

Introduction to Frequency Analysis for Experimental Engineering

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1. Overview

This document is intended to introduce basic understanding and usage of frequency analysis for analyzing experimental data conducted for the Experimental Engineering course. Some of the applicable labs include:

- Lab 1g: Horizontally Forced Pendulum and Chaotic Motion
- Lab 2a: Dynamic Response of a Mass-Spring System with Damping
- Lab 2b: Dynamic Response of a Rotor with Shaft Imbalance
- Lab 2c: Driven Mass-Spring System with Damping
- Lab 2d: Torsional Second-Order System
- Lab 2g: Dynamic Response of a Model Vehicle Suspension

Analyzing the data as a function of time (i.e. $x(t)$ in time domain) in frequency domain can be very useful for real-world applications, in which different factors could contribute to this data along with the unwanted environmental noise. Frequency analysis helps to distinguish contributions of the data and to provide insight information to understand the data. In this document, we will use the *Fourier series* as a frequency analysis tool.

2. Theoretical Background

The basic idea of *Fourier series* is very simple: a quantity of interest, $x(t)$, can be approximated by the summation of harmonic functions (sinusoidal functions) as a function of frequency

$$\begin{aligned}x(t) &= \sum_{n=0}^N C_n \cdot \sin(n\omega t + \phi_n) \\ &= \frac{a_0}{2} + \sum_{n=1}^N (a_n \cos n\omega t + b_n \sin n\omega t)\end{aligned}\tag{1}$$

where $\omega = \frac{2\pi}{T}$ is the angular frequency, T is the period, a_0, a_n, b_n ($n = 1, \dots, N$) are the *Fourier coefficients*.

Applying *Fourier transform* (continuous time) or *discrete-time Fourier transform* (DFT) for a discrete set of data, unknowns in Eq. (1) can be found and the given set of data can then be well represented by these few parameters in frequency domain.

3. DFT_GUI: Software for discrete Fourier transform analysis

The software DFT_GUI available on the course website can be used to conduct frequency analysis. It is strongly recommended that reader refer to the DFT_GUI manual (also available on the course website) for the software configuration and usage before performing the analysis for your data. This document only contains some important notes regarding the software.

Note 1: The data in the input file only allows 2 columns (time, data) with up to 5000 lines (rows), without any text (i.e., put only your data on the file, no header).

Note 2: A common task for the labs is to analyze the output data in terms of frequency (Hz) and its amplitude. This corresponds to the second and third column in the output file.

Note 3: As noted in the software manual, “When using the software, the results contained in the output file should be reported since they are more accurate than estimating the values from the plots”.

Note 4: There may be a circumstance that the data input to the software has a very small magnitude, for instance, a linear displacement of 3 mm, but then it was converted to SI unit as 0.003 m and as the input for the software. Nothing else needs to be done if the output frequency is discernible. However, if you find that your output frequency is all zero or not reasonable, this is probably due to the resolution limit (smallest value of the data) that the software can take. In this case, simply multiply all your data (not the time) with a large constant number (say, 1000). This is totally fine as long as the frequency is what you want, while the corresponding amplitude simply has the same scale-up as the data.

4. Examples

To assist in understanding what the frequency analysis does to the time history data, this section provides a set of simple examples as a demonstration. These examples along with the results may be found on the course website as excel documents.

4.1 Test 1

Create a data set x as a function of time t , $x(t) = 2 \times \cos t$, such that the data has an amplitude of 2, and the circular frequency $\omega = 1$ [rad/s]. The exact frequency in Hz is thus $f = \frac{\omega}{2\pi} \approx 0.159$ [Hz], and the corresponding period for this data set $T = 1/f \approx 6.283$ [s]. When creating the data, make sure that at least one period of data points are included (why? How many data points per period is needed? You should be able to answer questions like this after this tutorial is finished, and by testing).

