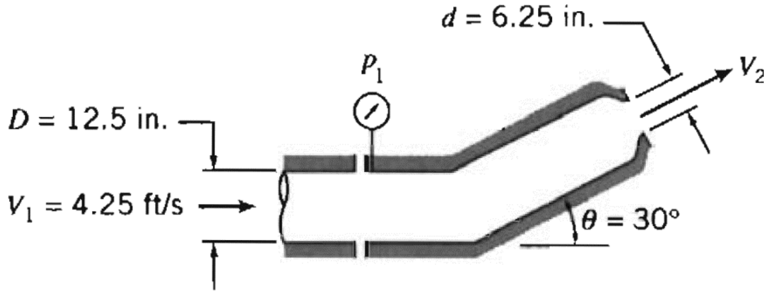
EXAM 2 Solutions

Problem 1: Control Volume Analysis [Momentum] (Chapter 5)



Information and assumptions

- Water properties $\rho = 1.94\ slugs/ft^3$ and $\gamma = 62.4\ lb/ft^3$
- $g = 32.2\ ft/s^2$
- Nozzle discharges to atmosphere ($p_2 = 0$)

Find

- Calculate (a) the jet velocity V2 at the nozzle end, (b) the pressure p1 at the flanged joint, and (c) the horizontal component of the anchoring force Fx to keep the nozzle in place

Solution

(a) Continuity equation

$$A_1V_1 = A_2V_2 \quad (+1.5)$$

$$V_2 = \frac{A_1}{A_2}V_1 = \left(\frac{D}{d}\right)^2 V_1 = \left(\frac{12.5\ in}{6.25\ in}\right)^2 (4.25\ ft/s) = \mathbf{17\ ft/s} \quad (+0.5)$$

(b) Bernoulli equation

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

Since $p_2 = 0$

$$p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2) + \gamma(z_2 - z_1) \quad (+2.5)$$

$$p_1 = \frac{1}{2}\left(1.94\frac{slugs}{ft^3}\frac{1\ lb\ s^2/ft}{1\ slug}\right)(17^2 - 4.25^2)ft^2/s^2 + (62.4\ lb/ft^3)\left(\frac{12}{12}\ ft\right)$$

$$\therefore p_1 = \mathbf{325.2\ lb/ft^2} \quad (+0.5)$$

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(c) Momentum equation

$$\dot{m}u_{\text{out}} - \dot{m}u_{\text{in}} = p_1 A_1 - F_x$$

$$F_x = p_1 A_1 - \dot{m}(u_{\text{out}} - u_{\text{in}})$$

where $\dot{m} = \rho Q = \rho V_1 A_1$, $u_{\text{out}} = V_2 \cos 30°$, $u_{\text{in}} = V_1$ and $A_1 = \frac{\pi D^2}{4} = \frac{\pi \times (12.5/12)^2}{4} = 0.852 \text{ ft}^2$.

$$\therefore F_x = p_1 A_1 - \rho(V_1 A_1)(V_2 \cos 30° - V_1) \quad \mathbf{(+4.5)}$$

$$F_x = (326)(0.852) - (1.94)(4.25 \times 0.852)(17 \cos 30° - 4.25) = \mathbf{204\ lb} \quad \mathbf{(+0.5)}$$

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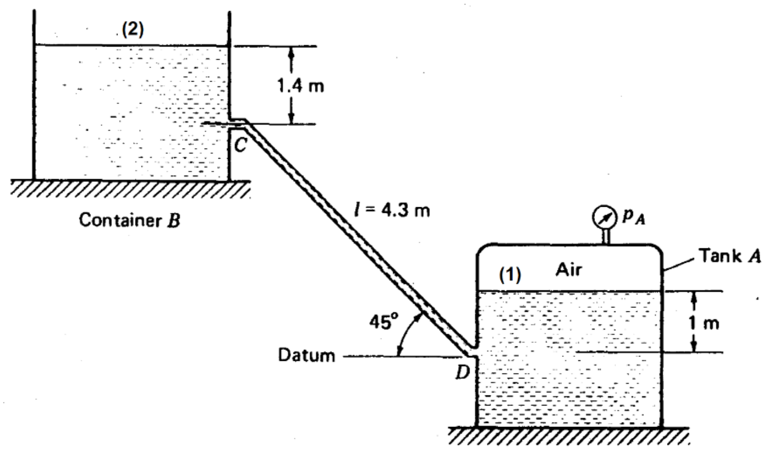
Problem 2: Laminar Pipe Flow (Chapter 8)



