Review for Exam 1

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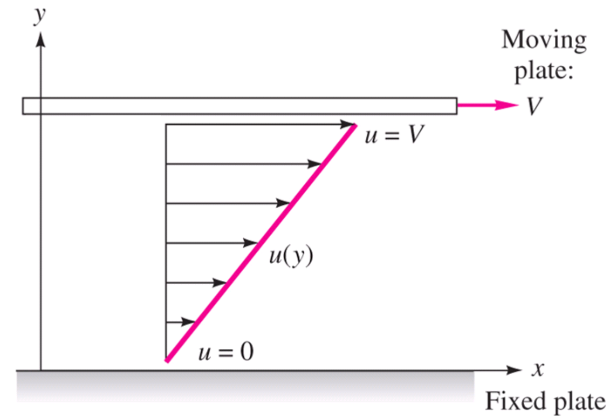
Definition of Fluid

- **Fluid**: Deforms continuously (i.e., flows) when subjected to a shearing stress
  - Solid: Resists to shearing stress by a static deflection

- **No-slip condition**: No relative motion between fluid and solid boundary at the contact
  - The fluid "sticks" to the solid boundaries



The fluid in contact with the lower plate is stationary, whereas the fluid in contact with the upper moving plate moves at speed *V*.

Dimensions and Units

- **Primary dimensions** (or fundamental dimensions): Mass {*M*}, Length {*L*}, Time {*T*}, and Temperature {*Θ*}.
- **Secondary dimensions** (or derived dimensions): All other dimensions expressed in terms of {*M*}, {*L*}, {*T*}, and {*Θ*}. For example,

$$\text{Force} = \text{Mass} \times \text{Acceleration},\ \{F\} = \{M \cdot L/T^2\}$$

- **SI units** (The International System) : The basic units are kilogram (kg), meter (m), and second (s). The force unit is the newton (N),

$$1\ N = 1\ kg \cdot 1\ m/s^2$$

- **BG units** (The British Gravitational System): The basic units are slugs (slug), foot (ft), and second (s). The force unit is the pound-force (lbf),

$$1\ lbf = 1\ slug \cdot 1\ ft/s^2$$

Primary dimension	SI unit	BG unit	Conversion factor
Mass $\{M\}$	Kilogram (kg)	Slug	1 slug = 14.5939 kg
Length $\{L\}$	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Time $\{T\}$	Second (s)	Second (s)	1 s = 1 s
Temperature $\{\Theta\}$	Kelvin (K)	Rankine (°R)	1 K = 1.8°R

Weight and Mass

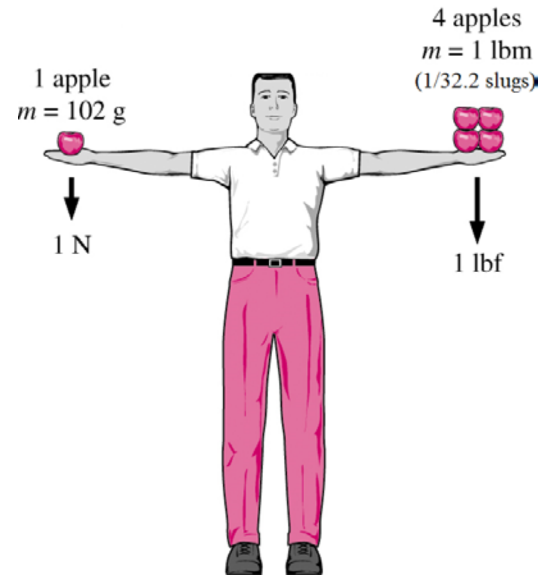
- Weight (W) is a force due to the gravity applied to a body,

$$W = m \cdot \mathrm{g}$$

where, m is the mass of the body and g is the gravitational acceleration:

  - SI unit system: g = 9.807 m/s^2
  - BG unit system: g = 32.174 ft/s^2

- Examples: If the mass of an apple is 102 g,
  - 1 N = 1 apple
  - 1 lbf = 4 apples



Note: Pound-mass (lbm)
1 lbm = 0.45359 kg
1 slug = 32.2 lbm

Measures of Fluid Mass and Weight

- Density (mass per unit volume)

$$\rho = \frac{m}{\forall} \quad (\text{kg/m}^3 \text{ or slugs/ft}^3)$$

- Specific Weight (weight per unit volume)

$$\gamma = \frac{W}{\forall} = \frac{mg}{\forall} = \rho g \quad (\text{N/m}^3 \text{ or lbf/ft}^3)$$

