Capacitated Plant Location

CROSS DECOMPOSITION

Dept of Mechanical & Industrial Engineering
The University of Iowa

Capacitated Plant Location Problem:

$$Minimize \sum_{i=1}^{m} F_i Y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

subject to

$$\sum_{i=1}^{m} X_{ij} \geq D_j, \quad j = 1, \ldots n$$

$$\sum_{j=1}^{n} X_{ij} \leq S_i Y_i, \quad i = 1, \ldots m$$

$$X_{ij} \geq 0 \text{ , } i = 1, \ldots m; \, j = 1, \ldots n$$

$$Y_i \in \{0,1\}, \quad i = 1, \ldots m$$

where $Y_i = \begin{cases} 1 & \text{if a plant is built at site i} \\ 0 & \text{otherwise} \end{cases}$

X_{ij} = quantity supplied by plant at site i to customer j

The following additional constraints are redundant but are potentially useful, depending upon how we choose the Lagrangian relaxation:

$$X_{ij} \le D_j Y_i, \quad i = 1, \ldots m; j = 1, \ldots n$$

These constraints, together with $\sum_{j=1}^{n} X_{ij} \le S_i, \quad i = 1, \ldots m$,

could be used instead of the constraints

$$\sum_{j=1}^{n} X_{ij} \le S_i Y_i, \quad i = 1, \ldots m$$

in order to force shipments from a plant to be zero if that plant has not been opened!

The constraint

$$\sum_{i=1}^{m} S_i Y_i \geq \sum_{j=1}^{n} D_j$$

is redundant, but is useful in order to generate trial solutions (Y) which are guaranteed to give feasible subproblems!

An *alternate formulation* of the CPL:

$$\textit{Minimize} \quad \sum_{i=1}^{m} F_i Y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

subject to

$$\sum_{i=1}^{m} X_{ij} \geq D_j, \quad j = 1, \ldots n$$

$$\sum_{j=1}^{n} X_{ij} \leq S_i, \quad i = 1, \ldots m$$

$$\sum_{i=1}^{m} S_i Y_i \geq \sum_{j=1}^{n} D_j$$

$$X_{ij} \leq D_j Y_i, \quad i = 1, \ldots m; \, j = 1, \ldots n$$ *(linking constraints)*

$$X_{ij} \geq 0 \; , \; i = 1, \ldots m; \, j = 1, \ldots n \, , \qquad Y_i \in \{0,1\}, \quad i = 1, \ldots m$$

Our goal is to separate the problem by relaxing the constraints linking the X and Y decisions.

A *Lagrangian Relaxation* of the CPL:

$$D(\mu) = Minimum \;\; \sum_{i=1}^{m} F_i Y_i + \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} \mu_{ij}\left[X_{ij} - D_j Y_i \right]$$

subject to

$$\sum_{i=1}^{m} X_{ij} \geq D_j, \quad j = 1, \ldots n$$

$$\sum_{j=1}^{n} X_{ij} \leq S_i, \quad i = 1, \ldots m$$

$$\sum_{i=1}^{m} S_i Y_i \geq \sum_{j=1}^{n} D_j$$

$$X_{ij} \geq 0 \;,\; i = 1, \ldots m; \; j = 1, \ldots n$$

$$Y_i \in \{0,1\}, \quad i = 1, \ldots m$$

Rearranging terms in the objective function:

$$D(\mu) = Minimum \sum_{i=1}^{m} \left(F_i - \mu_{ij} D_j\right) Y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} \left(C_{ij} + \mu_{ij}\right) X_{ij}$$

subject to

$$\sum_{i=1}^{m} X_{ij} \geq D_j, \quad j = 1, \ldots n$$

$$\sum_{i=1}^{m} S_i Y_i \geq \sum_{j=1}^{n} D_j$$

$$\sum_{j=1}^{n} X_{ij} \leq S_i, \quad i = 1, \ldots m$$

$$Y_i \in \{0,1\}, \quad i = 1, \ldots m$$

$$X_{ij} \geq 0 \ , \ i = 1, \ldots m; \ j = 1, \ldots n$$

This separates into ***two subproblems***: $D(\mu) = D_X(\mu) + D_Y(\mu)$

i.e., the ***transportation*** problem

$$D_X(\mu) = Minimum \sum_{i=1}^{m}\sum_{j=1}^{n}\left(C_{ij} + \mu_{ij}\right)X_{ij}$$

