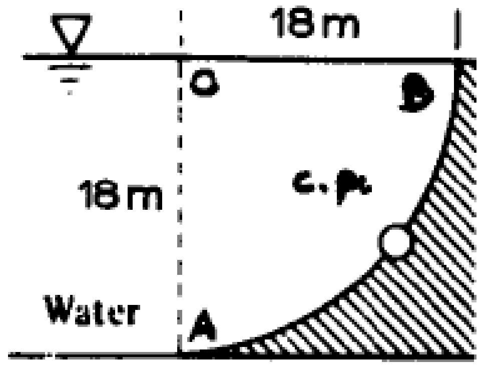
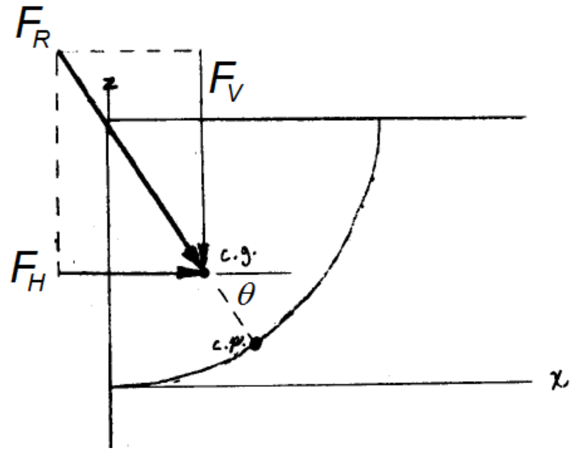
EXAM 3 Solutions

Problem 1: Hydrostatic pressure curved surface





Information and assumptions

- $\gamma = 62.4\ lb/ft^3$
- Depth into the paper is 40m

Find

- Determine (a) horizontal F_H and vertical F_V hydrostatic forces on the wall and (b) magnitude and angle of the resultant force F_R.

Solution

(a) Horizontal and vertical force

$$F_H = \gamma h_c A \ (+2.5)$$

$$F_H = (9.79)\left(\frac{18}{2}\right)(18 \times 40) = 63{,}439 \text{ kN} \quad (+0.5)$$

$$F_V = \gamma \forall \ (+2.5)$$

$$F_V = (9.79)\left(\frac{\pi}{4}\right)(18)^2(40) = 99{,}650 \text{ kN} \quad (+0.5)$$

(b) The resultant force

$$F_R = \sqrt{F_H^2 + F_V^2} \quad (+1.5)$$

$$F_R = \sqrt{(63{,}439)^2 + (99{,}650)^2} = 118{,}130 \text{ kN} \quad (+0.5)$$

$$\theta = \tan^{-1}\left(\frac{F_H}{F_V}\right) \quad (+1.5)$$

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$$\theta = \arctan\left(\frac{F_V}{F_H}\right) = \arctan\left(\frac{99{,}650}{63{,}439}\right) = 57.5^\circ \quad (+0.5)$$

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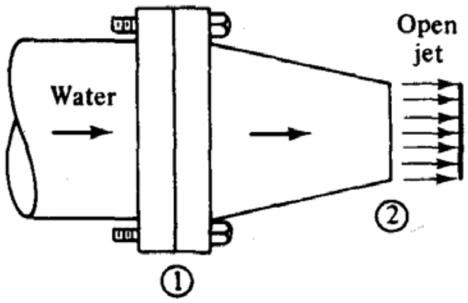
Problem 2: Momentum + energy equation



Information and assumptions

- $V_1 = 9.9 \frac{ft}{s}$
- $P_1 = 60psi \left(8{,}640 \frac{lb}{ft^2}\right)$
- $\rho = 1.94\ slugs/ft^3$
- $D_1 = 9in, D_2 = 3in$

Find

- Compute (a) the jet velocity V_2, (b) the axial force on the nozzle F_x, and (c) the head loss h_L in the nozzle. Use ρ = 1.94 slugs/ft^3 for the water.