- Specific Gravity

$$\text{SG} = \frac{\gamma}{\gamma_{\text{water}}} \quad \left(= \frac{\rho}{\rho_{\text{water}}} \right)$$

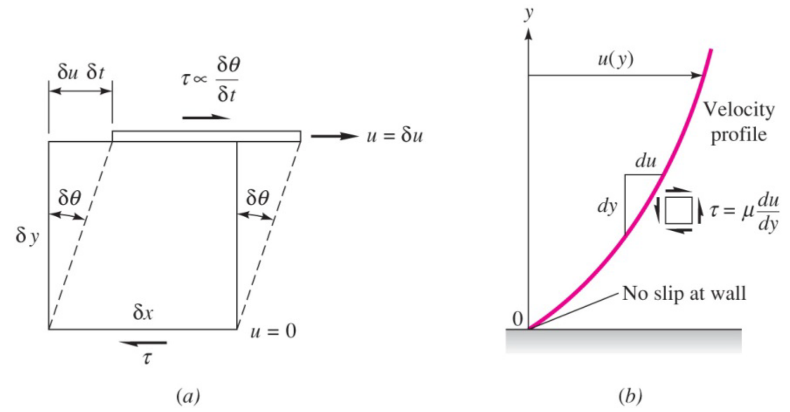
Ex) For mercury, SG = 13.6 and $\rho_{\text{mercury}} = \text{SG} \cdot \rho_{\text{water}} = (13.6)(1{,}000) = 13{,}600\,\text{kg/m}^3$

Viscosity

- Shear stress

$$\tau \propto \frac{\delta\theta}{\delta t}; \quad \tan\delta\theta = \frac{\delta u \delta t}{\delta y}$$

  - τ: Shear stress (N/m^2 or lbf/ft^2)
  - $\delta\theta$: Shear strain angle



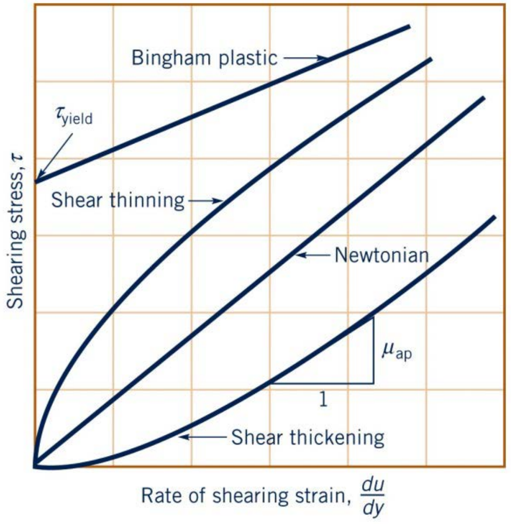
- Newtonian fluid

$$\tau = \mu \frac{du}{dy}$$

  - μ: Dynamic viscosity (N·s/m^2 or lbf·s/ft^2)
  - $\nu = \mu/\rho$: Kinematic viscosity (m^2/s or ft^2/s)
  - Shear force = $\tau \cdot A$

- Non-Newtonian fluid

$$\tau \propto \left(\frac{du}{dy}\right)^n$$



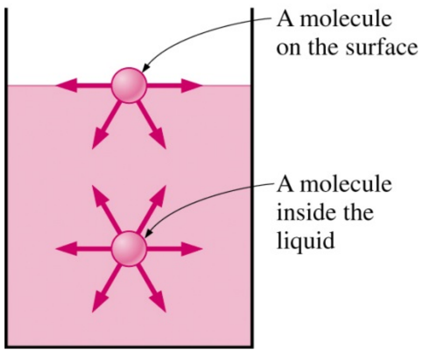
Vapor Pressure and Cavitation

- **Vapor pressure**: Below which a liquid evaporates, i.e., changes to a gas. If the pressure drop is due to
  - Temperature effect: Boiling
  - Fluid velocity: **Cavitation**

Cavitation formed on a marine propeller

Surface Tension

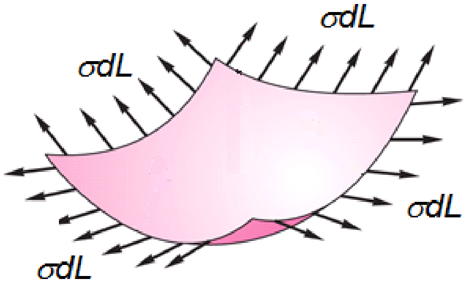
- **Surface tension force**: The force developed at the interface of two immiscible fluids (e.g., liquid-gas) due to the unbalanced molecular cohesive forces at the fluid surface.