$$\text{subject to } \sum_{i=1}^{m} X_{ij} \geq D_j, \quad j = 1,\ldots n$$

$$\sum_{j=1}^{n} X_{ij} \leq S_i, \quad i = 1,\ldots m$$

$$X_{ij} \geq 0 \text{ , } i = 1,\ldots m;\ j = 1,\ldots n$$

and the problem:

$$D_Y(\mu) = Minimum \sum_{i=1}^{m}\left(F_i - \mu_{ij} D_j\right)Y_i$$

$$subject\ \ to\ \sum_{i=1}^{m} S_i Y_i \geq \sum_{j=1}^{n} D_j, \quad Y_i \in \{0,1\}\ \forall i$$

By using the complements of the variables, i.e., $\overline{Y}_i \equiv 1 - Y_i$, the subproblem $D_Y(\mu)$ can be expressed as a ***0-1 knapsack*** problem:

$$D_Y(\mu) = \sum_{i=1}^{m} \left(F_i - \mu_{ij} D_j \right) - \text{ Maximum } \sum_{i=1}^{m} \left(F_i - \mu_{ij} D_j \right) \overline{Y}_i$$

$$\text{subject to } \sum_{i=1}^{m} S_i \overline{Y}_i \le \sum_{i=1}^{m} S_i - \sum_{j=1}^{n} D_j, \quad \overline{Y}_i \in \{0,1\} \quad \forall i$$

Consider still *another* formulation of CPL:

$$\textit{Minimize} \quad \sum_{i=1}^{m} F_i Y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

subject to

$$\sum_{i=1}^{m} X_{ij} \ge D_j, \quad j = 1, \ldots n$$

$$\sum_{j=1}^{n} X_{ij} \le S_i, \quad i = 1, \ldots m$$

$$\sum_{i=1}^{m} S_i Y_i \ge \sum_{j=1}^{n} D_j$$

$$\sum_{j=1}^{n} X_{ij} \le S_i Y_i, \quad i = 1, \ldots m \quad \textit{(linking constraints)}$$

$$X_{ij} \ge 0 \ , \ i = 1, \ldots m; \ j = 1, \ldots n$$

$$Y_i \in \{0,1\}, \quad i = 1, \ldots m$$

A *Lagrangian Relaxation* of the CPL, where $\mu \geq 0$:

$$D(\mu) = Minimum \ \sum_{i=1}^{m} F_i Y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} + \sum_{i=1}^{m} \mu_i \left[\sum_{j=1}^{n} X_{ij} - S_i Y \right]$$

or

$$D(\mu) = Minimum \ \sum_{i=1}^{m} (F_i - \mu_i S_i) Y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} (C_{ij} + \mu_i) X_{ij}$$

subject to

$$\sum_{i=1}^{m} X_{ij} \geq D_j, \quad j = 1, \ldots n$$

$$\sum_{j=1}^{n} X_{ij} \leq S_i, \quad i = 1, \ldots m$$

$$\sum_{i=1}^{m} S_i Y_i \geq \sum_{j=1}^{n} D_j$$

$$X_{ij} \geq 0 \ , \ i = 1, \ldots m; \ j = 1, \ldots n$$

$$Y_i \in \{0,1\}, \quad i = 1, \ldots m$$

This separates into ***two subproblems***: $D(\mu) = D_X(\mu) + D_Y(\mu)$

i.e., the ***transportation*** problem

$$D_X(\mu) = Minimum \sum_{i=1}^{m} \sum_{j=1}^{n} (C_{ij} + \mu_i) X_{ij}$$

$$\text{subject to} \quad \sum_{i=1}^{m} X_{ij} \geq D_j, \quad j = 1, \ldots n$$

$$\sum_{j=1}^{n} X_{ij} \leq S_i, \quad i = 1, \ldots m$$

$$X_{ij} \geq 0 \ , \ i = 1, \ldots m; \ j = 1, \ldots n$$

and the problem:

$$D_Y(\mu) = Minimum \ \sum_{i=1}^{m} (F_i - \mu_i S_i) Y_i$$

$$subject \ \ to \ \ \sum_{i=1}^{m} S_i Y_i \geq \sum_{j=1}^{n} D_j, \ \ Y_i \in \{0,1\} \forall i$$

