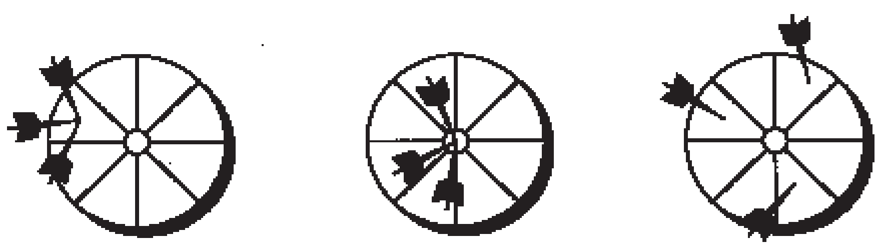
Appendix A: Determining Accuracy and Precision of Measurements: Using Calibration to Estimate Bias and Precision Errors

The write-up below is a guide for determining bias and precision errors in your measurements. In has two parts: Part A deals with calibration of measurements relative to a local standard; here you establish the accuracy of your measurements relative to the standard. Then Part B the total error of measurements are determined, in which the total instrument error established during calibration is combined with the errors determined from the measurements taken.

A) Determining Bias and Precision Errors During Calibration

When calibrating a sensor, you establish the total measurement error which is composed of bias errors and precision errors:

Precision errors are determined based on statistical data, and reflect errors due to random variations in repeated measurements. For example, precision errors can be due to variations in spatial location of a sensor or temporal variations. A low precision error means your repeated measurements fall close together as in the dart board analogy Figure a) shown below taken from Figliola et al., *Theory and Design for Mechanical Measurements*. Note that a small precision error does not necessarily mean your measurements have high accuracy, since they may fall close together but far from the true value.



(a) High repeatability gives low precision error but no direct indication of accuracy

(b) High accuracy means low precision and bias errors

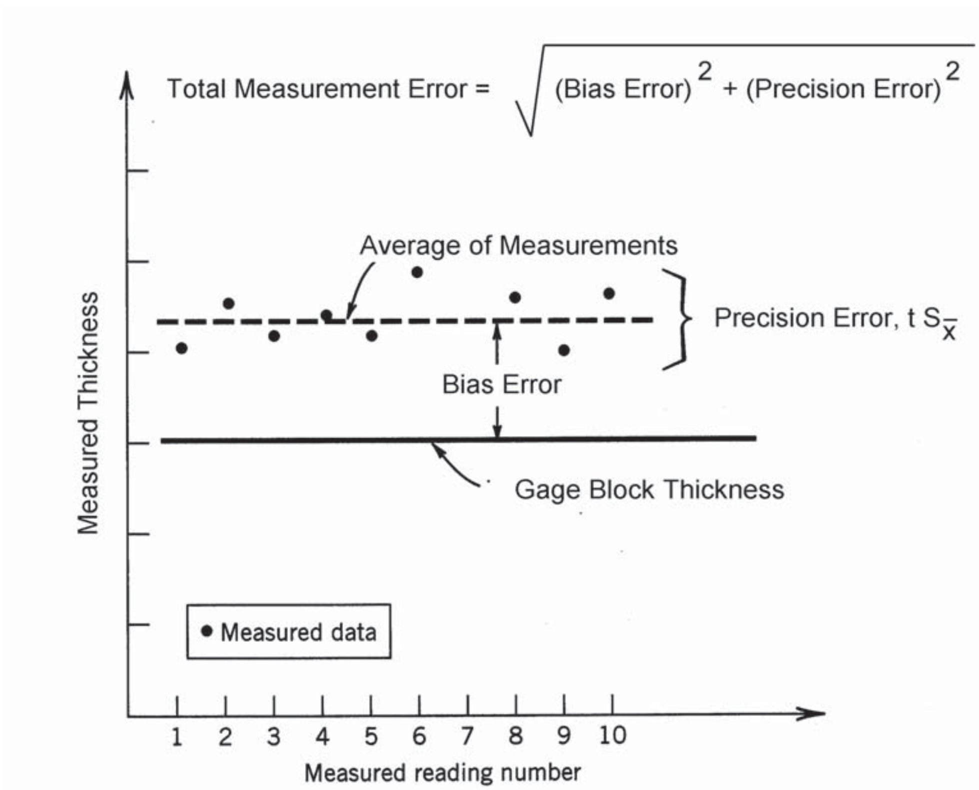
(c) Bias and precision errors lead to poor accuracy

Figure Throws of a dart: illustration of precision and bias errors and accuracy.

Bias errors are errors which have no statistical data associated with them. Resolution error (the so-called zero-order error) is the most obvious bias error; this is stated as +/- 1/2 the resolution of your sensor. Also, during calibration by comparison to a standard, you establish the bias error of your measurement with respect to a standard. This could be established by determining the difference between the mean of your sensor output and the value of the standard you are using for calibration. Another important bias error is the accuracy of your standard. In an absolute sense, the total error of your sensor cannot be less than the error of your standard used in calibrating your sensor. Considering the dart board analogy shown above, Figure a) demonstrates the even thought the precision error is small the bias error maybe large and the accuracy is unknown without a standard for comparison. High accuracy (Figure b) implies that both bias and precision errors are small. In Figure c) bias and precision errors are large and the accuracy is poor.