Information and assumptions

- γ = 9,780 N/m^3 and of μ = 0.0008 N·s/m^2
- $p_A = p_1$ =34.5 kPa gage
- $h_f = 32\mu LV/\gamma d^2$
- Neglecting the minor losses
- Assume laminar flow

Find

- Determine the flow rate Q through the capillary tube

Solution

Energy equation neglecting minor losses:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_f \ (+\mathbf{4})$$

Since p_2 = 0, $V_1 \approx$ 0, $V_2 \approx$ 0, h_p = 0, and h_t = 0,

$$\frac{p_1}{\gamma} + 0 + z_1 + 0 = 0 + 0 + z_2 + 0 + h_f$$

$$h_f = \frac{p_1}{\gamma} + z_1 - z_2 \ (+\mathbf{3})$$

$$h_f = \left(\frac{34.5 \times 1000}{9780}\right) + (1) - (1.4 + 4.3 \sin 45^\circ) = 0.087 \text{ m} \ (+\mathbf{0.5})$$

Thus,

$$\frac{32\mu LV}{\gamma d^2} = h_f$$

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$$V = \frac{h_f \gamma d^2}{32\mu L}$$

$$V = \frac{(0.087)(9780)(6/1000)^2}{(32)(0.0008)(4.3)} = 0.278\ m/s \quad (+\mathbf{1})$$

Therefore

$$Q = VA \quad (+\mathbf{1})$$

$$Q = (0.278)\left[\frac{(\pi)(6/1000)^2}{4}\right] = \mathbf{7.87 \times 10^{-6}\ m^3/s} \qquad (+\mathbf{0.5})$$

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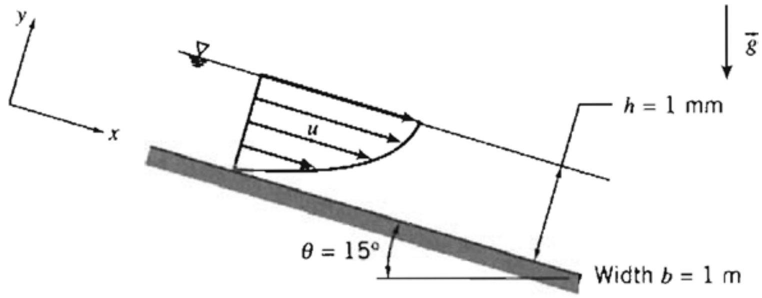

Problem 3: N-S (Chapter 6)

Information and assumptions

- $\mu \frac{d^2u}{dy^2} = -\rho g_x$
- ρ = 912 kg/m^3 and μ = 0.38 N·s/m^2
- h = 1 mm, b = 1 m, and θ = 15°
- $g = 9.81\ m/s^2$

Find

- (a) Derive an expression for the velocity distribution $u(y)$ and (b) find the volume flow rate $Q = \int_0^h u(y)bdy$

Solution

(a) Navier-Stokes equation

$$\frac{d^2u}{dy^2} = -\rho g \frac{\sin\theta}{\mu}$$

Integrating,

$$\frac{du}{dy} = -\rho g \frac{\sin\theta}{\mu} y + C_1 \quad (+2.5)$$

and integrating again,

$$u(y) = -\rho g \frac{\sin\theta}{\mu}\frac{y^2}{2} + C_1 y + C_2 \qquad (+2.5)$$