As before, by using the complements of the variables, i.e.,

$$\overline{Y}_i \equiv 1 - Y_i,$$

$D_Y(\mu)$ can be expressed as a ***0-1 knapsack*** problem:

$$D_Y(\mu) = \sum_{i=1}^{m} (F_i - \mu_i S_i) - \text{ Maximum } \sum_{i=1}^{m} (\mu_i S_i - F_i) \overline{Y}_i$$

$$\text{subject to } \sum_{i=1}^{m} S_i \overline{Y}_i \le \sum_{i=1}^{m} S_i - \sum_{j=1}^{n} D_j, \quad \overline{Y}_i \in \{0,1\} \quad \forall i$$

Cross-Decomposition

- Either Lagrangian subproblem can be used to generate trial solutions (Y) for the Benders' subproblems (which in turn generates Lagrangian multipliers for the Lagrangian subproblems) in a Cross-Decomposition scheme!

- Note that if lower bounds are not required, only the Lagrangian subproblem in the **Y** variables (i.e., the knapsack problem) needs to be solved at each iteration to provide the trial solution for Benders' subproblem

STALLING

"Stalling" can occur, i.e., the same trial Y variables or the same dual variables may be generated multiple times, in which case it is necessary to resort to the Benders' (or Lagrangian) Master Problem.

In order to avoid this, ***mean values*** may be passed from the primal subproblem to dual subproblem, and/or vice versa.

Note: "Cuts" or constraints for Benders' Master Problem may be derived as follows.

Assume that the subproblem is feasible, i.e., $\sum_{i=1}^{m} S_i Y_i \geq \sum_{j=1}^{n} D_j$

Transform the supply & demand constraints into equations by defining a "dummy" demand point *(#n+1)* :

$$\sum_{i=1}^{m} F_i Y_i + \underset{X \geq 0}{Minimize} \quad \sum_{i=1}^{m} \sum_{j=1}^{n+1} C_{ij} X_{ij}$$

subject to $\sum_{i=1}^{m} X_{ij} = D_j, \quad j = 1, \ldots n$

$$\sum_{j=1}^{n} X_{ij} = S_i Y_i, \quad i = 1, \ldots m$$

$$\sum_{i=1}^{m} X_{i,n+1} = \sum_{i=1}^{m} S_i Y_i - \sum_{j=1}^{n} D_j$$

Let U & V be the *optimal dual* variables.

Then the optimal value *v(Y)* is

$$\sum_{i=1}^{m} F_i Y_i + \sum_{i=1}^{m} U_i S_i Y_i + \sum_{j=1}^{n} V_j D_j + V_{n+1}\left(\sum_{i=1}^{m} S_i Y_i - \sum_{j=1}^{n} D_j\right)$$

$$= \sum_{i=1}^{m} (U_i + V_{n+1}) S_i Y_i + \sum_{j=1}^{n} \left(V_j - V_{n+1}\right) D_j$$

Thus, the linear *(under-)*estimate of the optimal value function *v(Y)* is $\alpha Y + \beta$, where

$$\alpha_i = \left(U_i + V_{n+1}\right) S_i, \qquad i = 1, \ldots m$$

$$\beta = \sum_{j=1}^{n} \left(V_j - V_{n+1}\right) D_j$$

EXAMPLE

CROSS DECOMPOSITION

Capacitated Plant Location

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Dept of Mechanical & Industrial Engineering
The University of Iowa

