- **Example 1 (LRFD Steel Design, Third Edition by William T. Segui, page 156-157.)**

Determine the design strength $\phi_b M_n$ for a W14 × 68 of A242 steel subject to

a. continuous lateral support.
b. unbraced length = 20 ft; C_b = 1.0.
c. unbraced length = 20 ft; C_b = 1.75.

a. From Part 2 of the *Manual*, a W14 × 68 is in shape group 2 and is therefore available with a yield stress, F_y, of 50 ksi. Determine whether this shape is compact, noncompact, or slender:

$$\frac{b_f}{2t_f} = 6.97 < 0.38\sqrt{\frac{29{,}000}{50}} = 9.15$$

This shape is compact and thus

$$M_n = M_p = F_y Z_x = 50(115) = 5750 \text{ in.-kips} = 479.2 \text{ ft-kips}$$

$$\phi_b M_n = 0.90(479.2) = 431 \text{ ft-kips}$$

b. L_b = 20 ft and C_b = 1.0. Compute L_p and L_r:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76(2.46)\sqrt{\frac{29{,}000}{50}} = 104.3 \text{ in.} = 8.689 \text{ ft}$$

From the torsion properties tables in Part 1 of the *Manual*,

$$J = 3.01 \text{ in.}^4 \quad \text{and} \quad C_w = 5370 \text{ in.}^6$$

Although X_1 and X_2 are tabulated in the dimensions and properties tables in Part 1 of the *Manual*, we compute them here for illustration:

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}} = \frac{\pi}{103}\sqrt{\frac{29{,}000(11{,}200)(3.01)(20.0)}{2}} = 3016 \text{ ksi}$$

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2 = 4\left(\frac{5370}{121}\right)\left(\frac{103}{11{,}200 \times 3.01}\right)^2 = 0.001657 \text{ (ksi)}^{-2}$$

$$L_r = \frac{r_y X_1}{(F_y - F_r)}\sqrt{1 + \sqrt{1 + X_2(F_y - F_r)^2}}$$

$$= \frac{2.46(3016)}{(50 - 10)}\sqrt{1 + \sqrt{1 + 0.001657(50 - 10)^2}} = 316.5 \text{ in.} = 26.37 \text{ ft}$$

Since $L_p < L_b < L_r$, the strength is based on inelastic LTB and

$$M_r = (F_y - F_r)S_x = \frac{(50 - 10)(103)}{12} = 343.3 \text{ ft-kips}$$

$$M_n = C_b\left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right]$$

$$= 1.0\left[479.2 - (479.2 - 343.3)\left(\frac{20 - 8.689}{26.37 - 8.689}\right)\right]$$

$$= 392.3 \text{ ft-kips} < M_p$$

$$\phi_b M_n = 0.90(392.3) = 353 \text{ ft-kips}$$

c. $L_b = 20$ ft and $C_b = 1.75$. The design strength for $C_b = 1.75$ is 1.75 times the design strength for $C_b = 1.0$. Therefore,

$$M_n = 1.75(392.3) = 686.5 \text{ ft-kips} > M_p = 479.2 \text{ ft-kips}$$

The nominal strength cannot exceed M_p; hence use a nominal strength of M_n = 479.2 ft-kips:

$$\phi_b M_n = 0.90(479.2) = 431 \text{ ft-kips.}$$

- **Example 2 (LRFD Steel Design, Third Edition by William T. Segui, page 159-161.)**

A simply supported beam with a span length of 40 feet is laterally supported at its ends and is subjected to the following service loads:

Dead load = 400 lb/ft (including the weight of the beam)

Live load = 1000 lb/ft

If $F_y = 50$ ksi, is a W14 × 90 adequate?

The factored load and moment are

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.400) + 1.6(1.000) = 2.080 \text{ kips/ft}$$

$$M_u = \frac{1}{8} w_u L^2 = \frac{2.080(40)^2}{8} = 416.0 \text{ ft-kips}$$

Determine whether the shape is compact, noncompact, or slender:

$$\lambda = \frac{b_f}{2t_f} = 10.2$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15$$

$$\lambda_r = 0.83\sqrt{\frac{E}{F_y - F_r}} = 0.83\sqrt{\frac{29{,}000}{50 - 10}} = 22.3$$

Since $\lambda_p < \lambda < \lambda_r$, this shape is noncompact. Check the capacity based on the limit state of flange local buckling:

$$M_p = F_y Z_x = \frac{50(157)}{12} = 654.2 \text{ ft-kips}$$

$$M_r = (F_y - F_r)S_x = \frac{(50 - 10)(143)}{12} = 476.7 \text{ ft-kips}$$

$$M_n = M_p - (M_p - M_r)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$= 654.2 - (654.2 - 476.7)\left(\frac{10.2 - 9.15}{22.3 - 9.15}\right) = 640.0 \text{ ft-kips}$$

Check the capacity based on the limit state of lateral-torsional buckling. From the Z_x table,

$L_p = 15.1$ ft and $L_r = 38.4$ ft

$L_b = 40 \text{ ft} > L_r$ $\quad\therefore$ failure is by *elastic* LTB.

From Part 1 of the *Manual*,

$I_y = 362 \text{ in.}^4$ (This can also be found in the Z_x table in Part 5 of the *Manual*.)

$J = 4.06 \text{ in.}^4$

$C_w = 16{,}000 \text{ in.}^6$

For a uniformly loaded, simply supported beam with lateral support at the ends,

$C_b = 1.14$

AISC Equation F1-13 gives

$$M_n = C_b \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w} \le M_p$$

$$= 1.14\left[\frac{\pi}{40(12)}\sqrt{29{,}000(362)(11{,}200)(4.06) + \left(\frac{\pi \times 29{,}000}{40 \times 12}\right)^2 (362)(16{,}000)}\right]$$

$$= 1.14(5421) = 6180 \text{ in.-kips} = 515.0 \text{ ft-kips}$$

$M_p = 654.2 \text{ ft-kips} > 515.0 \text{ ft-kips}$ (OK)

Because 515.0 < 640.0, LTB controls, and

$$\phi_b M_n = 0.90(515.0) = 464 \text{ ft-kips} > M_u = 416 \text{ ft-kips} \qquad \text{(OK)}$$

Since $M_u < \phi_b M_n$, the beam has adequate moment strength.

- **Example 3 (LRFD Steel Design, Third Edition by William T. Segui, page 165-166.)**

Check the beam in Example 5.6 for shear.

From Example 5.6, $w_u = 2.080$ kips/ft and $L = 40$ ft. A W14 × 90 with $F_y = 50$ ksi is used. For a simply supported, uniformly loaded beam, the maximum shear occurs at the support and is equal to the reaction

$$V_u = \frac{w_u L}{2} = \frac{2.080(40)}{2} = 41.6 \text{ kips}$$

From the dimensions and properties tables in Part 1 of the *Manual*, the web width-thickness ratio of a W14 × 90 is

$$\frac{h}{t_w} = 25.9$$

$$2.45\sqrt{\frac{E}{F_y}} = 2.45\sqrt{\frac{29{,}000}{50}} = 59.00$$

Since h/t_w is less than $2.45\sqrt{E/F_y}$, the strength is governed by shear yielding of the web:

$$V_n = 0.6F_y A_w = 0.6F_y(dt_w) = 0.6(50)(14.0)(0.440) = 184.8 \text{ kips}$$

$$\phi_v V_n = 0.90(184.8) = 166 \text{ kips} > 41.6 \text{ kips} \qquad \text{(OK)}$$

The shear design strength is greater than the factored load shear, so the beam is satisfactory. ∎