Attractive forces acting on a liquid molecule at the surface and deep inside the liquid

$$F_\sigma = \sigma \cdot L$$

- F_σ = Line force with direction normal to the cut
- σ = Surface tension [N/m], the intensity of the molecular attraction per unit length
- L = Length of cut through the interface



Capillary Effect

- **Capillary Effect**: The rise (or fall) of a liquid in a small-diameter tube inserted into a the liquid.

- Capillary rise:

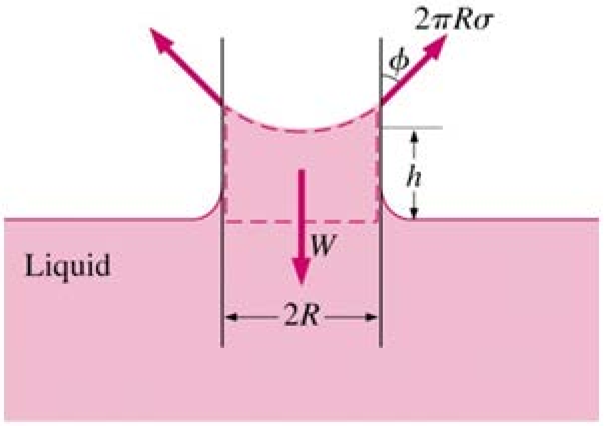
$$F_{\sigma,\mathrm{vertical}} = W$$

or

$$\sigma \cdot (2\pi R)\cos\phi = \rho \mathrm{g}(\pi R^2 h)$$

$$\therefore h = \frac{2\sigma}{\rho \mathrm{g} R}\cos\phi$$

Note: ϕ = contact angle



The forces acting on a liquid column that has risen in a tube due to the capillary effect

Equations of Fluid Motions

- Newton's 2nd law (per unit volume):

$$\rho \underline{a} = \sum \underline{f}$$

where, $\sum \underline{f}$ = $\underline{f}_{\text{body}}$ + $\underline{f}_{\text{surface}}$ and $\underline{f}_{\text{surface}} = \underline{f}_{\text{pressure}} + \underline{\mathrm{f}}_{\text{shear}}$

- Viscous fluids flow (Navier-Stokes equation):

$$\rho \underline{a} = -\rho \mathrm{g} \widehat{\boldsymbol{k}} - \nabla p + \mu \nabla^2 \underline{V}$$

- Inviscid fluids flow (μ = 0; Euler equation):

$$\rho \underline{a} = -\rho \mathrm{g} \widehat{\boldsymbol{k}} - \nabla p$$

- Fluids at rest (No motion, i.e.,$\underline{a}$ = 0):

$$\nabla p = -\rho \mathrm{g} \widehat{\boldsymbol{k}}$$

Absolute Pressure, Gage Pressure, and Vacuum

- **Absolute pressure**: The actual pressure measured relative to absolute vacuum
- **Gage pressure**: Pressure measured relative to local atmospheric pressure
- **Vacuum pressure**: Pressures below atmospheric pressure