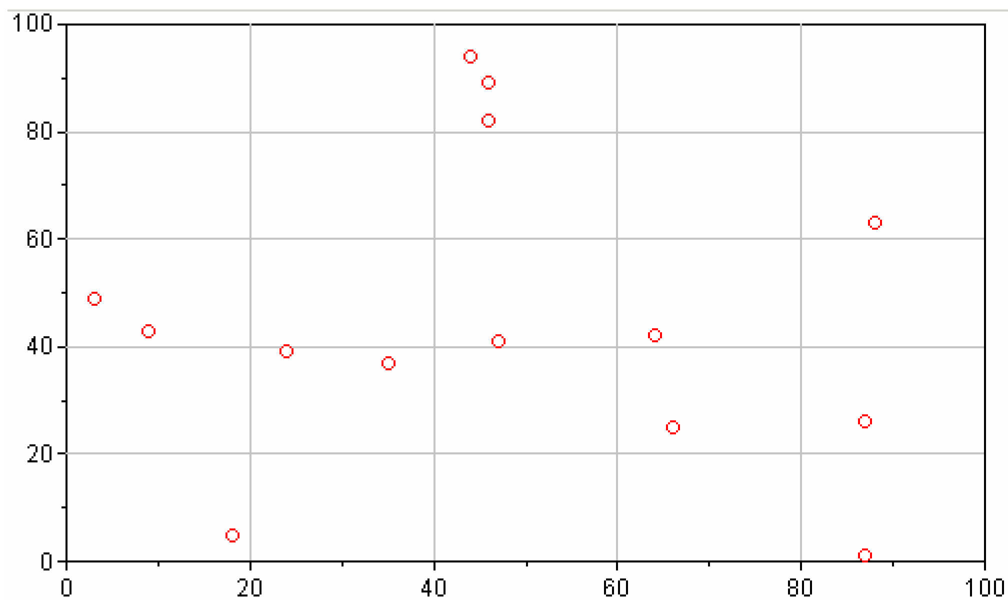
Randomly-generated problem with

- 7 potential plant sites and
- 14 demand points

Random number seed = 3432

i	X	Y	D	i	X	Y	D	i	X	Y	D	i	X	Y	D	i	X	Y	D
1	3	49	7	4	9	43	8	7	46	82	3	10	87	26	1	13	88	63	8
2	44	94	5	5	46	89	7	8	87	1	5	11	18	5	3	14	66	25	2
3	24	39	2	6	47	41	9	9	35	37	2	12	64	42	8				

Total demand: 70



Points 1 2 3 4 5 6 7 are potential plant sites, with capacities & fixed costs

i	Ki	Fi
1	37	405
2	63	344
3	25	330
4	59	116
5	82	292
6	48	498
7	95	281

(i = plant site #, K[i] = capacity, F[i] = fixed cost)

Costs, Supplies, & Demands:

i/j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	K	F
1	0	61	23	8	59	45	54	97	34	87	46	61	86	67	37	405
2	61	0	59	62	5	53	12	102	58	80	93	56	54	72	63	344
3	23	59	0	16	55	23	48	74	11	64	35	40	68	44	25	330
4	8	62	16	0	59	38	54	89	27	80	39	55	81	60	59	116
5	59	5	55	59	0	48	7	97	53	75	89	50	49	67	82	292
6	45	53	23	38	48	0	41	57	13	43	46	17	47	25	48	498
7	54	12	48	54	7	41	0	91	46	69	82	44	46	60	95	281
Demand:	7	5	2	8	7	9	3	5	2	1	3	8	8	2	409	0

K = capacity,
F = fixed cost

We solve this problem by

"Standard" Benders' decomposition *(optimizing master problem at each iteration)*

"Standard" Cross-Decomposition

First the problem is solved by Benders' decomposition algorithm:

<u>Benders Decomposition Algorithm</u>

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Master problem will be optimized at each iteration,
    providing the Y minimizing current approximation v(Y) and
    a lower bound
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Iteration #1

Trial Y for primal subproblems:
open #3 6 *(initial "guess")*

Primal subproblem results:

Transport costs	2030
Fixed costs	828
Total costs	2858

****** New incumbent! ******

Solution of Master Problem
Y: open < **2 7** >
Estimated V(X): ¯4579

Iteration #2

Trial Y for primal subproblems:
open #2 7

Primal subproblem results:

