General Antenna Formulas

G = Antenna Gain

λ = Wavelength (m)

$$A_e = \frac{G\lambda^2}{4\pi} \qquad \text{Antenna Effective Area (m}^2\text{)}$$

$$G = \frac{\pi^2 k^2}{\theta\phi} \qquad \theta, \phi \text{ are H and V beamwidths (radians), } k \text{ antenna factor}$$

$$G = \frac{\pi^2}{\theta^2} \qquad \text{For circular antennas, } \theta \text{ beamwidth (radians)}$$

Parabolic Dish Antennas

$$\theta = \frac{70\lambda}{D} \qquad \text{Beamwidth in degrees, } D = \text{dish diameter (m)}$$

$$\theta = \frac{\lambda}{D} \qquad \text{Beamwidth in radians}$$

Note that if we take the second formula and change it so the angle is measured in radians, then

$$\frac{2\pi}{360}\theta = \frac{\lambda}{D} \qquad \text{Beamwidth now in degrees}$$

$$= 57.1\frac{\lambda}{D} \approx \frac{60\lambda}{D}$$

This formula is also encountered in the literature.

Examples

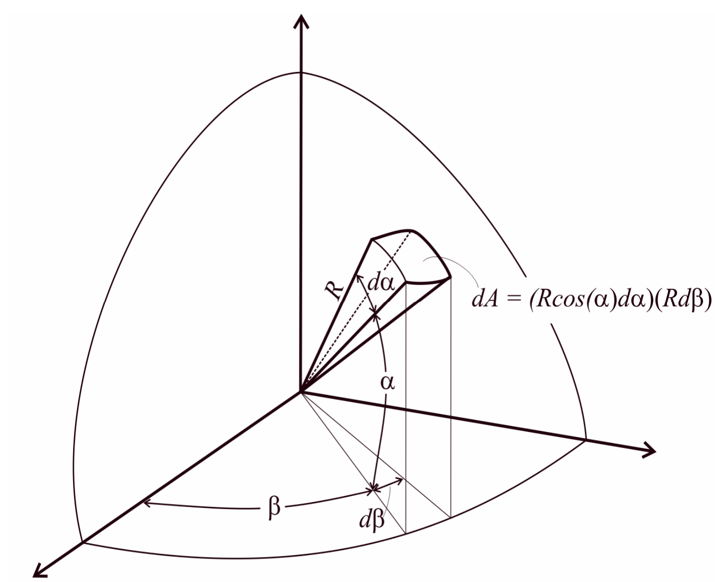
Problem 1

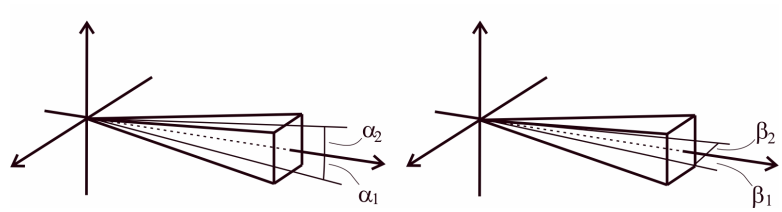
Show that the gain of an antenna with beam widths θ and ϕ

$$G = \frac{4\pi}{\phi\theta}, \quad \theta \text{ and } \theta \text{ measured in radians}$$

Approximate the antenna radiation pattern with a rectangle and assume the beam widths are small.

Solution





Without loss of generality, choose the coordinate system so that the beam is symmetrical with the *x*-axis. Consider a small patch on the surface on a sphere with radius *R*. The area of this patch is

$$dA = R\cos\alpha d\alpha \cdot Rd\beta$$

The area on the surface between angles α_1, α_2 and β_1, β_2 is

$$A = \int_{\alpha_1}^{\alpha_2} \int_{\beta_1}^{\beta_2} R \cos\alpha d\alpha \cdot R d\beta$$
$$= R^2 (\sin\alpha_2 - \sin\alpha_1)(\beta_2 - \beta_1)$$

Now set $\alpha_1 = -\theta/2$, $\alpha_2 = \theta/2$, and $\beta_1 = -\phi/2$, $\beta_2 = \phi/2$, and substitute into the equation above to find

$$A = R^2 (2\sin(\theta/2)) \cdot (2(\phi/2))$$

Assume the angles are small so $\sin(\theta/2) \approx \theta/2$, and we have

$$A = R^2 \theta\phi$$

The energy density at a distance R for an isotropic antenna is $4\pi R^2$ W/m^2. Consequently, the gain of the antenna is

$$G = \frac{4\pi R^2}{R^2 \theta\phi} = \frac{4\pi}{\theta\phi}$$

Problem 2

In the literature one encounters formulas for the gain of antennas that have similar form, but have different numerical values. For example one formula is:

$$G = \frac{4\pi}{\theta\phi}$$

so if one sets the beam widths $\theta = \phi$ then the gain for a circular antenna is

$$G = \frac{4\pi}{\theta^2} = \frac{12.57}{\theta^2}$$

However, some texts give the following expression for the gain of a circular antenna

$$G = \frac{\pi^2}{\theta^2} = \frac{9.89}{\theta^2}$$

Explain.

Solution

The first formula, i.e., $G = \dfrac{4\pi}{\theta\phi}$ assumes the so-called rectangular model where the energy is radiated into a pyramid, while the second assumes an elliptical model where the energy is radiated into a conic section. These and other assumptions about the antenna lead to different formulas. The key, however, is that they have the same form and that the gain is inversely proportional to the product of the beam widths.

Problem 3

What is the 3 dB beamwidth of an 8-foot parabolic dish antenna at 1.3 GHz?

Problem 4

The manufacturer says its 8-foot parabolic disk antenna has a 3.6° beamwidth at 2.5 GHz. (a) What is the gain of the antenna at 2.5 GHz? (b) What is the beamwidth at 5.5 GHz? (c) What is the gain at 5.5 GHz?

Problem 5

The beam width of a WSR-88D (NEXRAD) radar is 0.95 degrees. (a) Make an estimate of (parabolic) antenna gain. (b) Estimate the dish diameter. State assumptions you make.

Solution

(a) Assume the antenna is symmetric/circular and ignore losses so

$$G = \frac{4\pi}{\theta^2} = \frac{4\pi}{\left(\frac{2\pi \times 0.95}{360}\right)^2} = 46.6\,\text{dB}$$

The actual value is 45.5 dB.)

The WSR-88Ds operate in the S-band at ~ 3 GHz, so the wavelength is ~ 0.1 m. For a parabolic antenna,

$$\theta = \frac{70\lambda}{D} \quad \text{Beamwidth in degrees, } D = \text{dish diameter (m)}$$

So that D ~ 7.3 m. (The actual value is 9 m).

Problem 6

The manufacturer of the WSD-88D radar states that the antenna has a 9 m disk and the gain is 45.6 dB. Compare this with the values from the previous problem, and write a short explanation (3 sentences) for each of the differences.