22M:034, Engineering Math IV: Differential Equations

Computer Assignment 2: Amplitude modulation, resonance, and damping

10/20/03, due 10/31

The solution of the initial value problem

$$\begin{cases} y'' + v^2 y = E\cos(wt), \\ y(0) = y'(0) = 0 \end{cases}$$

exhibits *amplitude modulation* (AM), the effect being most pronounced when the constant w is chosen to be near to (but not equal to) the constant v. The earmark of amplitude modulation is a high-frequency oscillation taking place "inside" a low-frequency one. When the *rest conditions* $y(0) = y'(0) = 0$ are replaced by other initial conditions, the phenomenon at small times is a little less pronounced – the "bottleneck" the solution must pass through as the low-frequency oscillation passes through equilibrium are more relaxed. (There will typically be large times t_0, though, for which the initial conditions at t_0 are nearly rest conditions, resulting in tight bottlenecks at times near t_0.)

When w tends toward v, we approach *undamped resonance.*

Problem 1 Do a graphical analysis of AM and undamped resonance along the general lines of the attached sample Maple session. Here are the numbers you should use: Suppose your

$$\text{your University number} = 00\text{-}A_3A_4\text{-}A_5A_6\text{-}A_7A_8.$$

For the equation $y'' + v^2 y = E\cos(wt)$, use

$$\begin{aligned} v &= 4 + \frac{A_4}{2}, \\ E &= 7 + \frac{A_5}{4}. \end{aligned} \tag{1}$$

Let w range through various values (near to the value of v) to get the best graphical effect. For the case of non-rest conditions, take

$$y(0) = -A_6, \qquad y'(0) = A_7.$$

You won't automatically get a good picture – you will probably need to tweak some of the tweakable parameters. In the DEplot instruction, the numbers in

$$\mathsf{t} = -10..10$$

$$\mathsf{y} = -2..2$$

$$\mathsf{stepsize} = .05$$

might all be productively changed to something else. This is part of the art of graphically exploring and demonstrating physical phenomena.

Note that, instead of solving the initial value problems in closed form and then graphing, we just asked Maple directly for the graph. This will be a useful technique later, when we have nonlinear equations that *don't have* closed form solutions. In these cases you can't write an algebraic expression for the solution, but you know it's there.

The other Maple session attached demonstrates the transient nature of the effect of initial conditions, when damping is present.

Problem 2 Do your own demonstration of this principle, using the equation

$$y'' + by' + v^2 y = E\cos(vt),$$

using your numbers v and E from (1) above, and using

$$b = \frac{A_8 + 5}{10}.$$