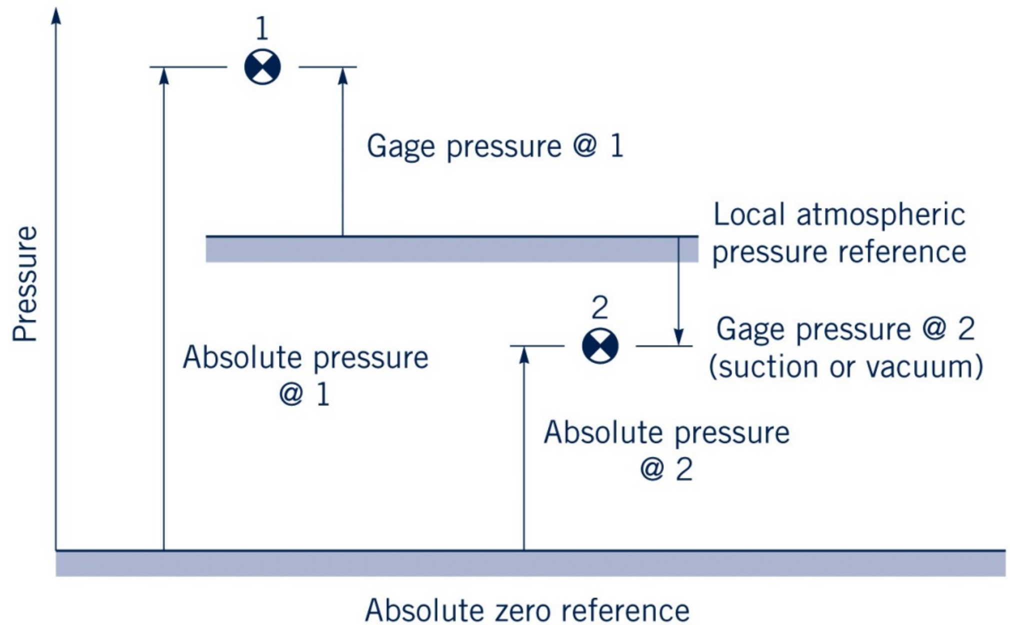


Figure 2.7

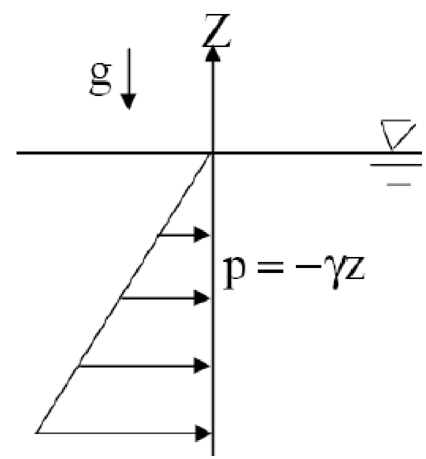
Pressure Variation with Elevation

For fluids at rest,

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$$

and

$$\frac{\partial p}{\partial z} = -\gamma$$



For constant γ (e.g., liquids), by integrating the above equations,

$$p = -\gamma z + C$$

At z = 0, $p = C = 0$ (gage),

$$\therefore p = -\gamma z$$

⇒ The pressure increases linearly with depth.

Pressure Measurements
(1) U-Tube manometer

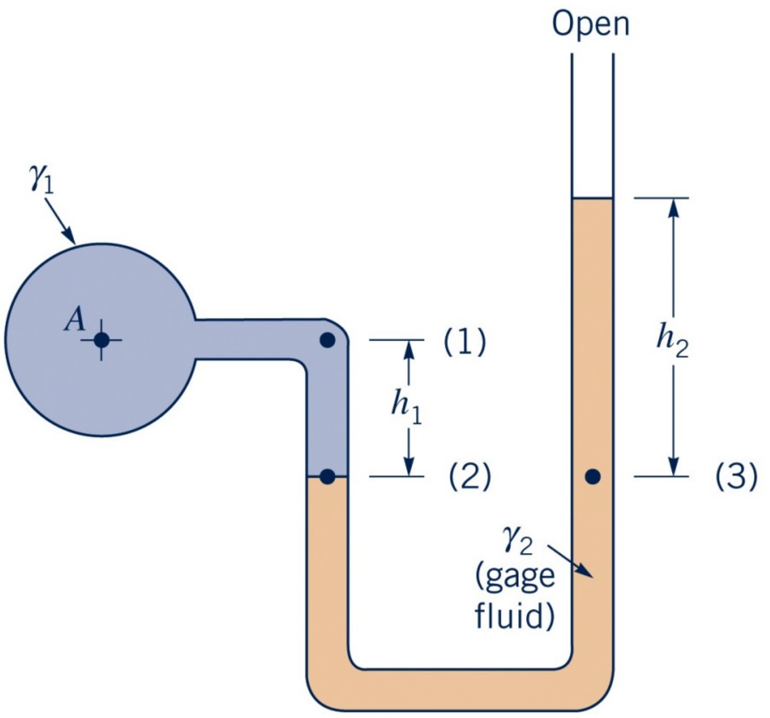


Figure 2.10

- Starting from one end, add pressure when move downward and subtract when move upward:

