## **Pulse Doppler Radar**

Assume a target at a distance *R* and has a radial velocity component of  $V_r$ . The roundtrip distance to target is 2*R*. This is equivalent to  $2R/\lambda$  wavelengths or  $(2R/\lambda)2\pi = 4\pi R/\lambda$ radians. If the phase of the transmitted signal is  $\phi_0$ , then the phase of the received signal is

$$\phi = \phi_0 + \frac{4\pi R}{\lambda}$$

The change in phase *between* pulses is  $\frac{d\phi}{dt} = \frac{4\pi}{\lambda} \frac{dR}{dt} = \frac{4\pi}{\lambda} V_r$ . The left hand side of the equation is equal to the frequency  $2\pi f_d$ , so that

$$2\pi f_d = \frac{4\pi}{\lambda} V_r \Longrightarrow f_d = \frac{2V_r}{\lambda} \tag{1}$$

Alternatively, let the transmitted frequency be  $f_t$ . The received signal can be represented as  $A_{rec} = K \sin(2\pi f_t(t - T_R))$ . The round-trip time  $T_R$  is equal to 2R/c. With a radial velocity of  $V_r$  the round-trip time is changing as  $R = R_0 - V_r t$ . Thus, the received signal is

$$A_{\rm rec} = K \sin\left(2\pi f_t \left(1 + \frac{2V_r}{c}\right)t - \frac{4\pi f_t R_0}{c}\right)$$

Thus, the received frequency changes by a factor  $2f_t V_r/c = 2V_{r/\lambda}$ , which is the same as before. The Nyquist criterion says that  $f_{\text{max}} = \text{PRF}/2$ , combining this with Equation (1), results in

$$V_{\rm max} = \frac{PRF\lambda}{4} \tag{2}$$

This is the **maximum unambiguous velocity**. Higher velocities cause *velocity folding* or *velocity aliases*.

Maximum **unambiguous range** is  $R_{max} = \frac{c}{2PRF}$ . This causes *range folding* or *range aliases*. Combining this with Equation (1) results in

$$V_{\max}R_{\max} = \frac{c\lambda}{8}$$

This summarizes the Doppler dilemma: a large  $R_{\text{max}}$  implies a small  $V_{\text{max}}$  and vice versa.

## Recognizing and dealing with range an velocity aliases.

- Look outside.
- Examine horizontal and vertical shapes of object. For example range-aliased storms become skinny close-by objects. Storm heights are suspicious convective storms are 10–15 km tall. Aliased they would be say 2 km tall, which is unrealistic.
- Examine reflectivities in conjunction with other factors,
- Change PRF-real echoes will not change position, but aliases will. This is not alwas an option.
- Velocity folding causes a change in sign, which is relatively easy to spot if the folding is within a larger region.
- Watch out, one can get multiple velocity foldings.



**Example 1.** A 3-cm radar with PRF of 1000 Hz is pointed towards a storm located 200 km from the radar. The radar display will show an alias at 200 km - c/(2PRF) = 50 km. The storm is moving *away* from the radar with a radial velocity of 25 miles per hour, or 11.1 m/s. The radar will display the storm velocity as  $(PRF \times \lambda)/4 - 11.11 = 7.5 - 11.1 = 3.6$  m/s *towards* the radar.

*Example 2.* A scanning radar with a PRF of 1000 Hz observes two identical distributed targets located at  $R_1 = 80$  km and another at  $R_2 = 210$  km (see below). Sketch and dimension a PPI up to 200 km.



**Answer**. The unambiguous range for the radar is c/(2PRF) = 150 km. The first target will not alias, the radar measures  $P_1$  and  $R_1$  and its processor outputs a reflectivity factor

 $z_1 = c_3 P_1 R_1^2$ . The radars measures a power  $P_2 = P_1 (R_1/R_2)^2$  for the second target. However, this target aliases to  $R_a = 210-150 = 60$  km, so the processor outputs

$$z_{2} = c_{3}P_{2}R_{a}^{2}$$
  
=  $c_{3}P_{1}(R_{1} / R_{2})^{2}R_{a}$   
=  $c_{3}P_{1}R_{1}\left(\frac{R_{a}}{R_{2}}\right)^{2} = z_{1}\left(\frac{R_{a}}{R_{2}}\right)^{2}$ 

Substituting the numerical values we get  $z_2 = 0.082z_1$ . Thus,  $z_2$  is about -10 dBZ less that  $z_1$ .



## Example 3



Reflectivity (dBZ)

Velocity (m/s)