

Midterm Answers

Question 1. A coupling $C = 40$ dB is equal to a ratio of 10^4 . Even though the sign is positive, it is implied that this ratio is an attenuation. This was pointed out in class. Thus, of the $10 \text{ kW} = 10^4 \text{ W}$ flowing into the coupler, $10^4/10^4 = 1 \text{ W}$ flows out P_f . The rest ($10^4 - 1 = 9,999 \text{ W}$) flows out P_o . (4 points total)

Question 2. The circulator acts as a transmit/receive switch. When transmitting, it directs the high amplitude transmit pulse out to the antenna and keep the receiver isolated and protected. It directs the received energy into the receiver. Key concepts, T/R switch, protection. (2 points total)

Question 3. The power flowing out C can only come from B, and A is not in the picture. Thus, $P_A =$ don't know, but is an error to say $P_A = 0$. $P_B = 15 \text{ W}$. (2 points total)

Question 4. Estimate dish diameter as ~ 8 feet ~ 2.5 meter. S-band corresponds to 2–4 GHz, or 15–7.5 cm wavelength. Pick a wavelength of 10 cm or 0.1 m. For a parabolic antenna:

$$\theta \approx \frac{70\lambda}{D} = \frac{70 \times 0.1}{2.5} = 2.8^\circ$$

Now 2.8° is equal to 0.059 radians, and the gain is then

$$G = \frac{\pi^2}{\theta^2} = 4.13 \times 10^3 = 36.2 \text{ dB}$$

Key points: Estimate diameter (1 point), estimate wavelength (1 point), compute beamwidth (2 points), compute gain (2 point). (6 points total)

Question 5. Use the radar equation for point targets to determine the target area

$$P_r(R_1) = \frac{G^2 \lambda^2 P_t \sigma}{64\pi^3 R_1^4} \Rightarrow \sigma = P_r(R_1) \frac{64\pi^3 R_1^4}{G^2 \lambda^2 P_t} \text{ m}^2$$

where $R_1 = 110 \text{ km}$. Use the radar equation for point targets to determine the received power at $R_2 = 180 \text{ km}$:

$$\begin{aligned}
P_r(R_2) &= \frac{G^2 \lambda^2 P_t \sigma}{64\pi^3 R_2^4} = \frac{G^2 \lambda^2 P_t}{64\pi^3 R_2^4} \cdot P_r(R_1) \frac{64\pi^3 R_1^4}{G^2 \lambda^2 P_t} \\
&= P_r(R_1) \frac{R_1^4}{R_2^4} \\
&= 10^{-14} \left(\frac{110}{180} \right)^4
\end{aligned}$$

This is the received power from one target. The received power from two point targets is twice this, or 2.78×10^{-15} W. Key Points: one can express the received power as a ratio of two distances (2 points), add power from point targets (1 point), correct computation (1 point). (4 points total)

Question 6. Use the various equations from the “cheat sheet”:

(a) $\theta \approx \frac{70\lambda}{D} = \frac{70 \times 0.107}{8.23} = 0.91^\circ = 16 \times 10^{-3} \text{ rad}$

(b) $\Delta R = \frac{c\tau}{2} = \frac{3 \times 10^8 \cdot 0.8 \times 10^{-6}}{2} = 120 \text{ m}$

(c) Dead zone = 120 m

(d) Assume symmetric antenna. The sample volume is then

$$\begin{aligned}
V &= \frac{\pi R^2 \theta^2 h}{8} \text{ where } h = c\tau. \\
V &= \frac{\pi R^2 \theta^2 h}{8} \\
&= \frac{\pi \cdot (3 \times 10^4)^2 (16 \times 10^{-3})^2 (3 \times 10^8)(0.8 \times 10^{-6})}{8} \\
&= 21.4 \times 10^6 \text{ m}^3
\end{aligned}$$

If one assumes symmetric, Gaussian beam, the sample volume is half this. Points: (a)–(c) 1 point, (d) state assumption and proper formula: 1 point, proper calculation: 2 points. (6 points total)

Question 7.

(a)

$$\begin{aligned}\text{Drop} &= \int_{0.2}^6 N(D)dD = \int_{0.2}^6 701e^{-1.75D} dD \\ &= 701 \left[\frac{e^{-1.75D}}{-1.75} \right]_{0.2}^6 \\ &\approx 400e^{-1.75 \times 0.2} = 282 \text{ drops/m}^3\end{aligned}$$

(b)

$$\begin{aligned}Z &= \int_{0.2}^6 D^6 N(D)dD \\ &= \int_{0.2}^6 D^6 e^{-1.75D} dD\end{aligned}$$

Numerically integrate using $\Delta D = 1.8 \text{ mm}$

$$\begin{aligned}Z &\approx 701((0.2)^6 e^{-1.75 \times 0.2} + (2)^6 e^{-1.75 \times 2} + (3.8)^6 e^{-1.75 \times 3.8} + (5.6)^6 e^{-1.75 \times 5.6}) \times 1.8 \\ &= 701(45.1 \times 10^{-6} + 1.933 + 3.8796 + 1.71) \times 1.8 \\ &= 9513 \text{ m}^3\end{aligned}$$

Alternatively, integrate analytically (integrate by parts). Key points: correct equation: 1 point each (a) and (b), clearly showing computation: 2 points each. It is NOT acceptable just to put answer down. (6 points total)

Question 8: One possibility is $9.4 + 0.06 = 9.46 \text{ GHz}$, the other possibility if $9.4 - 0.06 = 9.34 \text{ GHz}$. (1 point total)

Question 9. Method 1: measure gain/loss of each individual component and then do a budget. Method 2: use a target with known radar cross section, placed at a known distance (tether a calibration sphere, corner reflector). Method 2: measure drop size distribution of rain, compute Z , and compare with what radar measures. Method 3: point radar at sun, and measure received power. Then compare with other independent measurements from sun. (3 points total)

Question 10. Matched filters are used to suppress as much noise as possible. Averaging is used to improve signal-to-noise ratio: in the long run, random noise averages to zero. Another technique is coding the transmitted signal to make it distinctive from noise: chirp signal. Gating can be used, waiting for the signal in a certain time window. (3 points total).