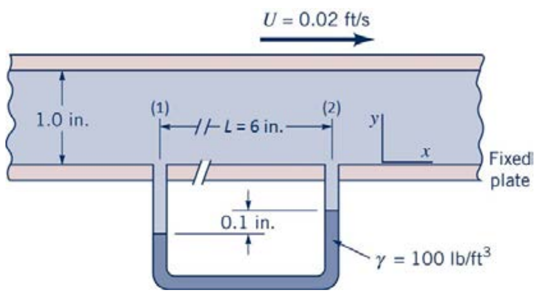
EXAM 1 Solutions

Problem 1: Manometer and Shear Stress



Information and assumptions

- γ_f = 80 lb/ft^3
- μ_f = 0.03 lb·s/ft^2
- U = 0.02 ft/s
- *b = 1 in.*
- *h = 0.1 in.*
- γ = 100 lb/ft^3
- $u(y) = \frac{Uy}{b} + \frac{1}{2\mu}\left(\frac{\Delta p}{L}\right)(by - y^2)$

Find

- Pressure drop between the two points (1) and (2), $\Delta p = p_1 - p_2$
- Shear stress on the fixed plate

Solution

(a) Manometer

$$p_1 - p_2 = (\gamma - \gamma_f)\Delta h \ (\mathbf{+4\ points})$$

$$\therefore \Delta p = (100 - 80)\left(\frac{0.1}{12}\right) = \mathbf{0.167\ lb/ft^2} \quad (\mathbf{+1\ points})$$

(b) Wall shear stress

$$\tau = \mu \frac{du}{dy} \ (\mathbf{+3\ points})$$

$$\tau = \mu \frac{du}{dy} = \mu\left[\frac{U}{b} + \frac{1}{2\mu}\left(\frac{\Delta p}{L}\right)(b - 2y)\right] \ (\mathbf{+1\ points})$$

$$\therefore \tau_w = \mu \left.\frac{du}{dy}\right)_{y=0} = (0.03)\left[\left(\frac{0.02}{1/12}\right) + \left(\frac{1}{2 \times 0.03}\right)\left(\frac{0.167}{6/12}\right)\left(\frac{1}{12} - 2 \times 0\right)\right] = \mathbf{0.021\ lb/ft^2} \quad (\mathbf{+1\ points})$$

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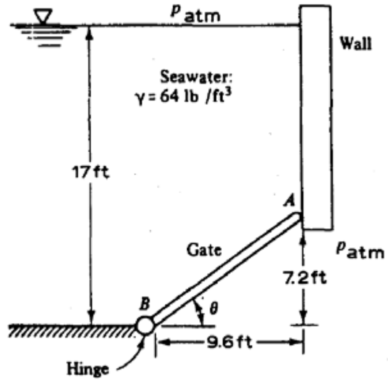
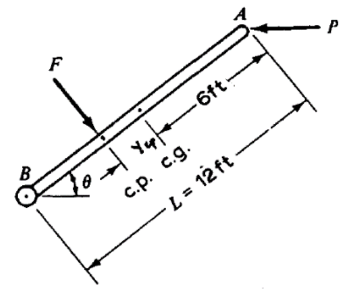
Problem 2: Hydrostatic Pressure





Information and assumptions

- γ = 64 lb/ft^3
- Gate width = 4 ft
- Seawater depth = 17 ft
- Gate length L = 12 ft
- Gate angle, tanθ = 7.2 ft / 9.6 ft

Find

- Pressure force, F_R
- Pressure center, y_{cp}
- Force by the wall at point A, P

Solution

(a) Pressure force

$$F_R = \bar{p}A = \gamma h_{cg} A \quad \textbf{(+4 points)}$$

$$F_R = (64)\left(17 - \frac{7.2}{2}\right)(4 \times 12) = \mathbf{41,165\ lbf} \quad \textbf{(+1 points)}$$

(b) Pressure center

$$y_{cp} = \frac{I_x}{\bar{y}A} = \frac{I_x \sin\theta}{h_{cg}A} \quad \textbf{(+2 points)}$$

$$y_{cp} = \frac{\left[\frac{(4)(12)^3}{12}\right]\left(\frac{7.2}{12}\right)}{\left(17 - \frac{7.2}{2}\right)(4 \times 12)} = \mathbf{0.537\ ft} \quad \textbf{(+1 points)}$$