Figure 1(a) shows the data created, and Fig. 1(b) shows the corresponding frequency results using the software. From the output file, the frequency with the peak is 0.145 Hz with amplitude of 1.92. This is the natural frequency and its contribution to the data that the frequency analysis

obtained with the provided discrete data set. Comparing the exact solution of $f = 0.159$ and amplitude of 2, the relative error is 9% and 4%, respectively. Do you know why these are not the exact values?

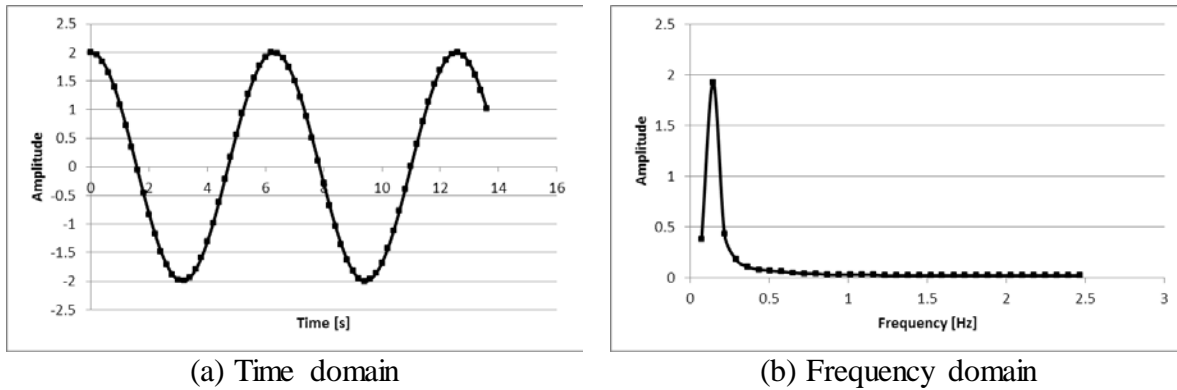


Figure 1: Test 1

4.2 Test 2

Now create another set of data $x(t) = 2 \times \cos t + 3 \times \cos 2t$ as shown in Fig. 2(a), therefore we have 2 different contributions with $\omega_1 = 1 [rad/s]$ and amplitude of 2, and $\omega_2 = 2 [rad/s]$ with amplitude of 3 for the data. The corresponding exact frequency in Hz is thus $f_1 \approx 0.159 [Hz]$ and $f_2 \approx 0.318 [Hz]$. From the frequency output file and Fig. 2(b), there are 2 peaks with amplitude of 2.17 and 2.25. The corresponding frequencies are 0.15 Hz and 0.3 Hz. It seems that the frequency results agree well with the exact solutions. How can you improve the results?