Solution

(a) Flow rate

$$Q = AV \quad (+1.5)$$

$$V_2 = \left(\frac{A_1}{A_2}\right) V_1 = \left(\frac{D_1}{D_2}\right)^2 V_1 = \left(\frac{9}{3}\right)^2 (9.9) = 89.1 \text{ ft/s} \quad (+0.5\ )$$

(b) Nozzle force

Momentum equation

$$\sum F = \sum (\dot{m}V)_{out} - \sum (\dot{m}V)_{in} \quad (+2.5)$$

$$F_x = p_1 \cdot \left(\frac{\pi}{4} D_1^2\right) + \rho \left[V_1^2 \left(\frac{\pi}{4} D_1^2\right) - V_2^2 \left(\frac{\pi}{4} D_2^2\right)\right] \ (+2)$$

$$= (8640) \left(\frac{\pi}{4}\right) \left(\frac{9}{12}\right)^2 + (1.94) \left[(9.9)^2 \left(\frac{\pi}{4}\right) \left(\frac{9}{12}\right)^2 - (89.1)^2 \left(\frac{\pi}{4}\right) \left(\frac{3}{12}\right)^2\right] = 3145 \text{ lb} \quad (+0.5)$$

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(3) Head loss

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} + h_L \text{ (+2.5)}$$

Thus,

$$h_L = \frac{8640}{(1.94)(32.2)} + \frac{(9.9)^2}{(2)(32.2)} - \frac{(89.1)^2}{(2)(32.2)} = 16.7 \text{ ft} \qquad \text{(+0.5)}$$

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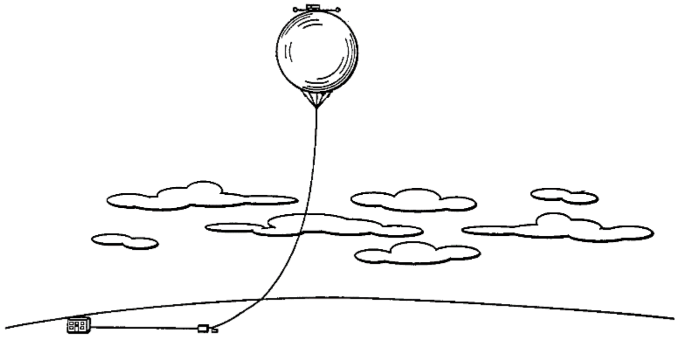
Problem 3: Dimensional analysis similarity



Information and assumptions

- $\frac{D}{\rho d^2 V^2} = \phi\left(\frac{\rho V d}{\mu}\right)$
- $V = 5\frac{m}{s},\ \rho = 1.24\frac{kg}{m^3}, \mu = 1.8 \times 10^{-5}\frac{Ns}{m^2}$
- $D_m = 2\ kN, \rho_m = 999\ kg/m^3, \mu_m = 10^{-3} Ns/m^2$
- $\frac{L_m}{L} = \frac{1}{20}$

Find

- What water speed is required to model the prototype?
- What will be the corresponding drag on the prototype?

Solution

Similarity requirement

For dynamic similarity,

$$\frac{\rho_m V_m d_m}{\mu_m} = \frac{\rho V d}{\mu} \qquad (+4)$$

Thus,

$$V_m = \left(\frac{\rho}{\rho_m}\right)\left(\frac{d}{d_m}\right)\left(\frac{\mu_m}{\mu}\right)V = \left(\frac{1.24}{999}\right)(20)\left(\frac{10^{-3}}{1.8\times 10^{-5}}\right)(5) = 6.9\,\mathrm{m/s} \qquad (+1)$$

Prediction equation

$$\frac{D}{\rho d^2 V^2} = \frac{D_m}{\rho_m d_m^2 V_m^2} \qquad (+4)$$

Thus,

$$D = \left(\frac{\rho}{\rho_m}\right)\left(\frac{d}{d_m}\right)^2\left(\frac{V}{V_m}\right)^2 D_m = \left(\frac{1.24}{999}\right)(20)^2\left(\frac{5}{6.9}\right)^2(2000) = 522\ \mathrm{N} \qquad (+1)$$

