Combined Bending and Axial Loads

Related Material

AISC LRFD Manual: Part 6
AISC LRFD Specifications: Chapter C, Chapter H

Two New Concepts

Superposition of stresses due to bending and axial loads
Secondary moment due to axial loads; moment amplification

Interaction Equations

- Strength interaction equations relating axial compression P_u to bending moment M_u have been recognized as the practical procedure for design.

- Axial compression strength requirement

  Required axial strength ≤ Design axial strength of the section

  $$P_u \le \phi_c P_n, \text{ or } \frac{P_u}{\phi_c P_n} \le 1$$

- Bending moment strength requirement

  Required bending strength ≤ Design bending strength of the section

  $$M_u \le \phi_b M_n, \text{ or } \frac{M_u}{\phi_b M_n} \le 1$$

- Combined bending and axial compression: Interaction equation

  $$\frac{P_u}{\phi_c P_n} + \frac{M_u}{\phi_b M_n} \le 1$$

LRFD Criteria

- For $\frac{P_u}{\phi_c P_n} \geq 0.2$,

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_u}{\phi_b M_n}\right) \leq 1$$

- For $\frac{P_u}{\phi_c P_n} < 0.2$,

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_u}{\phi_b M_n}\right) \leq 1$$

$$\phi_c = 0.85,\ \phi_b = 0.90$$

$P_u =$ Factored axial compression load

$M_u =$ Factored bending moment (moment magnification used)

$\frac{P_u}{\phi_c P_n} \geq 0.2$, large axial load, bending term is slightly reduced.

$\frac{P_u}{\phi_c P_n} < 0.2$, small axial load, axial load term is reduced.

P_n = Nominal axial strength of the section

M_n = Nominal bending strength of the section

Moment Amplification

- *Beam-column*: the member subjected to axial compression and bending. Axial load induces additional moment, called secondary moment that must be accounted for in design. Problem is nonlinear, requiring second order analysis. AISC permits use of *moment amplification method* or second order analysis.

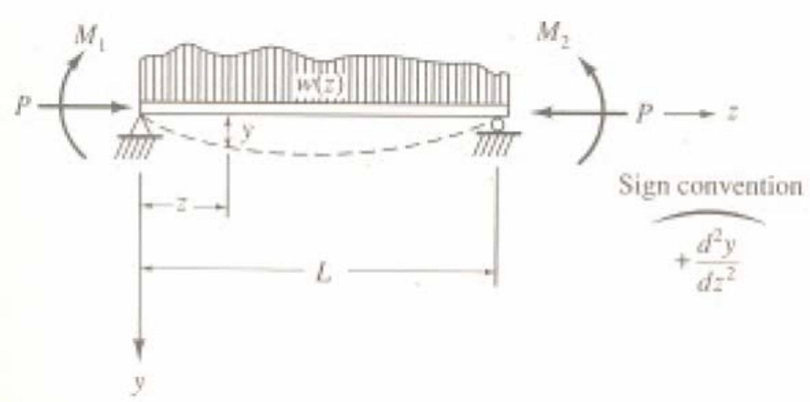

Figure 12.2.1 General loading of beam-column.

- Maximum moment: Note that the coordinate system here is different from the one used earlier.

$$M_z = M_i + Py = -EI\frac{d^2y}{dz^2}$$

Solution:

-- (1) If $w(z) = 0$ and $M_1 = M_2 = M$,

$$M_{z\max} = M\sec\frac{kL}{2}, \text{ where } k = \sqrt{P/EI}$$

-- (2) If $w(z) = w$ and $M_1 = M_2 = 0$,

$$M_{z\,max} = \frac{wL^2}{8}\left(\frac{8}{(kL)^2}\right)\left(sec\frac{kL}{2} - 1\right)$$

It can be shown that

$$M_{max} = M_0\left(\frac{1}{1-\alpha}\right); \;\; \alpha = \frac{P_u}{P_e}$$

Moment amplification factor

$$\left(\frac{1}{1-\alpha}\right); \ \alpha = \frac{P_u}{P_e}, \qquad P_e = \frac{\pi^2 E A_g}{(KL/r)^2}$$

- **Two Types of Amplification Factors – Braced vs Unbraced Frames**

1. The first one accounts for only member deflection; side sway is prevented.
2. The second one accounts for the effect of sway when the member is part of an unbraced frame.

