Rajan's Book Chapter 4: Computation of Deflections

Deflection of truss, beam and frame systems

Moment Area Method for Beams

1. The *change* in angle between two arbitrary points A and B on the elastic curve is equal to the area under the M/EI diagram between those points.
2. The deflection of point B on the elastic curve with respect to the *tangent* through point A on the elastic curve is equal to the moment of the area under the M/EI diagram between those two points taken about B.

Conjugate Beam Method for Beams

1. The slope θ at a point in the real beam is given by the shear V at the corresponding point in the conjugate beam.
2. The displacement Δ of a point in the real beam is given by the bending moment M at the corresponding point in the conjugate beam.

Energy Principles

For a conservative static system, total potential energy is given as

$$\Pi = U + V$$

U = strain energy; V = potential energy of external loads

Principle of stationary potential energy: $\delta\Pi = \delta U + \delta V = 0$

Consevation of energy: First law of thermodynamics

Work done = Change in energy

$$W_e = U; \quad W_e + W_i = 0$$

Axial force element: If a force of N is applied slowly resulting in the final displacement as Δ, the work done by the force is given as

$$W_e = \tfrac{1}{2} N\Delta$$

Beam moment: If M is the final moment and θ is the final rotation, then the work done is given as

$$W_e = \tfrac{1}{2} M\theta$$

Strain Energy

Axial deformation: $U = \dfrac{N^2 L}{2AE}$

Bending deformation: $U = \int_0^L \dfrac{M^2}{2EI} dx$

Shear deformation: $U = \int_0^L \dfrac{KV^2}{2GA} dx;\ K =$ form factor for the cross-sectional shape

Principle of Virtual Work

$$\delta W = \delta W_e + \delta W_i = 0$$

$$\sum_{i=1}^{n} P_i (\delta D)_i = \int_V \sigma(\delta\varepsilon) dV$$

Principle of Virtual Work: A deformable body that is in equilibrium under the action of external loads P if the external virtual work is equal to the internal virtual work due to compatible virtual displacements.

Principle of complementary virtual work:

$$\sum_{i=1}^{n} D_i (\delta P)_i = \int_V \varepsilon(\delta\sigma) dV$$

$$(\delta F)\Delta = \sum (f)\Delta L$$

δF is the virtual load in the direction in which Δ is desired; f is the virtual force, and ΔL is the internal deformation due to actual loads.

Unit Load Method for Beams and Frames

$$(1)(\Delta) = \int_0^L \frac{mM}{EI} dx$$

m = moment due to unit load, M = actual moment

$$(1)(\theta) = \int_0^L \frac{m_\theta M}{EI} dx$$

m_θ = moment due to a unit moment

Unit Load Method for Trusses

$$(1)(\Delta) = \sum en;\ e = \frac{NL}{AE}$$

e = extension of members due actual loads
n = member forces due to unit loads