## Pulse Doppler Radar

Assume a target at a distance $R$ and has a radial velocity component of $V_{r}$. The roundtrip distance to target is $2 R$. This is equivalent to $2 R / \lambda$ wavelengths or $(2 R / \lambda) 2 \pi=4 \pi R / \lambda$ radians. If the phase of the transmitted signal is $\phi_{0}$, then the phase of the received signal is

$$
\phi=\phi_{0}+\frac{4 \pi R}{\lambda}
$$

The change in phase between pulses is $\frac{d \phi}{d t}=\frac{4 \pi}{\lambda} \frac{d R}{d t}=\frac{4 \pi}{\lambda} V_{r}$. The left hand side of the equation is equal to the frequency $2 \pi f_{d}$, so that

$$
\begin{equation*}
2 \pi f_{d}=\frac{4 \pi}{\lambda} V_{r} \Rightarrow f_{d}=\frac{2 V_{r}}{\lambda} \tag{1}
\end{equation*}
$$

Alternatively, let the transmitted frequency be $f_{t}$. The received signal can be represented as $A_{\text {rec }}=K \sin \left(2 \pi f_{t}\left(t-T_{R}\right)\right)$. The round-trip time $T_{R}$ is equal to $2 R / c$. With a radial velocity of $V_{r}$ the round-trip time is changing as $R=R_{0}-V_{r}$ t. Thus, the received signal is

$$
A_{\mathrm{rec}}=K \sin \left(2 \pi f_{t}\left(1+\frac{2 V_{r}}{c}\right) t-\frac{4 \pi f_{t} R_{0}}{c}\right)
$$

Thus, the received frequency changes by a factor $2 f_{t} V_{r} / c=2 V_{r} / \lambda_{\text {. , which }}$ is the same as before. The Nyquist criterion says that $f_{\max }=\mathrm{PRF} / 2$, combining this with Equation (1), results in

$$
\begin{equation*}
V_{\max }=\frac{P R F \lambda}{4} \tag{2}
\end{equation*}
$$

This is the maximum unambiguous velocity. Higher velocities cause velocity folding or velocity aliases.
Maximum unambiguous range is $R_{\max }=\frac{c}{2 P R F}$. This causes range folding or range aliases. Combining this with Equation (1) results in

$$
V_{\max } R_{\max }=\frac{c \lambda}{8}
$$

This summarizes the Doppler dilemma: a large $R_{\max }$ implies a small $V_{\max }$ and vice versa.

## Recognizing and dealing with range an velocity aliases.

- Look outside.
- Examine horizontal and vertical shapes of object. For example range-aliased storms become skinny close-by objects. Storm heights are suspicious convective storms are $10-15 \mathrm{~km}$ tall. Aliased they would be say 2 km tall, which is unrealistic.
- Examine reflectivities in conjunction with other factors,
- Change PRF-real echoes will not change position, but aliases will. This is not alwas an option.
- Velocity folding causes a change in sign, which is relatively easy to spot if the folding is within a larger region.
- Watch out, one can get multiple velocity foldings.


Example 1. A 3-cm radar with PRF of 1000 Hz is pointed towards a storm located 200 km from the radar. The radar display will show an alias at $200 \mathrm{~km}-c /(2 P R F)=50 \mathrm{~km}$. The storm is moving away from the radar with a radial velocity of 25 miles per hour, or $11.1 \mathrm{~m} / \mathrm{s}$. The radar will display the storm velocity as $(P R F \times \lambda) / 4-11.11=7.5-11.1=$ $3.6 \mathrm{~m} / \mathrm{s}$ towards the radar.

Example 2. A scanning radar with a PRF of 1000 Hz observes two identical distributed targets located at $R_{1}=80 \mathrm{~km}$ and another at $R_{2}=210 \mathrm{~km}$ (see below). Sketch and dimension a PPI up to 200 km .


Answer. The unambiguous range for the radar is $c /(2 P R F)=150 \mathrm{~km}$. The first target will not alias, the radar measures $P_{1}$ and $R_{1}$ and its processor outputs a reflectivity factor
$z_{1}=c_{3} P_{1} R_{1}^{2}$. The radars measures a power $P_{2}=P_{1}\left(R_{1} / R_{2}\right)^{2}$ for the second target. However, this target aliases to $R_{a}=210-150=60 \mathrm{~km}$, so the processor outputs

$$
\begin{aligned}
z_{2} & =c_{3} P_{2} R_{a}{ }^{2} \\
& =c_{3} P_{1}\left(R_{1} / R_{2}\right)^{2} R_{a} \\
& =c_{3} P_{1} R_{1}\left(\frac{R_{a}}{R_{2}}\right)^{2}=z_{1}\left(\frac{R_{a}}{R_{2}}\right)^{2}
\end{aligned}
$$

Substituting the numerical values we get $z_{2}=0.082 z_{1}$. Thus, $z 2$ is about -10 dBZ less that $z_{1}$.


## Example 3



