Z, Z and Z-R Relationships

$$P_r = \left(\frac{G^2 \lambda^2 P_t \theta \phi c \tau}{1024 \ln(2) \pi^2} \right) \frac{\eta}{R^2} = C \frac{\eta}{R^2}$$

Radar equation for distributed targets

$$\eta = \frac{\pi^5 |\kappa|^2}{\lambda^4} z$$

Radar reflectivity

$$z = \sum_{\textit{Unit Volume}} D^6$$

Radar reflectivity factor (mm^6/mm^3)

Combining these equations, we can write: $P_r = c_2 \frac{z}{R^2}$, and rearrange to get:

$$z = c_3 P_r R^2$$

c_3 is the so-called *radar constant*. It has units $mm^6/m^3\ mW^{-1}\ km^{-2}$. The radar reflectivity factor z has a tremendous dynamic range so it is convenient to express it on a decibel scale with a reference $z = 1\ mm^6/m^3$

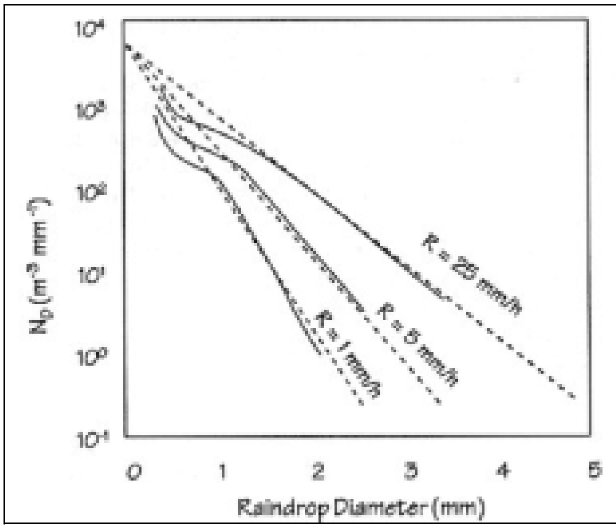
$$Z = 10 \log_{10} \left(\frac{z\ (\text{mm}^6/\text{m}^3)}{1\ (\text{mm}^6/\text{m}^3)} \right) \text{ with units dBZ}$$

Using this, we can write

$$z = c_3 P_r R^2$$
$$Z = C_3 + P_r + 20 \log_{10}(R)$$

where Z is measured in dBZ, C3 = $10\log_{10}(c_3)$, Pr is measured in dBm, and R is in km.

Marshall-Palmer DSD.

<table>
<tr><td rowspan="2">
</td><td>$N_D = N_o e^{-\lambda D}$
$\lambda = 4.1 R^{-0.21}$</td></tr>
<tr><td>N_o = 8000 $m^{-3}\ mm^{-1}$
R = rainrate in mm/h
D = drop diameter in mm</td></tr>
</table>

Given a DSD one can compute a Z-R relationship. In practice, empirical relationships are used:

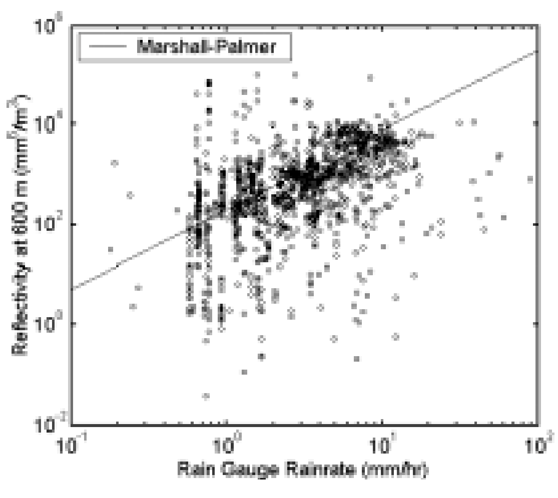
$$Z = AR^{b}$$

$$Z = 200R^{1.6} \quad \text{Marshall - Palmer or MP relationship}$$

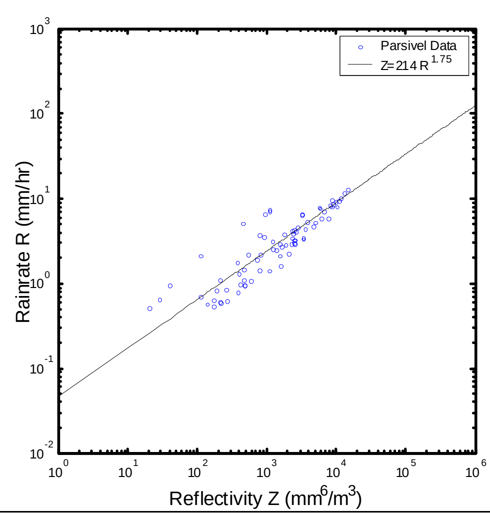
There are many relationship in use. The table below is from the NOAA ROC:

Table 1. Z-R RECOMMENDATIONS		
RELATIONSHIP	**Optimum for:**	**Also recommended for:**
Marshall-Palmer ($z=200R^{1.6}$)	General stratiform precipitation	
East-Cool Stratiform ($z=130R^{2.0}$)	Winter stratiform precipitation - east of continental divide	Orographic rain - East
West-Cool Stratiform ($z=75R^{2.0}$)	Winter stratiform precipitation - west of continental divide	Orographic rain - West
WSR-88D Convective ($z=300R^{1.4}$)	Summer deep convection	Other non-tropical convection
Rosenfeld Tropical ($z=250R^{1.2}$)	Tropical convective systems	

Sample *Z-R* relationships measured in Iowa (note the X-Y axis are switched between the graphs).



Sample: 1-minute integration interval



Sample: 5-minute integration interval.