Infiltration Example: Philip's Equation

Given

Constant rainrate storm i = 20 mm/hr for 4 hours

$S = 20$ mm $hr^{-1/2}$

$K = 10$ mm/hr

Find

f and F at t = 1 hr

f and F at t = 4 hr (end of storm)

1) Compute t_p

$$t_p = \frac{S^2 (i - (K/2))}{2i\,(i-K)^2} = \frac{(20)^2\,(20 - (10/2))}{2(20)\,(20-10)^2} = 1.5 \text{ hr}$$

2) Compute t_0

$$t_0 = t_p - \frac{1}{4K^2}\left(\sqrt{S^2 + 4KF_p} - S\right)^2 = 1.5 - \frac{1}{4(10)^2}\left(\sqrt{(20)^2 + 4(10)(1.5 * 20)} - 20\right)^2 = 0.5 \text{ hr}$$

3) f and F at t = 1 hr?

$t < t_p$ (Case 1: Supply Limited), therefore

$f(1) = i = 20$ mm/hr

$F(1) = it = 20(1) = 20$ mm

4) f and F at t = 4 hr?

$t > t_p$ (Case 2: Transport Limited), therefore

$$f(t) = f^*(t - t_0) = f^*(4 - 0.5) = f^*(3.5)$$

$$f(4) = \frac{1}{2}(20)(3.5)^{-1/2} + 10 = 15.35 \text{ mm/hr}$$

$$F(4) = (20)(3.5)^{1/2} + 10(3.5) = 72.42 \text{ mm/hr}$$