Transport costs	3012
Fixed costs	625
Total costs	3637

Solution of Master Problem
Y: open <**4 6 7** >
Estimated V(X): ¯1247

Iteration #3

Trial Y for primal subproblems:
open #4 6 7

Primal subproblem results:

Transport costs	1222
Fixed costs	895
Total costs	2117

****** New incumbent! ******

Solution of Master Problem
Y: open <1 2 3 5 >
Estimated V(X): 775

Iteration #4

Trial Y for primal subproblems:
open #1 2 3 5

Primal subproblem results:

Transport costs	1723
Fixed costs	1371
Total costs	3094

Solution of Master Problem
Y: open <2 3 6 >
Estimated V(X): 1311

Iteration #5

Trial Y for primal subproblems:
open #2 3 6

Primal subproblem results:

Transport costs	1377
Fixed costs	1172
Total costs	2549

Solution of Master Problem
Y: open <1 2 5 6 7 >
Estimated V(X): 1544

Iteration #6

Trial Y for primal subproblems:
open #1 2 5 6 7

Primal subproblem results:

Transport costs	1156
Fixed costs	1820
Total costs	2976

Solution of Master Problem
Y: open <4 5 6 >
Estimated V(X): 1590

Iteration #7

Trial Y for primal subproblems:
open #4 5 6

Primal subproblem results:

Transport costs	1167
Fixed costs	906
Total costs	2073

****** New incumbent! ******

Solution of Master Problem
Y: open <4 7 >
Estimated V(X): 1619

Iteration #8

Trial Y for primal subproblems:
open #4 7

Primal subproblem results:

Transport costs	2064
Fixed costs	397
Total costs	2461

Solution of Master Problem
Y: open <4 6 >
Estimated V(X): 1836

Iteration #9

Trial Y for primal subproblems:
open #4 6

Primal subproblem results:

Transport costs 1863
Fixed costs 614
Total costs 2477

Solution of Master Problem
Y: open <4 5 >
Estimated V(X): 1909

Iteration #10

Trial Y for primal subproblems:
open #4 5

Primal subproblem results:

Transport costs 2079
Fixed costs 408
Total costs 2487

Solution of Master Problem

Y: open <2 3 4 6 >
Estimated V(X): 1740

Iteration #11

Trial Y for primal subproblems:
open #2 3 4 6

Primal subproblem results:

Transport costs 1144
Fixed costs 1288
Total costs 2432

Solution of Master Problem
Y: open <3 4 5 6 7 >
Estimated V(X): 1765

Iteration #12

Trial Y for primal subproblems:
open #3 4 5 6 7

Primal subproblem results:

Transport costs 1090
Fixed costs 1517
Total costs 2607

Solution of Master Problem

Y: open <5 6 >
Estimated V(X): 1957

Iteration #13

Trial Y for primal subproblems:
open #5 6

Primal subproblem results:

Transport costs	1779
Fixed costs	790
Total costs	2569

Solution of Master Problem
Y: open <**2 3 4 7** >
Estimated V(X): 1979

Iteration #14

Trial Y for primal subproblems:
open #2 3 4 7

Primal subproblem results:

Transport costs	1663
Fixed costs	1071
Total costs	2734

Solution of Master Problem
Y: open <6 7 >
Estimated V(X): 2001

Iteration #15

Trial Y for primal subproblems:
open #6 7

Primal subproblem results:

Transport costs	1820
Fixed costs	779
Total costs	2599

Solution of Master Problem
Y: open <2 4 6 7 >
Estimated V(X): 2014

Iteration #16

Trial Y for primal subproblems:
open #2 4 6 7

Primal subproblem results:

Transport costs	1148
Fixed costs	1239
Total costs	2387

Solution of Master Problem
Converged at iteration #16!
no trial solution

Incumbent Solution

Random Problem (Seed = 3432)

(Found at iteration #7!)