The factored moment is given as

$$M_u = B_1 M_{nt} + B_2 M_{lt} \qquad \text{(AISC Equation C1-1)}$$

$M_{nt} =$ maximum moment assuming that no side sway occurs, whether the frame is actually braced or not (subscript nt is for "no translation")

$M_{lt} =$ maximum moment caused by side sway (subscript lt is for "lateral translation"). This moment can be caused by lateral loads or by unbalanced gravity loads.

$B_1 =$ amplification factor for the moment occurring in the member when it is braced against side sway.

$B_2 =$ amplification factor for the moment resulting from side sway.

- **Members in Braced Frames**

The amplification factor $1/(1-\alpha)$ is for members braced against side sway. The members can bend in single curvature or reverse-curvature. Thus, the maximum moment in a member depends on distribution of the moment. This distribution is accounted for by a factor C_m applied to the amplification factor. This factor applies only for braced conditions.

$$B_1 = \frac{C_m}{1-\alpha} \geq 1;\ \alpha = \frac{P_u}{P_{e1}};\ P_{e1} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 EA_g}{(KL/r)^2}$$

Evaluation of C_m

1. When no transverse loads are acting on the member

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) \qquad \text{(AISC Equation C1-2)}$$

where M_1/M_2 is the ratio of the bending moments at the ends; M_1 is smaller in absolute value. The ratio is ***positive*** for members bent in reverse curvature and ***negative*** for single curvature.

2. For transversely loaded members, C_m can be taken as 0.85 if the ends are restrained against rotation and 1.0 if they are unrestrained (pinned; the amplification factor was derived for this condition)

- **Members in Unbraced Frames**

Maximum secondary moment is always at the end, and the primary and secondary moments are additive. Therefore, there is no need for C_m; i.e., C_m = 1.

Amplification Factor for Side sway Moments

$$B_2 = \frac{1}{1 - \frac{\left(\sum P_u\right)}{\left(\sum H\right)}\left(\frac{\Delta_{oh}}{L}\right)} \quad \text{(AISC Equation C1-4)}$$

Or

$$B_2 = \frac{1}{1 - \left(\frac{\sum P_u}{\sum P_{e2}}\right)} \quad \text{(AISC Equation C1-5)}$$

where

$\sum P_u =$ sum of factored loads on all columns in the story under consideration

$\Delta_{oh} =$ drift (side sway displacement) of the story under consideration

$\sum H =$ sum of all horizontal forces causing Δ_{oh}

$L =$ story height

$\sum P_{e2} =$ sum of the Euler loads for all columns in the story (when computing P_{e2}, use KL/r for the axis of bending and a value of K corresponding to the unbraced conditions)

- Aminmansour (2000) developed a simplified design procedure, which was later adopted by LRFD. Ref: *Engineering Journal*, AISC 37, No. 2, 41-72.

- Refer to Pages 6-10 and 6-11 of the AISC Manual for more details.

$$\begin{cases} bP_u + mM_u \le 1.0, & P_u/(\phi_c P_n) \ge 0.2 \\ bP_u/2 + 9mM_u/8 \le 1.0, & P_u/(\phi_c P_n) < 0.2 \end{cases}$$

where $b = \dfrac{1}{\phi_c P_n}$, $\quad m = \dfrac{8}{9(\phi_b M_n)}$ which can be obtained from AISC-LRFD Manual Table 6.1 and Table 6.2 for all W-shape steels.

Procedure:

1. Select an average b value from Table 6.1
2. Solve for m from the AISC interaction equation
3. Select a shape from Table 6.2 that has values of b and m close to those needed.
4. Use the values of b and m for the shape selected to check the AISC interaction equation.