯99999999.99            6      0.65        0.68¯99999999.99
 3      0.80        0.96        0.99            7      0.67        0.81        0.71
 4      0.80        0.96        0.99            8      0.67        0.84        0.85
 5      0.80        0.96        0.99            9                  0.84        0.88
 6      0.80        0.96        0.99           10      0.74        0.89        0.88
 7      0.80        0.96        0.99           11      0.74        0.92        0.94
 8      0.80        0.96        0.99           12      0.74        0.92        0.97
 9      0.80        0.96        0.99
10      0.80        0.96        0.99
11      0.80        0.96        0.99
12      0.80        0.96        0.99
```

```
         ———Stage 2———

 s \ x:  1            2            3
———————————————————————————————————————
 4 │      0.72¯99999999.99¯99999999.99
 5 │      0.86¯99999999.99¯99999999.99
 6 │      0.89¯99999999.99¯99999999.99
 7 │      0.89         0.79¯99999999.99
 8 │      0.89         0.95¯99999999.99
 9 │      0.89         0.98¯99999999.99
10 │      0.89         0.98         0.80
11 │      0.89         0.98         0.96
12 │      0.89         0.98         0.99
```

```
         ———Stage 4———

 s \ x:  1            2            3
———————————————————————————————————————
 7 │      0.46¯99999999.99¯99999999.99
 8 │      0.57¯99999999.99¯99999999.99
 9 │      0.69         0.53¯99999999.99
10 │      0.72         0.66¯99999999.99
11 │      0.75         0.79         0.54
12 │      0.76         0.83         0.67
```

Optimal values & decisions at each stage:

```
Stage 4:
        Optimal   Optimal   Resulting
 State  Values   Decisions   State
   7     0.46       1          5
   8     0.57       1          6
   9     0.69       1          7
  10     0.72       1          8
  11     0.79       2          7
  12     0.83       2          8
```

```
Stage 2:
             Optimal   Optimal   Resulting
      State  Values   Decisions   State
        4     0.72       1          1
        5     0.86       1          2
        6     0.89       1          3
        7     0.89       1          4
        8     0.95       2          2
        9     0.98       2          3
       10     0.98       2          4
       11     0.98       2          5
       12     0.99       3          3
```

```
Stage 3:
        Optimal    Optimal   Resulting
 State  Values    Decisions    State
   5     0.54        1           4
   6     0.68        2           4
   7     0.81        2           5
   8
   9     0.88        3           6
  10     0.89        2           8
  11     0.94        3           8
  12     0.97        3           9
```

```
Stage 1:
          Optimal   Optimal   Resulting
   State  Values   Decisions   State
     1     0.80       1          0
     2     0.96       2          0
     3     0.99       3          0
     4     0.99       3          1
     5     0.99       3          2
     6     0.99       3          3
     7     0.99       3          4
     8     0.99       3          5
     9     0.99       3          6
    10     0.99       3          7
    11     0.99       3          8
    12     0.99       3          9
```

b. What is the maximum reliability that can be achieved allowing 12 kg. total weight? _____%

c. How many units of each device should be included in the system?

Device #	# of units
1	____
2	____
3	____
4	____

d. Four values have been blanked out in the output. Fill in the correct values below.

i. the optimal value $f_3(8)$ ________

ii. the optimal decision $x_3^*(8)$ _________

iii. the state which results from the optimal decision $x_3^*(8)$ ________

iv. the value associated with the decision to include 1 unit of device #3, given that 9 kg. is still available _______

e. Suppose that only 10 kg. of capacity were available. What is the achievable system reliability? _____%
How many units of each device should be included?

Device #	# of units
1	____
2	____
3	____
4	____

<><><><><><><><><><><><><><><>