Summary

Transport cost=	1167
Fixed costs=	906
Total costs=	2073
Low bound	2073
Gap (%)	0

Plant	Fixed Cost	Supply	Surplus
4	116	59	39
5	292	82	67
6	498	48	13

Total fixed costs= 906 = 43.70% of total cost

Optimal Shipments

	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5
4	7	0	2	8	0	0	0	0	0	0	3	0	0	0	39
5	0	5	0	0	7	0	3	0	0	0	0	0	0	0	67
6	0	0	0	0	0	9	0	5	2	1	0	8	8	2	13

(*Demand pt #15 is dummy demand for excess capacity.)*

Dual Solution of Transportation Problem

Supply constraints

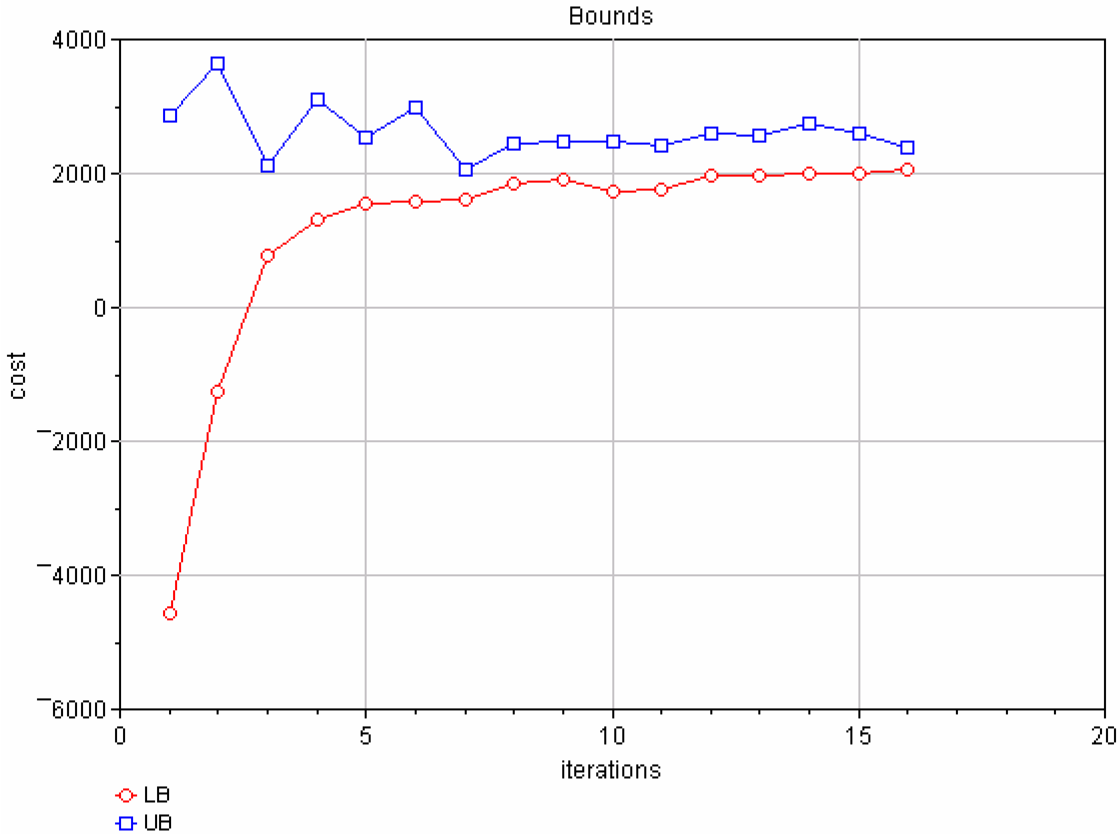
i	Ui	i	Ui
1	8	4	16
2	11	5	16
3	0	7	9

Demand constraints

j	Vj	j	Vj	j	Vj	j	Vj	j	Vj	j	Vj	j	Vj
1	¯8	3	0	5	¯16	7	¯9	9	¯3	11	23	13	31
2	¯11	4	¯16	6	¯16	8	41	10	27	12	1	14	9

Reduced costs: COST - U°.+V

I \ J=	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	64	15	16	67	53	55	48	29	52	15	52	47	50
2	58	0	48	67	10	58	10	50	50	42	59	44	12	52
3	31	70	0	32	71	39	57	33	14	37	12	39	37	35
4	0	57	0	0	59	38	47	32	14	37	0	38	34	35
5	51	0	39	59	0	48	0	40	40	32	50	33	2	42
6	37	48	7	38	48	0	34	0	0	0	7	0	0	0
7	53	14	39	61	14	48	0	41	40	33	50	34	6	42



Lower bound is monotonically increasing!

