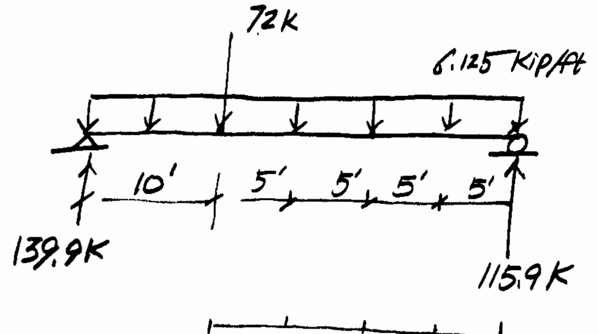
Soln: with the self-weight of the beam:

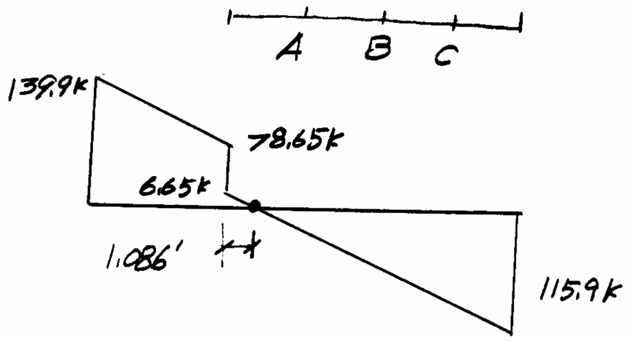
$W_u = 1.2 W_D + 1.6 W_L = 1.2(1+0.104) + 1.6(3) = 6.125$ kip/ft

$P_u = 1.2 P_D + 1.6 P_L = 1.2(12) + 1.6(36) = 72.0$ kips

($1.4 W_D$ & $1.4 P_D$ do not control)



the points for use in the C_b formula



shear force diagram

$M_u = 139.9(11.086) - 6.125(11.086)^2/2 - 72.0(1.086) = 1096$ ft-kips (shear force = 0)

Verify that the shape is compact:

flange: $\lambda_p = 0.38\sqrt{\frac{29000}{50}} = 9.152$ $\quad \lambda = \frac{b_f}{2t_f} = 8.50 < \lambda_p$ compact

Web: $\lambda_p = 3.76\sqrt{\frac{29000}{50}} = 90.55$ $\quad \lambda = \frac{h}{t_w} = 43.1 < \lambda_p$ compact

So the shape is compact. No need to consider web or flange buckling.

For a W24x104, $A = 30.6\ in^2$, $S_x = 258\ in^3$, $Z_x = 289\ in^3$

$I_y = 259\ in^4$, $r_y = 2.91\ in$, $J = 4.72\ in^4$, $C_w = 35{,}200\ in^6$

$L_p = 1.76\, r_y\sqrt{\frac{E}{F_y}} = 1.76 \cdot 2.91\sqrt{\frac{29000}{50}} = 123.3\ in = 10.28\ ft$

$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}} = \frac{\pi}{258}\sqrt{\frac{29000 \cdot 11200 \cdot 4.72 \cdot 30.6}{2}} = 1865$ ksi

$X_2 = \frac{4C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2 = \frac{4(35{,}200)}{259}\left(\frac{258}{11200 \cdot 4.72}\right)^2 = 1.295 * 10^{-2}/ksi^2$

$$L_r = \frac{r_y X_1}{(F_y - F_r)} \sqrt{1 + \sqrt{1 + X_2 (F_y - F_r)^2}}$$

$$= \frac{2.91 \cdot 1865}{50 - 10} \sqrt{1 + \sqrt{1 + 0.01295 \cdot (50-10)^2}} = 322.8 \text{ in} = 26.90 \text{ ft}$$

For $L_b = 20$ ft $L_p < L_b < L_r$ so inelastic LTB

$$M_n = C_b \left[M_p - (M_p - M_r) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \le M_p$$

compute C_b first.

$M_{max} = M_u = 1096$ ft-kips

$M_A = 115.9\ (15) - 6.125\ (15)^2/2 = 1049$ ft-kips

$M_B = 115.9\ (10) - 6.125\ (10)^2/2 = 852.8$ ft-kips

$M_C = 115.9\ (5) - 6.125\ (5)^2/2 = 502.9$ ft-kips

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C}$$

$$= \frac{12.5 * 1096}{2.5 * 1096 + 3 * 1049 + 4 * 852.8 + 3 * 502.9} = 1.268$$

$M_p = F_y Z_x = 50 * 289 = 14450$ in.-kips

$M_r = (F_y - F_r) S_x = (50-10)(258) = 10320$ in.-kips

$$M_n = 1.268 \left[14450 - (14450 - 10320) \left(\frac{20 - 10.28}{26.90 - 10.28} \right) \right]$$

$= 15260$ in-kips $> M_p = 14450$ in.-kips

So use $M_n = M_p = 14450$ in.-kips

$\phi_b M_n = 0.9\ (14450) = 13010$ in.-kips $= 1080$ ft-kips

Since $M_u = 1096$ ft-kips $> \phi_b M_n = 1080$ ft-kips

W24 x 104 is not adequate. ✱

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HW2

Soln:

Because the beam has continuous lateral support, LTB does not need to be checked.

