Russell-Yasuda Kasai Model Multi-stage

Asset / Liability Management

- Cariño, D. R., T. Kent, et al. (1994). "The Russel-Yasuda Kasai model: An asset-liability model for a Japanese insurance company using multistage stochastic programming." *Interfaces* **24**(1): 29-49.
- Cariño, D. R., D. H. Myers, et al. (1998). "Concepts, technical issues, and uses of the Russell-Yasuda Kasai financial planning model." *Operations Research* **46**(4): 450-462.
- Cariño, D. R. and W. T. Ziemba (1998). "Formulation of the Russell-Yasuda Kasai financial planning model." *Operations Research* **46**(4): 433-449.

- Developed by Frank Russell Company and The Yasuda Fire & Marine Insurance Company.
- Decisions are made on how best to invest in assets to meet a random liability stream over time, with random investment returns.
- Goal is to produce a high-income return to pay annual interest on savings-type insurance policies while maximizing the long-term wealth of the firm.
- Handles complex regulations imposed by Japanese insurance laws and practices.

Model is multistage stochastic LP with recourse

R-Y Kasai Model page 2 D.L.Bricker

Random Variables

 rp_{it} = price return of asset j in period t

 ri_{jt} = income return of asset j in period t

 F_t = deposit inflow in period t

 P_t = principal payout in period t

 I_t = income payout in period t

 g_t = rate of interest paid on policies in period t

 L_t = liability valuation at end of period t

Decision Variables

 w_{jt} = market value held in asset j in period t

 W_t = total fund market value in period t

 u_t = income shortfall in period t

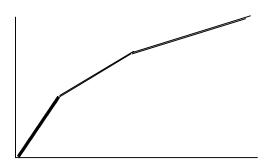
 v_t = income surplus in period t

Objective:

Maximize
$$E\left[W_{H} - \sum_{t=1}^{H} c_{t}\left(u_{t}\right)\right]$$

where $c_t(u_t)$ is a piecewise-linear concave function which specifies the penalties for income shortfalls.

(Converts to LP with introduction of additional variables.)



Constraints include:

$$\begin{split} \sum_{j=1}^{J} w_{jt} &= W_{t} \\ W_{t+1} - \sum_{j=1}^{J} \left(1 + r p_{j,t+1} + r i_{j,t+1}\right) w_{jt} &= F_{t+1} - P_{t+1} - I_{t+1} \\ \sum_{j=1}^{J} r i_{j,t+1} w_{jt} + u_{t+1} - v_{t+1} &= g_{t+1} L_{t} \end{split}$$

$$\sum_{j=1}^{J} r i_{j,t+1} w_{jt} + u_{t+1} - v_{t+1} = g_{t+1} L_{t}$$

$$w_{jt} \ge 0, \quad u_{t} \ge 0, \quad v_{t} \ge 0$$

Periods are of varying length:

8 branches in period 1 (first quarter)

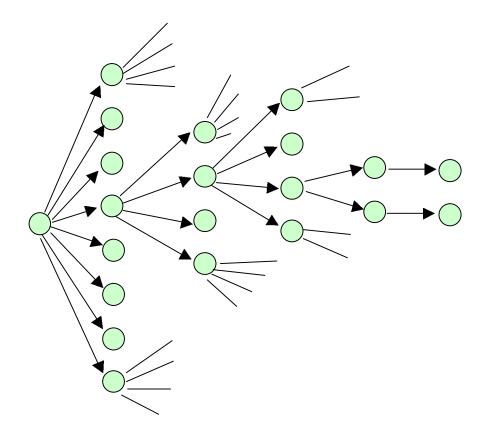
4 branches in period 2 (remainder of first year)

4 branches in period 3 (year 2)

2 branches in period 4 (years 3-5)

1 branch in period 6 (terminal conditions)

Total number of scenarios: $8 \times 4 \times 4 \times 2 \times 1 = 256$



Size of Problem (# asset classes=7)

rows: 263

columns: 431

Size of Deterministic Equivalent Problem

rows ~ 31,000

columns ~ 44,000

Nested Benders' Decomposition was used.

Three hours of computation was required to solve full model.

Contributed 79 million US\$ in first two years of use.