Cross-Decomposition Algorithm

Current parameters for cross-decomposition

Method for generating Y for primal subproblems
    Most recent

Method for updating Lagrangian multipliers for use in dual subproblems
    Most recent

Iteration #1

Dual subproblem results:

Using multipliers:

i	1	2	3	4	5	6	7
Mu[i]	2.621	1.619	2.96	1.508	1.182	1.187	0.957

Subproblem in X:

Optimal cost= 1009, X=

	1	2	3	4	5	6	7	8	9	0	1	2	3	4	sum
1	7	0	0	0	0	0	0	0	0	0	0	0	0	0	7
2	0	5	0	0	0	0	0	0	0	0	0	0	0	0	5
3	0	0	2	0	0	0	0	0	2	0	3	0	0	0	7
4	0	0	0	8	0	0	0	0	0	0	0	0	0	0	8
5	0	0	0	0	7	0	0	0	0	0	0	0	0	0	7
6	0	0	0	0	0	9	0	5	0	1	0	8	0	2	25
7	0	0	0	0	0	0	3	0	0	0	0	0	8	0	11

Subproblem in Y: Objective coefficients:

i	1	2	3	4	5	6	7
cost	308	242	256	27	195	441	190

Optimal cost= 697, by opening plants **3 6**

Total cost (Lower bound): 1706

Primal Subproblem

Trial Y for primal subproblem is:
open plants #3 6
with fixed costs 828
Primal subproblem solution:
Transportation cost = 2030
Total cost = 2858
Dual variables: **30 0 53 37 5 53 12**
***** new incumbent! *****

Iteration #2

Dual subproblem results:

Using multipliers:

i	1	2	3	4	5	6	7
Mu[i]	30	0	53	37	5	53	12

Subproblem in X:

Optimal cost= 2222, X=

	1	2	3	4	5	6	7	8	9	0	1	2	3	4	sum
1	7	0	2	0	0	0	0	0	0	3	0	0	0	0	12
2	0	5	0	0	7	0	0	5	0	1	0	0	8	0	26
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	8	0	0	0	0	0	0	0	0	0	0	8
5	0	0	0	0	0	0	0	0	0	0	8	0	0	0	8
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	9	3	0	2	0	0	0	0	2	16

Subproblem in Y: Objective coefficients:

i	1	2	3	4	5	6	7
cost	¯705	344	¯995	¯2067	¯118	¯2046	¯859

Optimal cost= ¯6790,
by opening plants **1 3 4 5 6 7**

Total cost (Lower bound): ¯4568

Primal Subproblem

Trial Y for primal subproblem is:
open plants #1 3 4 5 6 7
with fixed costs 1922
Primal subproblem solution:
Transportation cost = 1034
Total cost = 2956
Dual variables: **5 0 5 5 5 5 5**

Iteration #3

Dual subproblem results:
Using multipliers:

i	1	2	3	4	5	6	7
Mu[i]	5	0	5	5	5	5	5

Subproblem in X:
Optimal cost= 1044, X=

	1	2	3	4	5	6	7	8	9	0	1	2	3	4	sum
1	7	0	0	0	0	0	0	0	0	0	0	0	0	0	7
2	0	5	0	0	7	0	0	0	0	0	0	0	0	0	12
3	0	0	2	0	0	0	0	0	2	0	3	0	0	0	7
4	0	0	0	8	0	0	0	0	0	0	0	0	0	0	8
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	9	0	5	0	1	0	8	0	2	25
7	0	0	0	0	0	0	3	0	0	0	0	0	8	0	11