For L.B., first determine the shape classification,

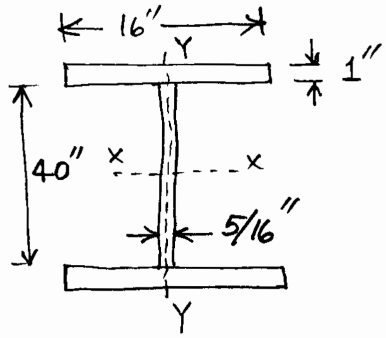
flange, $\lambda = \frac{b_f}{2t_f} = \frac{16}{2*1} = 8.0 \qquad \lambda_p = 0.38\sqrt{\frac{29000}{50}} = 9.152$

$\lambda < \lambda_p$ compact.

Web $\quad \lambda = \frac{h}{t_w} = \frac{40}{5/16} = 128.0 \qquad \lambda_p = 3.76\sqrt{\frac{29000}{50}} = 90.55$

$\lambda_r = 5.70\sqrt{\frac{29000}{50}} = 137.3$

$\lambda_p < \lambda < \lambda_r$ non-compact



Therefore, W.L.B. must be investigated.

Compute the properties of the cross section

flange area $= 1.0*16 = 16.0\ in^2$, Half-web area $= (\frac{5}{16})(\frac{40}{2}) = 6.25\ in^2$

Taking moments about the PNA (mid-depth)

$$\bar{y} = \frac{16.0(20.5) + 6.25(10)}{16.0 + 6.25} = 17.55\ in$$

$$Z_x = \frac{A}{2} \cdot a = \frac{A}{2} \cdot 2\bar{y} = (16.0 + 6.25)(2*17.55) = 781.0\ in^3$$

$$M_p = F_y Z_x = 50\,(781.0) = 39050\ in\text{-}kips$$

(Note: It is also ok to compute M_p based on definition: F_y everywhere on the cross section, that is

$$M_p = 2*50*(16 \times 20.5 + 6.25 \times 10) = 39050\ in\text{-}kips$$)

$$I_x = \frac{1}{12}\left(\frac{5}{16}\right)(40)^3 + 2\left[\frac{1}{12}(16)(1.0)^3 + 16.0\cdot(20.5)^2\right]$$
$$= 15120 \text{ in}^3$$

$$S_x = \frac{I_x}{c} = \frac{15120}{\frac{40}{2}+1.0} = 720.0 \text{ in}^3$$

(Note that $S_x = S_{x1} + S_{x2} + \cdots$ is not applicable)

$$M_r = (F_y - F_r)S_x = (50-16.5)(720.0) = 24120 \text{ in.-kips}$$

$$M_n = M_p - (M_p - M_r)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$
$$= 39050 - (39050 - 24120)\left(\frac{128.0 - 90.55}{137.3 - 90.55}\right)$$
$$= 27090 \text{ in.-kips} < M_p$$

$$\phi_b M_n = 0.9\,(27090) = 24{,}380 \text{ in.-kips} = 2030 \text{ ft-kips}$$

so $\phi_b M_n = 2030$ ft-kips.

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HW3

If the column table (4-21 ~ 4-151) & beam design chart (5-71 ~ 5-102) are not used, the equations in the manual can be consulted.

Soln: flange $\lambda = \frac{b_f}{2t_f} = 9.34 < \lambda_r = 0.56\sqrt{\frac{E}{F_y}} = 13.5$ not slender

Web $\lambda = \frac{h}{t_w} = 23.5 < \lambda_r = 1.49\sqrt{\frac{E}{F_y}} = 35.9$ not slender.

$$\max \frac{KL}{r} = \frac{K_yL}{r_y} = \frac{1.0(14)*12}{3.71} = 45.283$$

$$\frac{K_xL}{r_x} = \frac{0.9(14)*12}{6.17} = 24.51 < \frac{K_yL}{r_y}$$

$$\lambda_c = \frac{K_yL}{r_y\pi}\sqrt{\frac{F_y}{E}} = \frac{45.283}{3.14}\sqrt{\frac{50}{29000}} = 0.599 < 1.5$$

Use AISC Eq E 2-2

$$F_{cr} = 0.658^{\lambda_c^2}F_y = 0.658^{(0.599)^2}(50) = 43.03 \text{ ksi}$$

$$\phi_c P_n = 0.85 A_g F_{cr} = 0.85 * 43.03 * 29.1 = 1064.3 \text{ kips}$$ (1060 from Table) P4-23

$$\frac{P_u}{\phi_c P_n} = \frac{500}{1064.3} = 0.47 > 0.2$$ So Use AISC Eq H1-1a.

