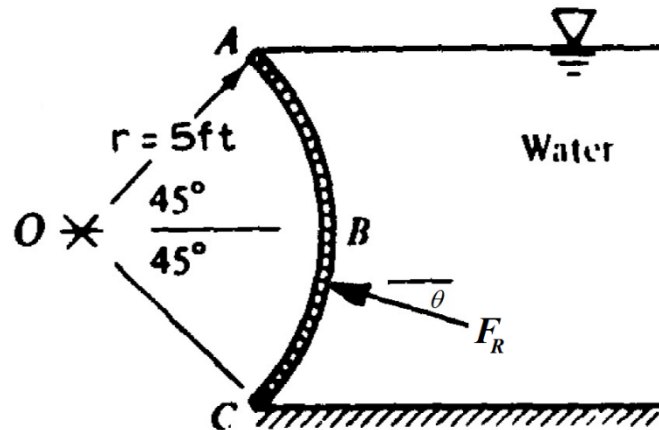


EXAM 3 Solutions

Problem 1: Hydrostatic Force



Information and assumptions

- $\gamma_{\text{water}} = 62.4\text{ lb/ft}^3$

Find

- Compute the (a) horizontal F_H and (b) vertical F_V hydrostatic forces on the wall and (c) the line of action (i.e., the angle θ) of the resultant force F_R .

Solution

(a) Horizontal force

$$F_H = \gamma h_c A_{\text{proj}} \quad (+4)$$

$$F_H = (62.4)(5 \sin 45^\circ)(2 \times 5 \sin 45^\circ \times 9) = \mathbf{14,040\text{ lb}} \quad (+1)$$

(b) Vertical force

$$F_V = \gamma V = \gamma(V_{\text{quarter circle}} - V_{\text{triangle}}) \quad (+3)$$

$$F_V = \gamma V = (62.4)(9)[(\pi)(5)^2/4 - (2)(5 \sin 45^\circ)(5 \cos 45^\circ)/2] = \mathbf{4007\text{ lb}} \quad (+1)$$

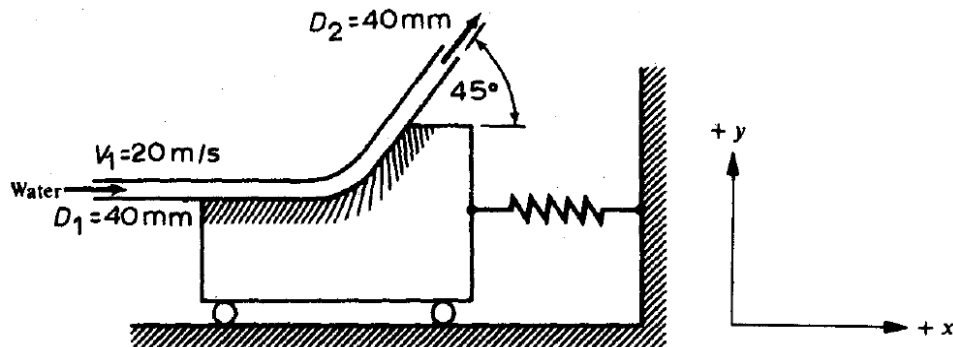
(c) Line of action of the resultant force

$$\theta = \tan^{-1}\left(\frac{F_H}{F_V}\right)$$

$$\theta = \tan^{-1}\left(\frac{4007}{14,040}\right) = \mathbf{15.9^\circ} \quad (+1)$$

EXAM 3 Solutions

Problem 2: Momentum



Information and assumptions

- $\rho = 998 \text{ kg/m}^3$
- $k = 1.6 \text{ kN/m}$
- $F_x = -k\Delta x$
- *Steady state*
- *Circular cross section area for water jet*

Find

- Find the x-component of the force on the wheels, F_x , caused by deflecting the water jet and (b) the compression of the spring, Δx .