Subproblem in Y: Objective coefficients:

i	1	2	3	4	5	6	7
cost	220	344	205	¯179	¯118	258	¯194

Optimal cost= ¯491, by opening plants **4 5 7**

Total cost (Lower bound): 553

Primal Subproblem

Trial Y for primal subproblem is: open plants #4 5 7
with fixed costs 689
Primal subproblem solution:
Transportation cost = 1980
Total cost = 2669
Dual variables: **30 33 22 38 38 0 38**

***** new incumbent! *****

Iteration #4

Dual subproblem results:
Using multipliers:

i	1	2	3	4	5	6	7
Mu[i]	30	33	22	38	38	0	38

Subproblem in X:
Optimal cost= 1153, X=

	1	2	3	4	5	6	7	8	9	0	1	2	3	4	sum
1	7	0	0	8	0	0	0	0	0	0	0	0	0	0	15
2	0	5	0	0	7	0	0	0	0	0	0	0	0	0	12
3	0	0	2	0	0	0	0	0	0	0	0	0	0	0	2
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	9	0	5	2	1	3	8	8	2	38
7	0	0	0	0	0	0	3	0	0	0	0	0	0	0	3

Subproblem in Y: Objective coefficients:

i	1	2	3	4	5	6	7
cost	¯705	¯1735	¯220	¯2126	¯2824	498	¯3329

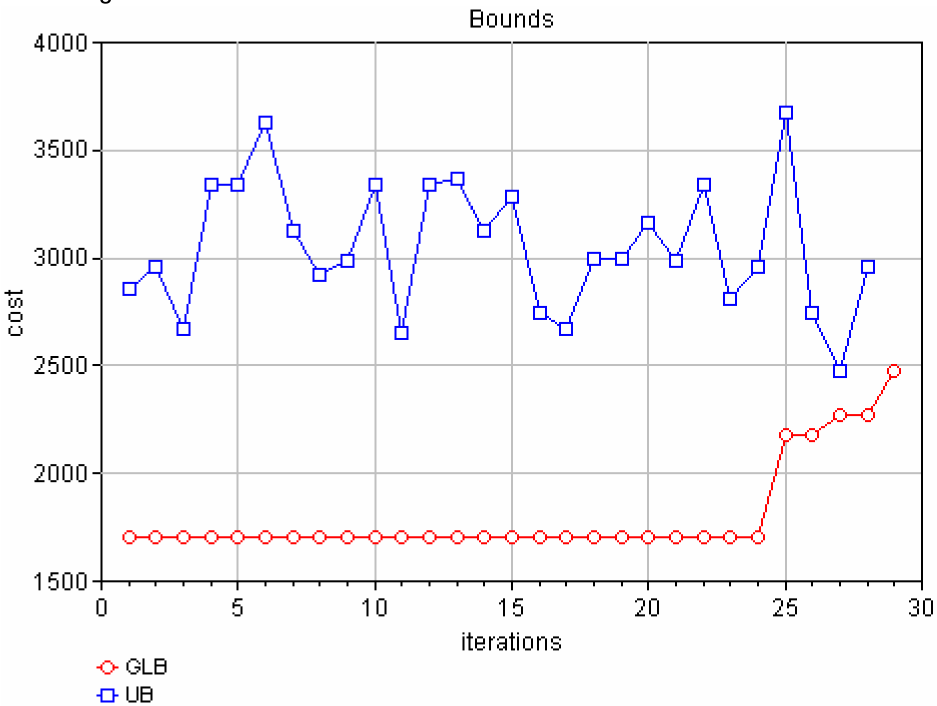
Optimal cost= ¯10939,
by opening plants **1 2 3 4 5 7**

Total cost (Lower bound): ¯9786

Primal Subproblem

Trial Y for primal subproblem is:
open plants #1 2 3 4 5 7
with fixed costs 1768
Primal subproblem solution:
Transportation cost = 1572
Total cost = 3340
Dual variables: **27 27 23 27 27 0 27**

Convergence occurs in iteration #29:



Upper Bound
vs
Greatest Lower Bound