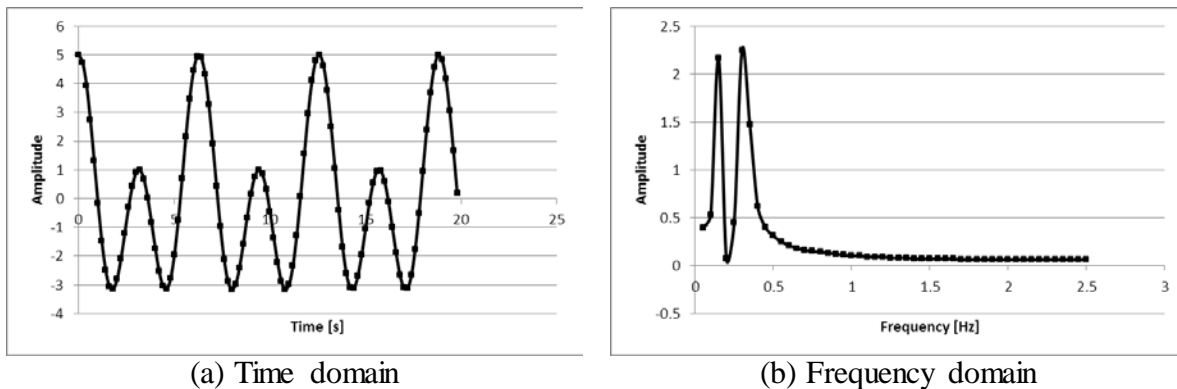


Figure 2: Test 2

4.3 Test 3

This test uses the same data set as test 1, except that an initial value of 0 is included before elapsed time of 4 s. This is to demonstrate the importance of inputting the appropriate data for analysis, instead of simply copy whatever collected data during the experiment. In this example, obviously the data before $t = 4 [s]$ is not meaningful for the targeted data set $x(t) = 2 \times \cos t$. When inputting such data as shown in Fig. 3(a) for frequency analysis, shown in Fig. 3(b), the peak amplitude is 1.38 with frequency $f = 0.145 [Hz]$. Comparing with results from test 1, the

frequency remains the same, attribute to the fact that the provided data set still has enough data points covering the period of the targeted data. However, the influence of this frequency is reduced significantly as indicated by the fact that the amplitude is reduced from 1.92 to 1.38, while fluctuations exist in the higher frequencies (compare the wavy curve in Fig. 3(b) in this test and the flat one shown in Fig.1 (b) for frequency higher than 0.5 Hz).

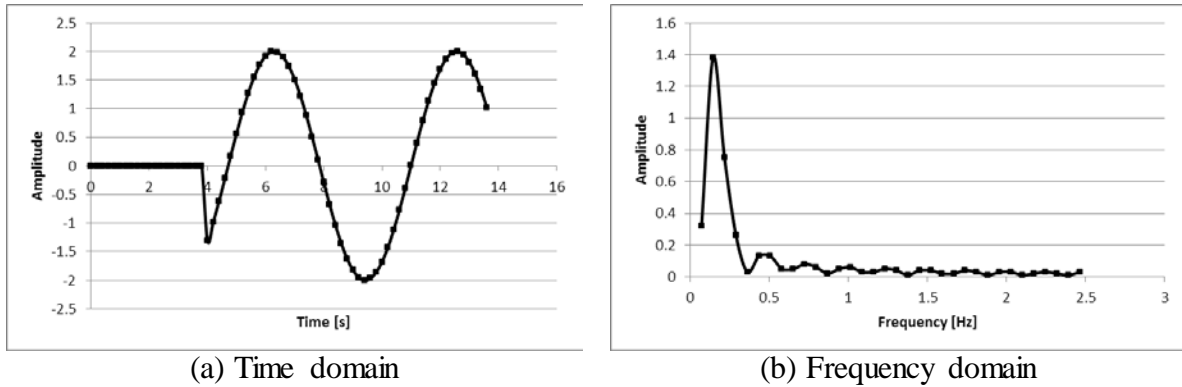


Figure 3: Test 3

5. Challenge test

A data set was created in the form $x(t) = a_1 \times \cos \omega_1 t + a_2 \times \cos \omega_2 t$ along with some random noise as shown in Fig. 4. The whole data set is included as file `<challenge_test.txt>`. Use the frequency analysis tool, find the coefficients $a_1, \omega_1, a_2, \omega_2$ that best approximate the exact solution. Plot this “exact” form and the original data set to show the validity of the results.

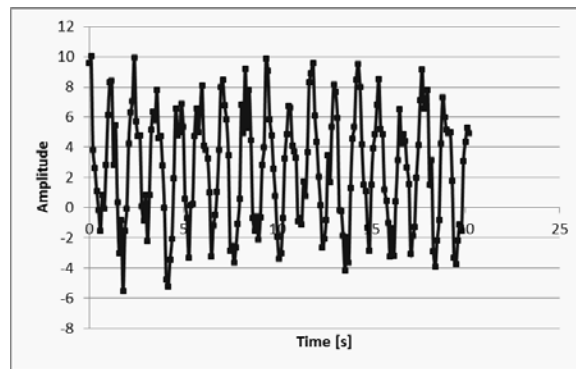


Figure 4: Challenge test