Solution

(a) Flow rate

$$Q = AV \quad (+1.5)$$

$$Q = \left[\frac{(\pi)(0.04)^2}{4} \right] (20) = 0.02513 \text{ m}^3/\text{s} \quad (+0.5)$$

Momentum equation

$$\sum F = \sum (\dot{m}V)_{out} - \sum (\dot{m}V)_{in} \quad (+4)$$

$$F_x = (\dot{m}u)_{out} - (\dot{m}u)_{in} = \rho Q(u_2 \cos 45^\circ - u_1) \quad (+2)$$

$$F_x = (998)(0.02513)(20 \cos 45^\circ - 20) = -147 \text{ N} \quad (+1)$$

EXAM 3 Solutions

(b) Spring compression

By Hook's law,

$$F_x = -k\Delta x$$

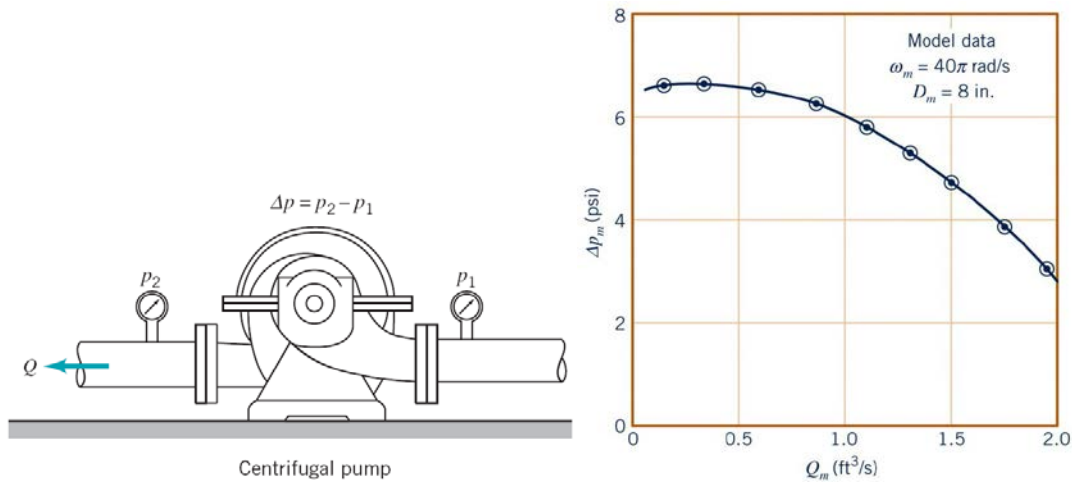
$$\Delta x = -\frac{F_x}{k}$$

Thus,

$$\Delta x = -\frac{(-147)}{1,600} = \mathbf{0.092\ m} \text{ (or 92 mm)} \quad \mathbf{(+1)}$$

EXAM 3 Solutions

Problem 3: PI and Similarity



Information and assumptions

- $\Delta p = f(D, \omega, \rho, Q)$
- $D_m = 8 \text{ in}$, $\rho_m = 998 \text{ kg/m}^3$, $\omega_m = 40\pi \text{ rad/s}$
- $D = 12 \text{ in}$, $\rho = 998 \text{ kg/m}^3$, $Q = 6 \text{ ft}^3/\text{s}$

Find

- (a) By using dimensional analysis, show that the two pi terms of this problem are $\Pi_1 = \Delta p / \rho \omega^2 D^2$ and $\Pi_2 = Q / \omega D^3$
- Use experimental data given (see figure) to predict the pressure rise across a geometrically similar pump (prototype) for a prototype flowrate of $6 \text{ ft}^3/\text{s}$.