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$$

Thus,

$$\therefore p_A = \gamma_2 h_2 - \gamma_1 h_1$$

- If $\gamma_1 \ll \gamma_2$ (e.g., γ_1 is a gas and γ_2 a liquid),

$$p_A = \gamma_2 \left(h_2 - \frac{\gamma_1}{\gamma_2} h_1 \right)$$

$$\therefore p_A \approx \gamma_2 h_2$$

Pressure Measurements
(2) Differential manometer

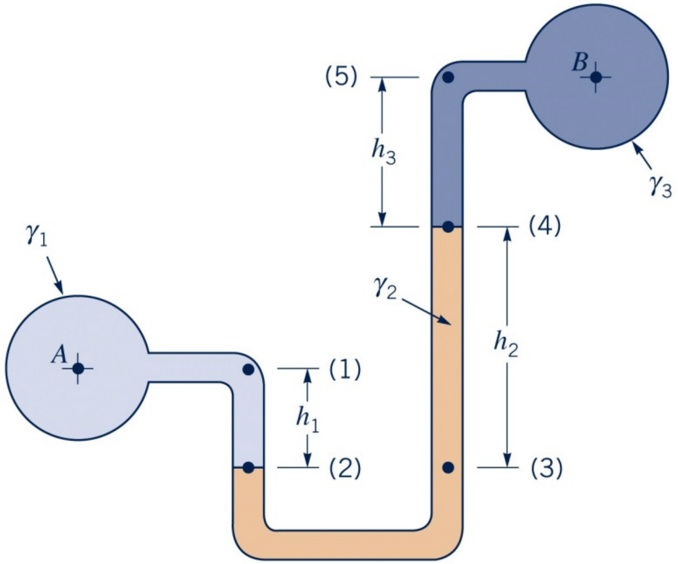


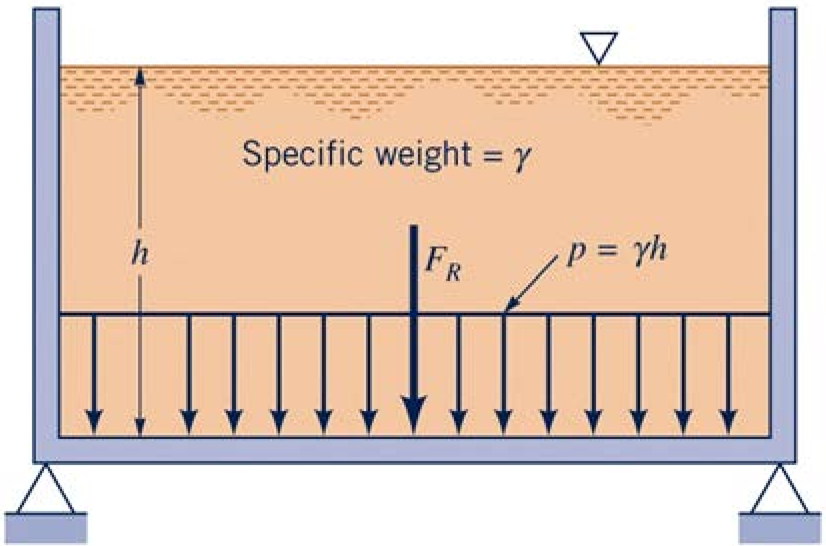
Figure 2.11

- To measure the *difference* in pressure:

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

$$\therefore \Delta p = p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

Hydrostatic Forces: (1) Horizontal surfaces



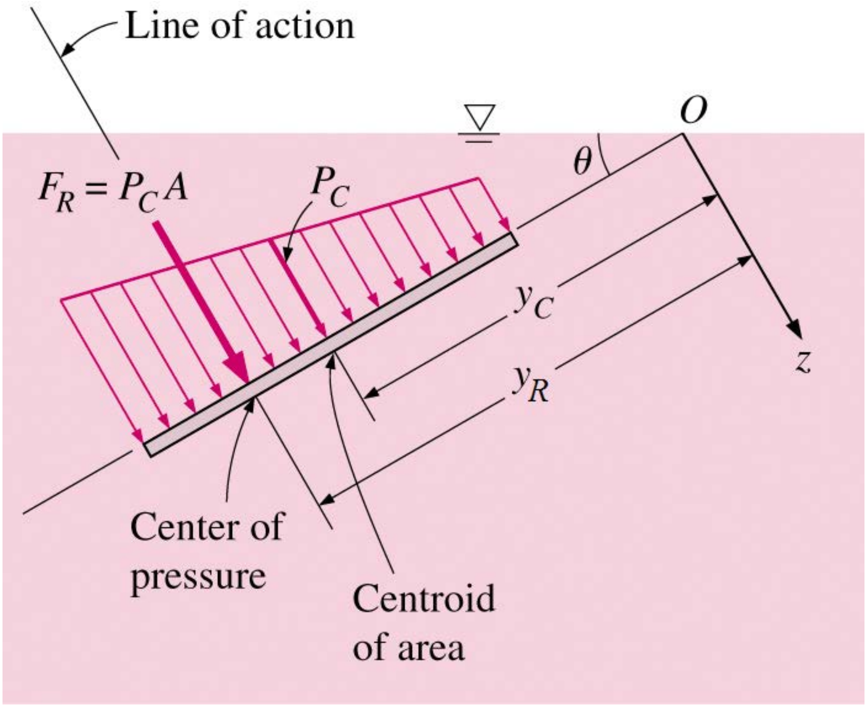
- Pressure is uniform on horizontal surfaces (e.g., the tank bottom) as

