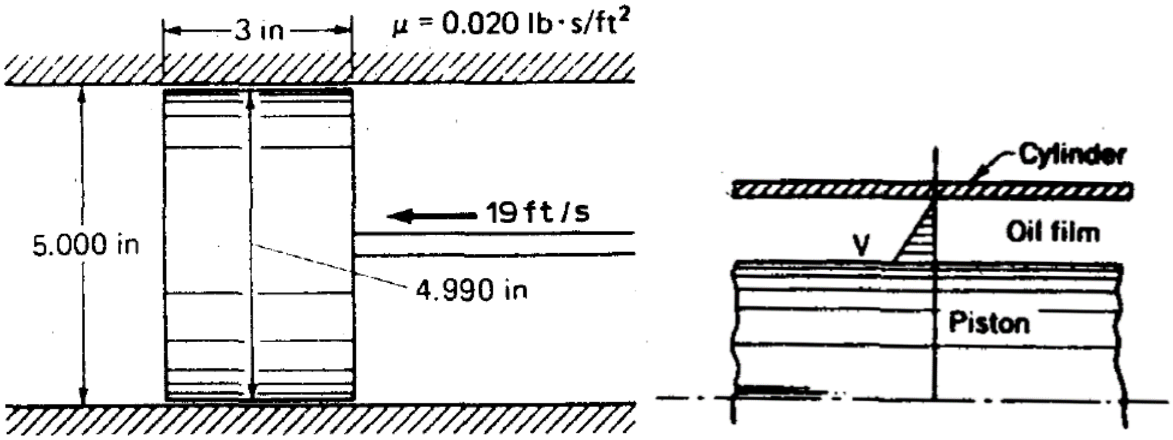
EXAM1 Solutions

Problem 1: Shear stress (Chapter 1)



Information and assumptions

- $\mu = 0.020\ lb\ s/ft^2$
- $V = 19\ ft/s$

Find

- Find (a) shear stress on piston surface, (b) required force

Solution

(a) Shear stress

$$\tau = \mu \frac{du}{dy}$$ +5 points

$$u = \frac{V}{h} y$$

$$\frac{du}{dy} = \frac{V}{h}$$ +2 points

$$\therefore \tau = \mu \frac{V}{h} = \left(0.020 \frac{lb\ s}{ft^2}\right) \left[\frac{19\ ft/s}{(5.000 - 4.990)\ \ /2\ in \times \frac{1\ ft}{12\ in}} \right] = \mathbf{912\ lb/ft^2}$$ +1 points

(b) Force required

$$F_f = \tau A$$ +1 points

$$F_f = (912\ lb/ft^2) \left[\pi \left(\frac{4.990}{12}\right) \left(\frac{3}{12}\right) \right] = \mathbf{298\ lb}$$ +1 points

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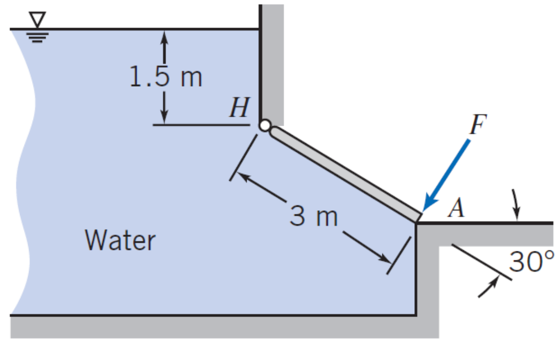


Problem 2: Hydrostatic force (Chapter 2)

Information and assumptions

- Gate width is 3 m
- $\gamma = 9.80\ kN/m^3$

Find

- Find (a) the hydrostatic force, (b) pressuce center and (c) the Force F

Solution

(a) Pressure force

$$F_R = \bar{p}A = \gamma h_c A$$ +3 points

$$F_R = (9800)\left(1.5 + \frac{3\sin 30^\circ}{2}\right)(3 \times 3) = \mathbf{198.45\ kN}$$ +1 points

(b) Pressure center

$$\bar{y} = \frac{1.5}{\sin 30^\circ} + \frac{3}{2} = 4.5\ \text{m}$$ +1 points

$$y_R = \bar{y} + \frac{I_x}{\bar{y}A}$$ +2 points

$$y_R = 4.5 + \frac{(3)(3)^3/12}{(4.5)(3 \times 3)} = \mathbf{4.667\ m}$$ +1 points