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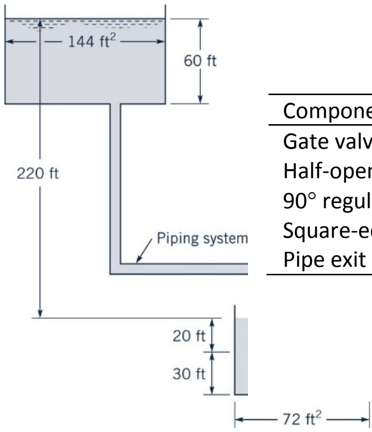
Problem 4: Pipe flow with iteration



Component	K_L
Gate valve	0.15
Half-open globe valve	20.0
90° regular elbow	1.5
Square-edged pipe entrance	0.5
Pipe exit	1.0

Information and assumptions

- $\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$
- $\epsilon = 0.00015\ ft, \nu = 1.21 \times 10^{-5}$

Find

- Determine flow rate

Solution

Energy equation between the upper water surface 1 to the lower water surface 2,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2\mathrm{g}} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2\mathrm{g}} + z_2 + h_L$$

Since $p_1 = p_2$ and assume $V_1 \approx 0$ and $V_2 \approx 0$,

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2\mathrm{g}} = z_1 - z_2 \ (+4)$$

Soving for V gives,

$$V = \sqrt{\frac{2\mathrm{g}(z_1 - z_2)}{f\frac{L}{D} + \sum K_L}} \quad [Eq. 1]$$

where, g = 32.2 ft/s2, $z_1 - z_2$ = 220 ft, L = 250 ft, D = 2 in., and $\sum K_L$ = 4×0.15 + 2×20 + 14×1.5 + 0.5 + 1.0 = 63.1. Thus,

$$V = \sqrt{\frac{14168}{1500f + 63.1}} \quad (+1)$$

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Reynolds number

$$Re = \frac{VD}{\nu} = \frac{(12.4)(2/12)}{1.21 \times 10^{-5}} = 13774\, V\ [Eq.2]\ (+1)$$

For a commercial steel pipe,

$$\frac{\varepsilon}{D} = \frac{0.00015}{2/12} = 0.0009 \quad (+1)$$

Friction factor

$$f = \left[-1.8 \log\left[\left(\frac{0.0009}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]\right]^{-2} => f = \left[-1.8 \log\left[9.7388 \times 10^{-5} + \frac{6.9}{Re}\right]\right]^{-2}\ [Eq.3]$$

Using equation 1, 2 and 3.

Assume f = 0.019,

$V = 12.4\ ft/s \rightarrow Re = 1.713 \times 10^5 \rightarrow f_{new} = 0.021$

$V_{new} = 12.2\ ft/s \rightarrow Re = 1.686 \times 10^5 \rightarrow f_{new} = 0.021 \rightarrow$ converged (+2)

$$Q = VA = (12.2)\frac{\pi}{4}\left(\frac{2}{12}\right)^2 = \mathbf{0.27\ ft^3/s} \quad (+1)$$

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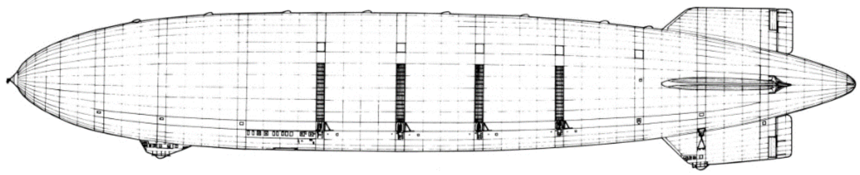
Problem 5: Boundary layer