$$p = \gamma h$$

- The magnitude of the resultant force is simply

$$F_R = pA = \gamma h A \ \ (= \gamma \forall)$$

Hydrostatic Forces: (2) Inclined surfaces



- Average pressure on the surface

$$\bar{p} = p_C = \gamma h_c$$

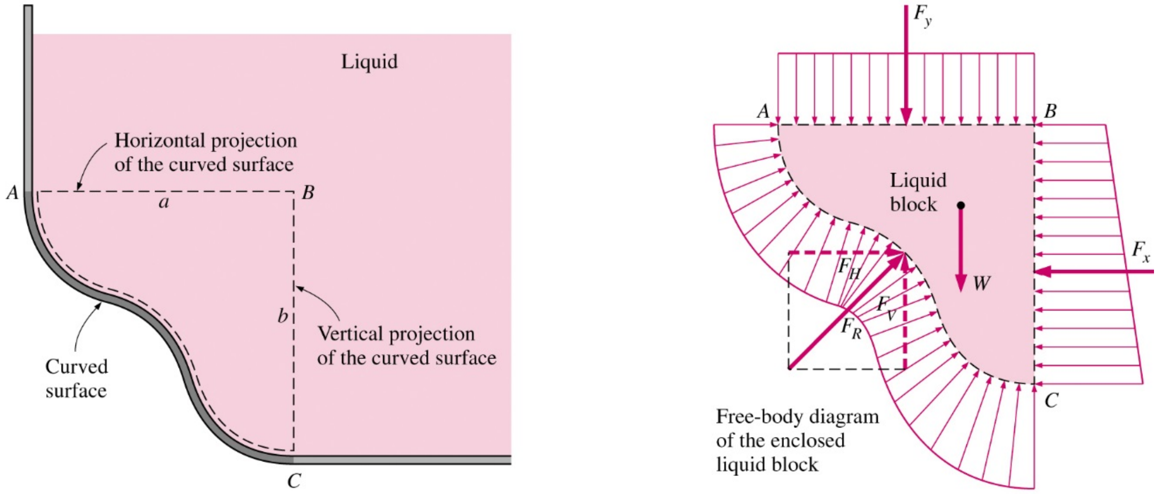
- The magnitude of the resultant force is simply