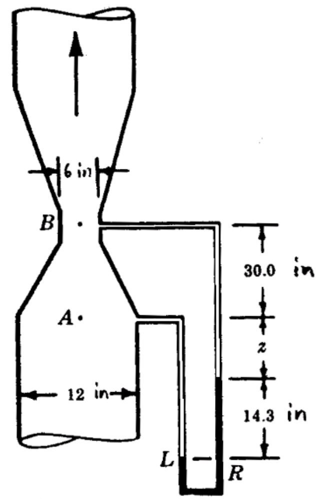
(c) Force to hold the gate closed

$$\sum M_H = \left(y_R - \frac{1.5}{\sin 30^\circ}\right)(198{,}480) - (3)(F) = 0$$ +1 points

$$\therefore F = \frac{(4.667 - 3)(198{,}480)}{3} = \mathbf{110.29\ kN}$$ +1 points

EXAM1 Solutions

Problem 3: Bernoulli equation (Chapter 3)



Information and assumptions

- $\gamma = 64.2\ lb/ft^3$
- Ignore friction loss
- SG = 13.6 for the manometer fluid
- $g = 32.2\ ft/s^2$

Find

- Determine (a) pressure drop, (b) flow rate

Solution

a) Manometer

$$p_A + \left(z + \frac{14.3}{12}\right)\gamma - \frac{14.3}{12}(\mathrm{SG}\cdot\gamma) - \left(z + \frac{30}{12}\right)\gamma = p_B$$

$$\therefore \Delta p = p_A - p_B = \left(\frac{30}{12} + \left(\frac{14.3}{12} - 1\right)\cdot(13.6)\right)(64.2) = \mathbf{1124.46\ lb/ft^2}$$ +2 points

b) Continuity equation

$$A_A V_A = A_B V_B$$ +1 points

$$V_A = \left(\frac{A_B}{A_A}\right)V_B = \left(\frac{6}{12}\right)^2 V_B = 0.25V_B$$ +1 points

Bernoulli equation

$$\frac{p_A}{\gamma} + \frac{v_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{v_B^2}{2g} + z_B$$ +3 points

$$\frac{p_A}{64.2} + \frac{(0.25V_B)^2}{(2)(32.2)} + z_A = \frac{p_B}{64.2} + \frac{V_B^2}{(2)(32.2)} + \left(z_A + \frac{30}{12}\right)$$ + 2 points

$$\therefore V_B = \sqrt{\frac{(2)(32.2)}{1 - 0.25^2}\left(\frac{p_A - p_B}{64.2} - \frac{30}{12}\right)} = \mathbf{32.12\ ft/s}$$

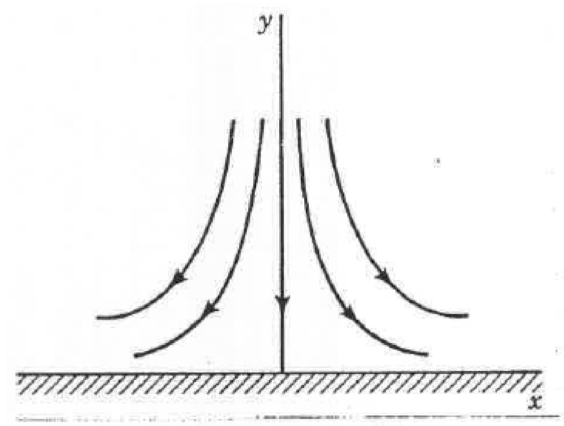
$$\therefore Q = A_B V_B = \left(\frac{\pi(6/12)^2}{4}\right)(32.12) = \mathbf{6.31\ ft^3/s}$$ + 1 points

EXAM1 Solutions

Problem 4: Acceleration and Euler equation (Chapter 4)

Information and assumptions

- Two dimensional flow
- $\underline{V} = Kx\,\hat{\boldsymbol{\imath}} - Ky\,\boldsymbol{j}$
- $K = 2\,s^{-1}$
- $\rho = 998\,kg/m^3$
- $\mu = 1.003 \times 10^{-3}\,Ns/m^2$
- $\rho a_y = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$



Find

- Calculate (a) the acceleration components, and (b) pressure gradient $\partial p/\partial y$, at $x = 0$, $y = 1$.

Solution

(a) Acceleration

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$ +2 points

$$a_x = 0 + (Kx)(K) + (-Ky)(0) = K^2 x$$ +1 points

At (0, 1) +0.5 points

$$a_x = (2)^2(0) = \mathbf{0\ m/s^2}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$ +2 points

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 0 + (Kx)(0) + (-Ky)(-K) = K^2 y$$ +1 points

At (0, 1) +0.5 points

$$a_y = (2)^2(1) = \mathbf{4\ m/s^2}$$

(b) Navier-Stokes equation

$$\rho a_y = -\frac{\partial p}{\partial y} + \mu(0 + 0)$$ +2 points

$$\therefore \frac{\partial p}{\partial y} = -\rho a_y = -(998)(4) = \mathbf{-3,992\ Pa/m}$$ +1 points