At $y = h$,

$$\left.\frac{du}{dy}\right)_{y=h} = -\rho g \frac{\sin\theta}{\mu} h + C_1 = 0 \qquad \therefore C_1 = \rho g \frac{\sin\theta}{\mu} h \ (+1.5)$$

At $y = 0$,

$$u(0) = 0 + 0 + C_2 = 0 \qquad \therefore C_2 = 0$$

$$\therefore \boldsymbol{u(y) = \rho g \frac{\sin\theta}{\mu}\left(hy - \frac{y^2}{2}\right)} \quad (+1.5)$$

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(b) Flow rate

$$Q = \int_0^h \rho g \frac{\sin\theta}{\mu}\left(hy - \frac{y^2}{2}\right) b\,dy = \frac{\rho g \sin\theta b}{\mu}\frac{h^3}{3} \quad (+\mathbf{1.5})$$

$$\therefore Q = \frac{(912)(9.81)(\sin 15°)(1)}{0.38}\frac{(1/1000)^3}{3} = \mathbf{2.03\times10^{-6}\ m^3/s} \qquad (+\mathbf{0.5})$$

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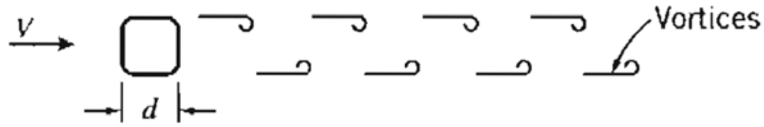
Problem 4: Dimensional Analysis (Chapter 7)



Information and assumptions

- $f = (\rho, d, V, \mu)$
- $dp/dm = 2$

Find

- (a) Use dimensional analysis to develop a functional relationship for f. (b) Vortex shedding occurs in standard air on two cylinders with diameters d_m and d_p

Solution

(a) Dimensional analysis

$$f = (\rho, d, V, \mu)$$

f	ρ	d	V	μ
$\{T^{-1}\}$	$\{ML^{-3}\}$	$\{L\}$	$\{LT^{-1}\}$	$\{ML^{-1}T^{-1}\}$

$(+2)$

$$r = n - m = 5 - 3 = 2 \qquad (+2)$$

$$\Pi_1 = \rho^a V^b d^c f \doteq (ML^{-3})^a (LT^{-1})^b (L)^c (T^{-1}) \doteq M^a L^{(-3+b+c)} T^{(-b-1)} \doteq M^0 L^0 T^0$$

$\Rightarrow a = 0, b = -1, c = 1$. Thus,

$$\Pi_1 = V^{-1} d f = \frac{\boldsymbol{fd}}{\boldsymbol{V}} \ (+2)$$

$$\Pi_2 = \rho^a V^b d^c \mu \doteq (ML^{-3})^a (LT^{-1})^b (L)^c (ML^{-1}T^{-1}) \doteq M^{(a+1)} L^{(-3+b+c-1)} T^{(-b-1)} \doteq M^0 L^0 T^0$$

$\Rightarrow a = -1, b = -1, c = -1$. Thus,

$$\Pi_2 = \rho^{-1} V^{-1} d^{-1} \mu = \frac{\boldsymbol{\mu}}{\boldsymbol{\rho V d}} \ (+2)$$

$$\therefore \frac{fd}{V} = \phi\left(\frac{\rho V d}{\mu}\right)$$

(b) Dynamic similarity

$$\frac{\rho V_p d_p}{\mu} = \frac{\rho V_m d_m}{\mu} \quad \Rightarrow \quad \frac{V_p}{V_m} = \frac{d_m}{d_p} = \frac{1}{2} \qquad (+1)$$

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$$\frac{f_p d_p}{V_p} = \frac{f_m d_m}{V_m} \quad \Rightarrow \quad \frac{f_p}{f_m} = \left(\frac{d_m}{d_p}\right)\left(\frac{V_p}{V_m}\right) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4} \qquad (+\mathbf{1})$$