Information and assumptions

- $V = 123.2\ ft/s$
- ρ = 0.001756 slugs/ft^3 and μ = 3.7 × 10^{-7} lb-s/ft^2
- Flat plate model with $L = 785\ ft$, $W = 132\pi$

Find

- Estimate the power needed to overcome skin friction

Solution

Reynolds number

$$Re_L = \frac{\rho U L}{\mu} = \frac{(0.001756)(123.2)(785)}{3.7 \times 10^{-7}} = 4.59 \times 10^8 \quad (+1.5)$$

$$Re_L = 4.59 \times 10^8 \quad (+0.5)$$

Skin drag coefficient,

$$C_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} \ (+4.5)$$

$$= \frac{0.455}{(\log_{10} 4.59 \times 10^8)^{2.58}} = 1.73 \times 10^{-3} \quad (+0.5)$$

Skin drag,

$$D_f = C_f \cdot \frac{1}{2} \rho U^2 A \ (+1.5)$$

$$= (1.73 \times 10^{-3}) \left(\frac{1}{2}\right) (0.001756)(123.2)^2 (785)[(\pi)(132)] = 7505 \text{ lb} \quad (+0.5)$$

Power

$$P = D_f \cdot U = (7505)(123.2)/550 = 1681 \text{ hp} \quad (+1)$$

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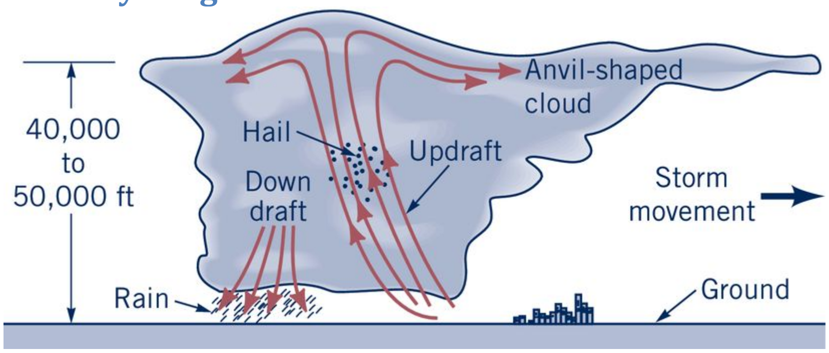
Problem 6: Bluff body drag



Information and assumptions

- $\rho = 2.28 \times 10^{-3}$ slugs/ft^3, $\nu = 1.57 \times 10^{-4}$ ft^2/s, $\rho_{ice} = 1.84$ slugs/ft^3.
- *D = 1.5 in*
- C_D=0.5
- Neglect the buoyant force on the hail

Find

- (a) Estimate the velocity U of the updraft needed to make D = 1.5-in.-diameter (i.e., "golf ball-sized") hail
- (b) Show whether this assumed C_D value is reasonable or not by using the chart in Appendix B.

Solution

(a) Terminal velocity

Since the weight and drag are in balance for the falling hail at its terminal velocity U,

$$Weight = Drag \quad (+2)$$

$$\rho_{\text{ice}} \text{g} \cdot \frac{\pi}{6} D^3 = \frac{1}{2} \rho U^2 \frac{\pi}{4} D^2 C_D \quad (+4)$$

By solving for the velocity,

$$U = \sqrt{\frac{4}{3} \cdot \frac{\rho_{\text{ice}}}{\rho} \cdot \frac{\text{g}D}{C_D}} = \sqrt{\frac{4}{3} \cdot \frac{1.84}{2.28 \times 10^{-3}} \cdot \frac{(32.2\,)(1.5/12)}{0.5}} = 93.1 \text{ ft/s} \quad (+2)$$

(b) Reynolds number

$$Re = \frac{UD}{\nu} = \frac{(91.2)(1.5/12)}{1.57 \times 10^{-4}} = 7.41 \times 10^4$$

For this value of Re, $C_D \approx 0.5$ from the chart in Appendix B. Thus, the assumed C_D value in (a) was reasonable. (+2)