$$F_R = \bar{p}A = \gamma h_c A$$

- Pressure center

$$y_R = y_c + \frac{I_{xc}}{y_c A}$$

Hydrostatic Forces: (3) Curved surfaces



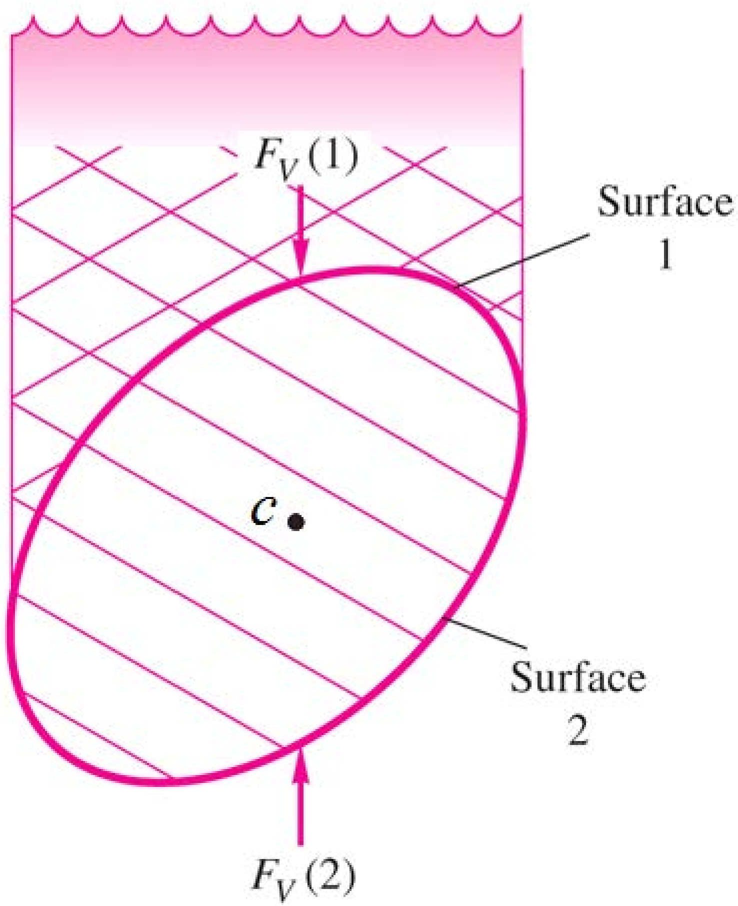
$$F_x = \bar{p}_{\text{proj}} \cdot A_{\text{proj}}$$

$$F_y = \gamma V\!\!\!\!/_{\text{above } AB}$$

$$W = \gamma V\!\!\!\!/_{ABC}$$

- Horizontal force component: $F_H = F_x$
- Vertical force component: $F_V = F_y + W = \gamma V\!\!\!\!/_{\text{total volume above } AC}$
- Resultant force: $F_R = \sqrt{F_H^2 + F_V^2}$

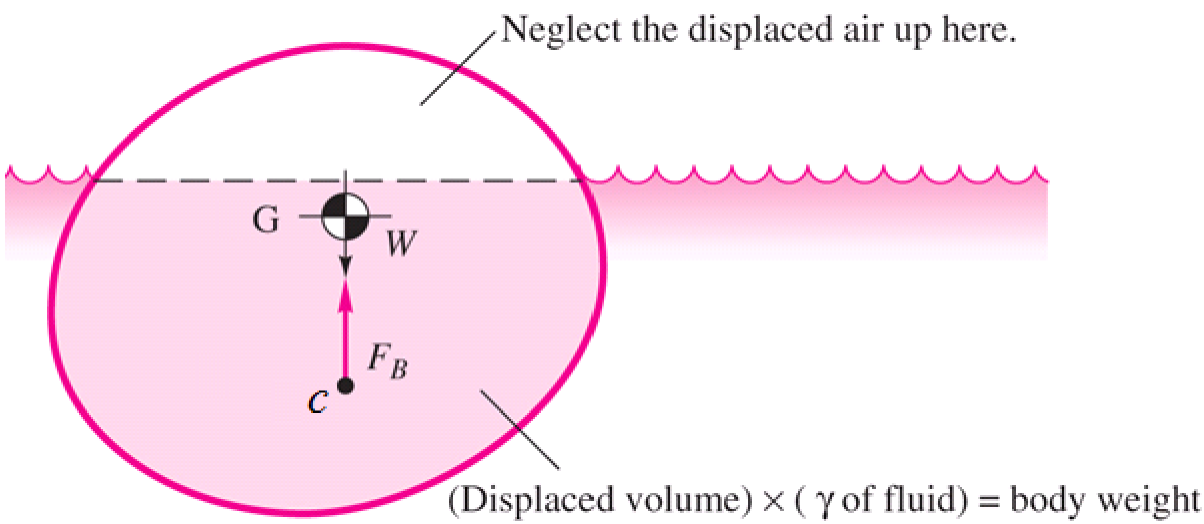
Buoyancy: (1) Immersed bodies



$$F_B = F_{V2} - F_{V1} = \gamma V\!\!\!/$$

- Fluid weight equivalent to body volume $V\!\!\!/$
- Line of action (or the center of buoyancy) is through the centroid of $V\!\!\!/$, c