For the axis of bending, $C_m = 0.85$ (since $K_x = 0.9$, ends are restrained)

$$P_{e1} = \frac{\pi^2 E A_g}{(K_xL/r_x)^2} = \frac{\pi^2(29000)(29.1)}{(24.51)^2} = 1.386 * 10^4 \text{ kips.}$$

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} = \frac{0.85}{1 - \frac{500}{13860}} = 0.882 < 1.0$$ Use $B_1 = 1.0$

$$M_u = B_1 M_{nt} + B_2 M_{lt} = 1.0(360) + 0 = 360.0 \text{ ft-kips}$$

<u>plastic</u>: $M_p = F_y Z = \frac{50 * 173}{12} = 720.8$ ft-kips

$\phi_b M_p = 0.9 * 720.8 = 648.75$ ft-kips

$M_r = (F_y - F_r) S_x = \frac{40 * 157}{12} = 523.3$ ft-kips

<u>LTB</u>: Check buckling condition

$$L_p = 1.76\, r_y \sqrt{\frac{E}{F_y}} = \frac{1.76 * 3.71}{12} \sqrt{\frac{29000}{50}} = 13.1 \text{ ft}$$

$$X_1 = \frac{\pi}{S_x} \sqrt{\frac{EGJA}{2}} = \frac{3.14}{157} \sqrt{\frac{29000 \cdot 11200 \cdot 29.1 \cdot 5.37}{2}} = 3186.1 \text{ ksi}$$

$$X_2 = \frac{4 C_w}{I_y} \left(\frac{S_x}{GJ}\right)^2 = \frac{4 \cdot 18000}{402} \left(\frac{157}{11200 \cdot 5.37}\right)^2 = 0.00122 \ 1/\text{ksi}^2$$

$$L_r = \frac{r_y X_1}{F_y - F_r} \sqrt{1 + \sqrt{1 + X_2 (F_y - F_r)^2}} = \frac{3.71 * 3186.1}{40} \sqrt{1 + \sqrt{1 + 0.00122 \cdot 40^2}}$$

$$= 487.22 \text{ in.} = 40.6 \text{ ft}$$

$L_p\ (13.1) < L_b\ (14) < L_r\ (40.6)$, Inelastic LTB.

$$M_n = C_b \left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

$$= 1.6 \left[720.8 - (720.8 - 523.3)\left(\frac{14 - 13.1}{40.6 - 13.1}\right)\right]$$

$$= 1142.88 \text{ ft-kips}$$

$\phi_b M_n = 0.9 * 1142.88 = 1028.6$ ft-kips $> \phi_b M_p$

LB:

Flange: $\lambda = \frac{b_f}{2t_f} = 9.34$ $\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 9.152$

$\lambda_r = 0.83\sqrt{\frac{E}{F_L}} = 22.35$

$\lambda_p < \lambda < \lambda_r$ flange not compact

need to check F.L.B.

$\frac{P_u}{\phi_b P_y} = \frac{500}{0.9(50)(29.1)} = 0.382 > 0.125$

Web: $\lambda = \frac{h}{t_w} = 23.5$, $\lambda_p = 1.12\sqrt{\frac{E}{F_y}}\left(2.33 - \frac{P_u}{\phi_b P_y}\right) \geq 1.49\sqrt{\frac{E}{F_y}}$

$= 1.49 * 24$

so $\lambda < \lambda_p$ Web is compact

no need for W.L.B.

F.L.B.: $M_n = M_p - (M_p - M_r)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$

$= 720.8 - (720.8 - 523.3)\left(\frac{9.34 - 9.152}{22.35 - 9.152}\right)$

$= 718$ ft-kips

$\phi_b M_n = 0.9 * 718 = 646.2$ ft-kips $\leq \phi_b M_p = 648.75$ ft-kips

so $\phi_b M_n = 646.2$ ft-kips (646 ft-kips from chart P5-88)

F.L.B. controls

AISC Eq. H1-1a:

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right)$$

$$= 0.47 + \frac{8}{9}\left(\frac{360}{646} + 0\right) = 0.965 < 1 \quad (OK)$$

the Member is satisfactory.

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