Solution

(a)

Δp	D	ω	ρ	Q
$ML^{-1}T^{-2}$	L	T^{-1}	ML^{-3}	L^3T^{-1}

(+1)

Pi terms

$$\Pi_1 = D^a \omega^b \rho^c \Delta p \doteq (L)^a (T^{-1})^b (ML^{-3})^c (ML^{-1}T^{-2}) \doteq L^0 T^0 M^0 \quad \mathbf{(+1)}$$

Thus, $a = -2$, $b = -2$, and $c = -1$. **(+0.5)**

$$\therefore \Pi_1 = \frac{\Delta p}{\rho \omega^2 D^2}$$

$$\Pi_2 = D^a \omega^b \rho^c Q \doteq (L)^a (T^{-1})^b (ML^{-3})^c (L^3T^{-1}) \doteq L^0 T^0 M^0 \quad \mathbf{(+1)}$$

Thus, $a = -3$, $b = -1$, and $c = 0$. **(+0.5)**

EXAM 3 Solutions

$$\therefore \Pi_2 = \frac{Q}{\omega D^3}$$

(b) Similarity

$$\frac{Q}{\omega D^3} = \frac{Q_m}{\omega_m D_m^3}$$

$$Q_m = \left(\frac{\omega_m}{\omega}\right) \left(\frac{D_m}{D}\right)^3 Q \quad (+2)$$

$$Q_m = \left(\frac{40\pi}{60\pi}\right) \left(\frac{8}{12}\right)^3 (6) = 1.19 \text{ ft}^3/\text{s} \quad (+1)$$

Prediction equation

$$\frac{\Delta p}{\rho \omega^2 D^2} = \frac{\Delta p_m}{\rho_m \omega_m^2 D_m^2}$$

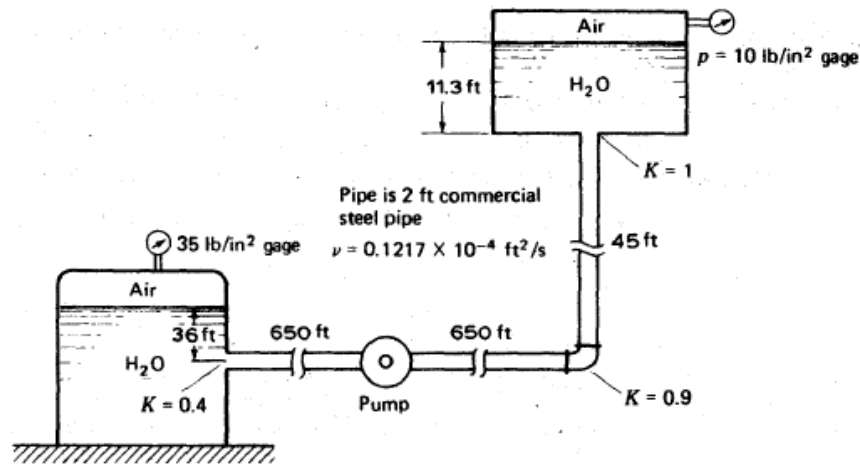
$$\Delta p = \left(\frac{\rho}{\rho_m}\right) \left(\frac{\omega}{\omega_m}\right)^2 \left(\frac{D}{D_m}\right)^2 \Delta p_m \quad (+2)$$

From Fig. 3(b), $\Delta p_m = 5.5$ psi at $Q_m = 1.19$ ft³/s.

$$\therefore \Delta p = (1) \left(\frac{60\pi}{40\pi}\right)^2 \left(\frac{12}{8}\right)^2 (5.5) = 27.8 \text{ psi} \quad (+1)$$

EXAM 3 Solutions

Problem 4: Pipe Flow (Iterative)



Information and assumptions

- $\frac{1}{\sqrt{f}} = -1.8 \log \left[\left(\frac{\epsilon/D}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right]$
- $g = 32.2 \text{ ft/s}^2$, $\rho = 1.94 \text{ slugs/ft}^3$, $\mu = 2.34 \times 10^{-5} \text{ lbf} \cdot \text{s/ft}^2$, $\gamma = 62.4 \text{ lbf/ft}^3$
- $h_p = 52 \text{ ft}$, $\epsilon = 0.00016 \text{ ft}$, $D = 2 \text{ ft}$

Find

- Determine flow rate

Solution

Energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(f \frac{L}{D} + \sum K \right) \frac{V^2}{2g} \quad (+3)$$