In the example that follows below, a gage block is used to calibrate the measurements of a micrometer. A gage block is a calibration standard for thickness measurement, it is a block of material, steel in this case, machined to a very high accuracy of thickness. In this case a Federal Grade 2 accuracy gage block is used; it meets certain tolerances for length, flatness and parallelism. The calibrated accuracy of the micrometer can be no greater than the total error of this standard accuracy.

As indicated in the figure below, the total error from the calibration will be the root sum of the squares (RSS) of the bias and precision errors. In the figure, the scatter of ten measured data points lies above the gage block thickness (the true thickness, or true value of the standard). In this figure, the only bias error shown is the average measurement bias errors (it is not the only one considered in the analysis below).



Consider a calibration procedure where forty measurements were made with the micrometer on the gage block.

$N := 40$

The mean thickness measured on the gage block is
$x_{ave} := 0.12621 in$

The standard deviation of the gage block is
$S_x := 0.0001463177 in$

The t-statistic for a sample size of 40 is (remember degrees of freedom is N-1)

$t_{39\_95} := 2.02268893$

The standard gage block thickness established the true thickness which is

$x_{true} := 0.12620 \cdot in$

From this data we can establish the **bias error of the mean of the measurements** with respect to the standard.

This bias error is:

$\varepsilon_{bias\_mean} := \left| x_{true} - x_{ave} \right|$

$\varepsilon_{bias\_mean} = 10 \times 10^{-6}\, in$

Also, the **precision error of the measurements** is:

$$\varepsilon_{precision} := t_{39\_95} \frac{S_x}{\sqrt{N}}$$

$\varepsilon_{precision} = 4.679 \times 10^{-5}\, in$

We should always consider the bias error associated with resolution of the measurements. In the case the resolution of the micrometer is

$Resolution := 0.00005 \cdot in$

So, the **resolution bias error is**:

$$\varepsilon_{resolution} := \frac{Resolution}{2}$$

$\varepsilon_{resolution} = 2.5 \times 10^{-5}\, in$

For a calibration analysis such as this, the calibration standard has some errors associated with its accuracy transferred from the "true" dimensional standard. From the standard's documentation, these are from length, flatness and parallelism tolerances for Federal Grade 2 accuracy gage blocks. Taking the largest possible errors in the standard, these errors are conservatively estimated to be:

$\varepsilon_{standard\_length} := 4 \cdot 10^{-6} \cdot in$

$\varepsilon_{standard\_flatness} := 4 \cdot 10^{-6} \cdot in$

$\varepsilon_{standard\_parallelism} := 4 \cdot 10^{-6} \cdot in$

$$\varepsilon_{standard\_total} := \sqrt{\varepsilon_{standard\_length}^{2} + \varepsilon_{standard\_flatness}^{2} + \varepsilon_{standard\_parallelism}^{2}}$$

Therefore the **bias error associated with our local standard**, or the error with respect to the true absolute dimensional measurement is

$\varepsilon_{standard\_total} = 6.928 \times 10^{-6}\, in$

Note that even if the bias and precision errors of our measurements were identically "zero", we could still not claim our measurements to be more accurate (in an absolute sense) than the error associated with the standard used for the calibration.

Note that the error of the standard is a fraction of our measurement resolution. This is desirable but not always possible. Highly accurate standards usually come at a high price!

In general, if you don't have a standard, your error must be estimated conservatively using information from your measurement system's/sensor's vendor, and any other available data. You should at least include the resolution error.

From this calibration process, the **total error or overall instrument error** of this micrometer measurement is calculated by the RSS of all the errors:

$$\varepsilon_{calibration\_total} := \sqrt{\varepsilon_{bias\_mean}^2 + \varepsilon_{precision}^2 + \varepsilon_{standard\_total}^2 + \varepsilon_{resolution}^2}$$

$$\varepsilon_{calibration\_total} = 5.443 \times 10^{-5}\,in$$

The **accuracy** of these measurements is determined relative to the standard as

$$Accuracy\% := \left(1 - \frac{|\varepsilon_{calibration\_total}|}{x_{true}}\right)$$

$$Accuracy\% = 99.957\%$$

From this point forward, in any error or uncertainty analysis you may use this overall instrument error as a **bias error**. It includes all the errors established during calibration. Note that it includes the resolution error, so you don't need to include that again later. You can include the resolution here or later, but don't account for it twice.

B) Using the Calibrated Error in Measurements

Consider now that you have made forty measurements with the micrometer that you calibrated in Part A). In this case you have measured the diameter of a cylinder.

The average diameter is

$$x_{cyl\_ave} := .32512\,in$$

$$N := 40$$

The standard deviation is

$$S_x := 0.0003\,in$$

The t-statistic is

$$t_{39\_95} := 2.02268893$$

You can now, as the experienced experimentalist that you are, state that the true diameter of the cylinder has a confidence interval of

$$\varepsilon_{cylinder} := \sqrt{\varepsilon_{calibration\_total}^2 + \left(t_{39\_95}\frac{S_x}{\sqrt{N}}\right)^2}$$

$$\varepsilon_{cylinder} = 1.103 \times 10^{-4}\,in$$

based on 95% probability.

So the "true" diameter of your cylinder is 0.32512 + or - 0.000057 inches.