(c) Force by the wall

$$\sum M_B = (P)(7.2) - (41{,}165)(12 - 6 - 0.537) = 0 \ \textbf{(+1 points)}$$

$$\therefore P = \frac{(41{,}165)(5.463)}{7.2} = \mathbf{31,234\ lbf} \quad \textbf{(+1 points)}$$

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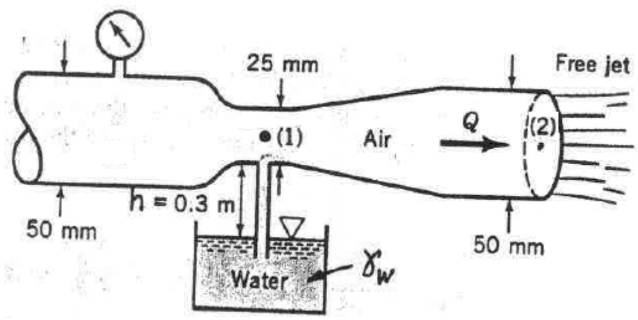
Problem 3: Bernoulli Equation



Information and assumptions

- γ = 12 N/m^3
- $p_1 = -\gamma_w h$
- $\gamma_w = 9.8 \times 10^3$ N/m^3
- h = 0.3 m
- D_1 = 25 mm
- D_2 = 50 mm
- P_2 = 0 (gage)

Find

- Flow rate Q to draw water into the section (1).
- Neglect compressibility and viscous effects

Solution

Bernoulli equation,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \ (\textbf{+3 points})$$

where, $z_1 = z_2$ and $p_2 = 0$.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} \ (\textbf{+2 points})$$

From continuity equation $A_1 V_1 = A_2 V_2$, $(\textbf{+1 points})$

$$V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{50}{25}\right)^2 V_2 = 4V_2 \quad (\textbf{+1 points})$$

Thus,

$$\frac{p_1}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = -\frac{15V_2^2}{2g}$$

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Also, $p_1 = -\gamma_w h$ so that

$$\frac{p_1}{\gamma} = -\frac{\gamma_w h}{\gamma} = -\frac{(9.80 \times 10^3)(0.3)}{12} = -245 \text{ m } (+\mathbf{1\ points})$$

Thus,

$$-245 = -\frac{15V_2^2}{2(9.81)} \ (+\mathbf{1\ points})$$

or

$$V_2 = 17.9 \text{ m/s}$$

Finally,

$$Q = A_2V_2 = \frac{\pi}{4}(0.05)^2(17.9) = \mathbf{0.03551\ m^3/s} \quad (+\mathbf{1\ points})$$

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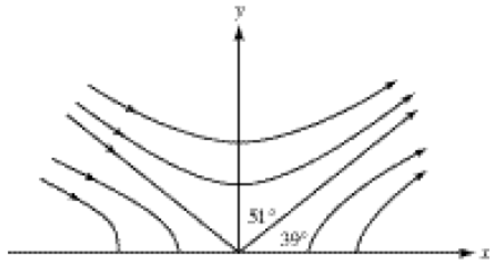
Problem 4: Acceleration



Information and assumptions

- u = 3y and v = 2x
- Euler equation, $\nabla p = -\rho \boldsymbol{a}$
- ρ = 998 kg/m^3

Find

- Acceleration components a_x and a_y at a point x = 1 m and $y = \sqrt{3/2}$ m
- Pressure gradient at a point x = 1 m and $y = \sqrt{3/2}$ m

Solution

(a) Acceleration

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\ (\mathbf{+2\ points})$$

$$a_x = 0 + (3y)(0) + (2x)(3) = 6x\ (\mathbf{+1\ points})$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\ (\mathbf{+2\ points})$$

$$a_y = 0 + (3y)(2) + (2x)(0) = 6y\ (\mathbf{+1\ points})$$

At $(1, \sqrt{3/2})$,

$$a_x = (6)(1) = \mathbf{6\ m/s^2}\ (\mathbf{+1\ points})$$

$$a_y = (6)\left(\sqrt{3/2}\right) = \mathbf{7.348\ m/s^2}\ (\mathbf{+1\ points})$$

(b) Pressure gradient

$$\frac{\partial p}{\partial x} = -\rho a_x = -(998)(6) = \mathbf{-5,988\ Pa/m}\ (\mathbf{+1\ points})$$

$$\frac{\partial p}{\partial y} = -\rho a_y = -(998)(7.348) = \mathbf{-7,333\ Pa/m}\ (\mathbf{+1\ points})$$