Since $V_1 = V_2 = 0$

$$\frac{p_1}{\gamma} + z_1 + h_p = \frac{p_2}{\gamma} + z_2 + \left(f \frac{L}{D} + \sum K \right) \frac{V^2}{2g} \quad (+1)$$

$$= \frac{35 \times 144}{62.4} + 36 + 52$$

$$= \frac{(10 \times 144)}{62.4} + (45 + 11.3) + \left(f \frac{(650 + 650 + 45)}{2} + (0.4 + 0.9 + 1) \right) \frac{V^2}{2 \times 32.2}$$

$$\therefore V = \sqrt{\frac{5756.9}{672.5f + 2.3}} \quad (\text{Eq. 1}) \quad (+1)$$

EXAM 3 Solutions

Reynolds number

$$Re = \frac{\rho V D}{\mu} = \frac{1.94 \times 2}{2.34 \times 10^{-5}} V \Rightarrow Re = 165812 V \text{ (Eq. 2) (+1)}$$

Friction factor

$$f = \left[-1.8 \log \left[\left(\frac{0.00016/2}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right] \right]^{-2} \Rightarrow f = \left[-1.8 \log \left[6.633 \times 10^{-6} + \frac{6.9}{Re} \right] \right]^{-2} \text{ (Eq. 3) (+1)}$$

Using equation 1, 2 and 3.

Assume $f = 0.01$,

$$V = 25.26 \frac{ft}{s} \rightarrow Re = 4.19 \times 10^6 \rightarrow f_{new} = 0.01195$$

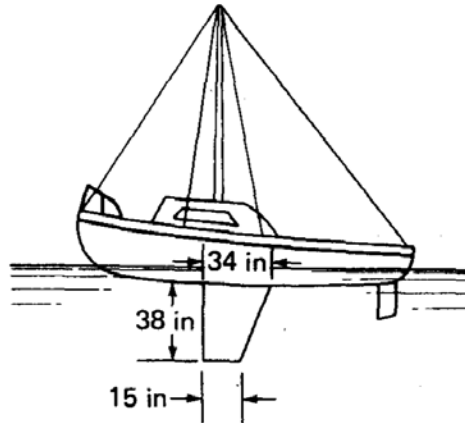
$$V_{new} = 23.60 \frac{ft}{s} \rightarrow Re = 3.91 \times 10^6 \rightarrow f_{new} = 0.01198$$

$$V_{new} = 23.58 \frac{ft}{s} \rightarrow Re = 3.91 \times 10^6 \rightarrow f_{new} = 0.01198 \rightarrow \text{converged (+2)}$$

$$Q = V \cdot \frac{\pi D^2}{4} = 23.58 \times \frac{\pi \times 2^2}{4} = 73.1 \text{ ft}^3/\text{s (+1)}$$

EXAM 3 Solutions

Problem 5: Boundary Layer



Information and assumptions

- Assume simplified geometry: plate of length 38 in and width 24.5 in
- $\nu = 1.664 \times 10^{-5} \text{ ft}^2/\text{s}$ and $\rho = 1.94 \text{ slugs/ft}^3$
- $h_p = 52 \text{ ft}$, $\varepsilon = 0.00016 \text{ ft}$, $D = 2 \text{ ft}$
- $Re_{crit} = 5 \times 10^5$

Find

- Determine skin friction drag D_f from the keel if the speed of sailboat is (a) 2 knots and (b) 10 knots

Solution

(a)

$$Re_L = \frac{UL}{\nu} = \frac{(2 \times 1.689)(24.5/12)}{1.664 \times 10^{-5}} = 4.14 \times 10^5 \quad (\text{Laminar}) \quad (+2)$$

$$C_f = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{4.14 \times 10^5}} = 0.002063 \quad (+2)$$

$$D_f = C_f \cdot \frac{1}{2} \rho U^2 b L \times 2 = (0.002063) \left(\frac{1}{2}\right) (1.94) (3 \times 1.689)^2 \left(\frac{38}{12}\right) \left(\frac{24.5}{12}\right) \times 2 = \mathbf{0.3 \text{ lb}} \quad (+1)$$

(b)

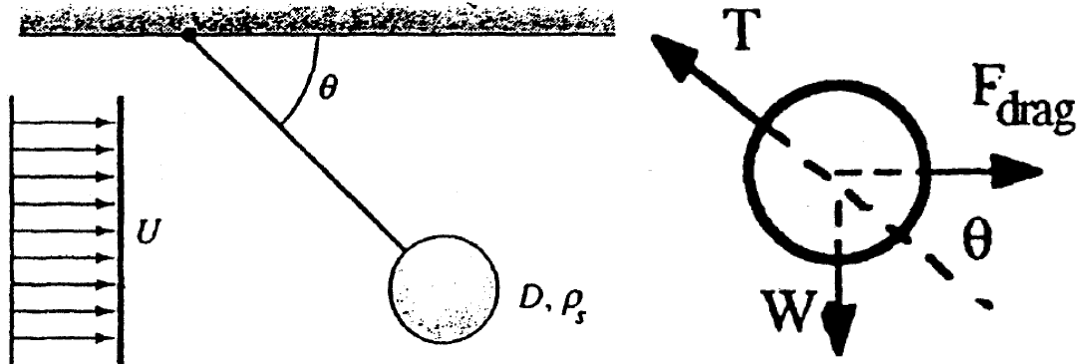
$$Re_L = \frac{UL}{\nu} = \frac{(10 \times 1.689)(24.5/12)}{1.664 \times 10^{-5}} = 2.07 \times 10^6 \quad (\text{Laminar} - \text{Turbulent}) \quad (+2)$$

$$C_f = \frac{0.0074}{Re_L^{1/5}} - \frac{1700}{Re_L} = \frac{0.0074}{(2.07 \times 10^6)^{1/5}} - \frac{1700}{2.07 \times 10^6} = 0.003216 \quad (+2)$$

$$D_f = C_f \cdot \frac{1}{2} \rho U^2 b L \times 2 = (0.003216) \left(\frac{1}{2}\right) (1.94) (10 \times 1.689)^2 \left(\frac{38}{12}\right) \left(\frac{24.5}{12}\right) = \mathbf{11.5 \text{ lb}} \quad (+1)$$

EXAM 3 Solutions

Problem 6: Drag



Information and assumptions

- $\rho_s = 7,844 \text{ kg/m}^3$
- $\rho_{\text{air}} = 1.225 \text{ kg/m}^3$ and $\mu_{\text{air}} = 1.78 \times 10^{-5} \text{ kg/m}\cdot\text{s}$
- $D = 3 \text{ cm}$
- $U = 40 \text{ m/s}$
- Neglect the string drag and the buoyancy force on the sphere

Find

- Find (a) the net weight W , (b) the drag, and (c) the angle θ

Solution

(a)

$$W = \rho_s g V \quad (+2)$$

$$W = (7844)(9.81) \left[\frac{(\pi)(0.03)^3}{6} \right] = 1.09 \text{ N} \quad (+1)$$

(b)

$$\text{Re} = \frac{\rho U D}{\mu} \quad (+2)$$

$$\text{Re} = \frac{(1.225)(40)(0.03)}{1.78 \times 10^{-5}} = 8.3 \times 10^4 \quad (+1)$$

$$C_D \approx 0.5 \quad (\text{From Appendix B}) \quad (+2)$$

$$F_{\text{drag}} = C_D \cdot \frac{1}{2} \rho U^2 \pi D^2 / 4$$

EXAM 3 Solutions

$$F_{\text{drag}} = (0.5) \left(\frac{1}{2} \right) (1.225)(40)^2 \pi (0.03)^2 / 4 = \mathbf{0.346 \text{ N}} \quad (+1)$$

(d)

$$\tan \theta = \frac{W}{F_{\text{drag}}} = \frac{1.09}{0.346} = 3.15$$

$$\therefore \theta \approx \mathbf{72^\circ} \quad (+1)$$