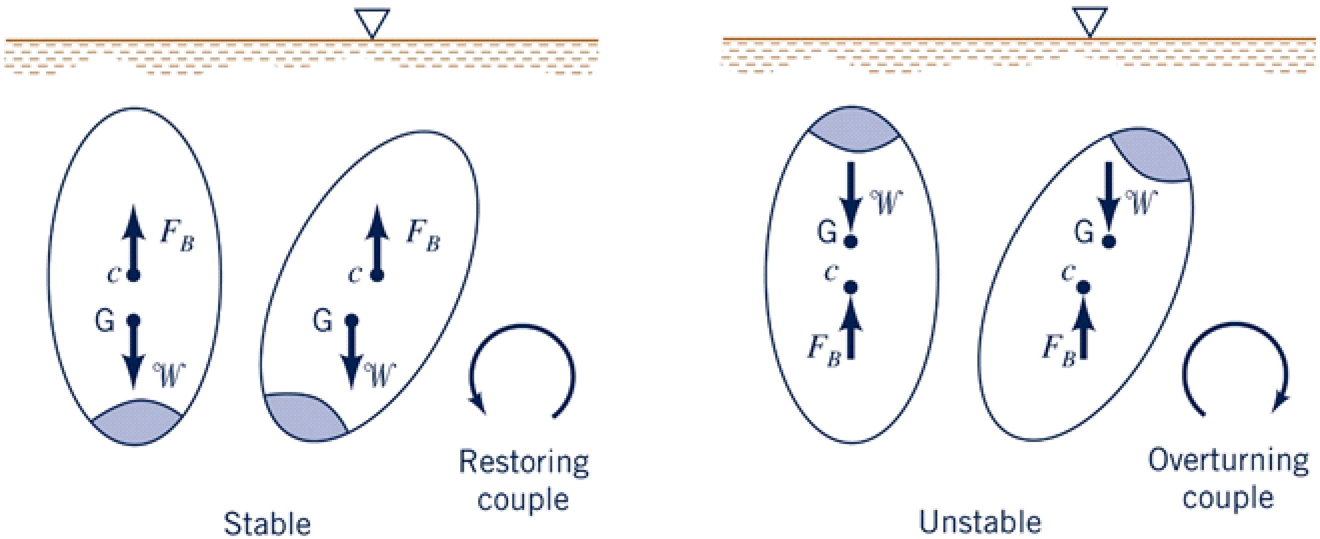
Buoyancy: (2) Floating bodies



$F_B = \gamma V\!\!\!/_{\text{displaced volume}}$ (i.e., the weight of displaced water)

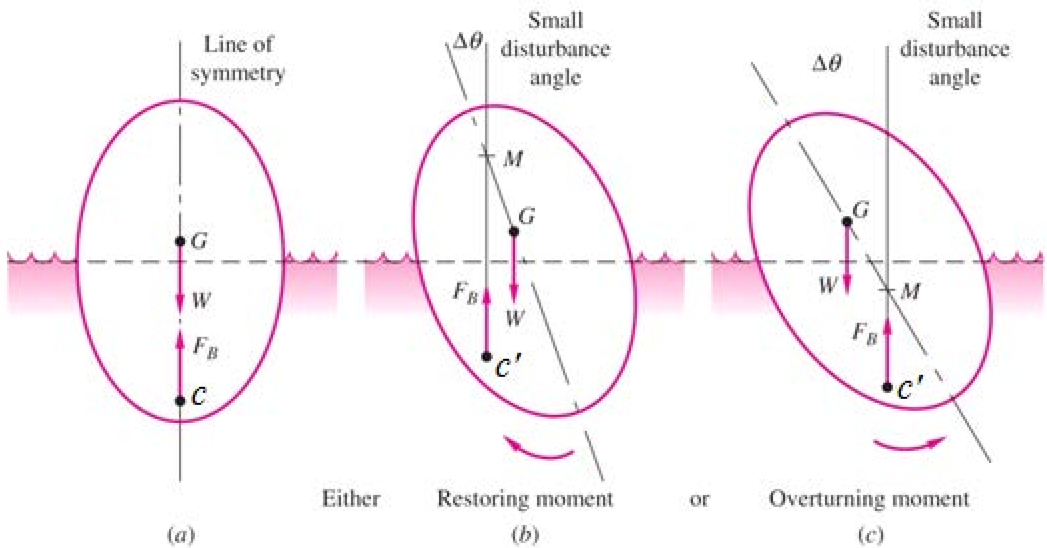
Line of action (or the center of buoyancy) is through the centroid of the displaced volume

Stability: (1) Immersed bodies



- If c is above G: Stable (righting moment when heeled)
- If c is below G: Unstable (heeling moment when heeled)

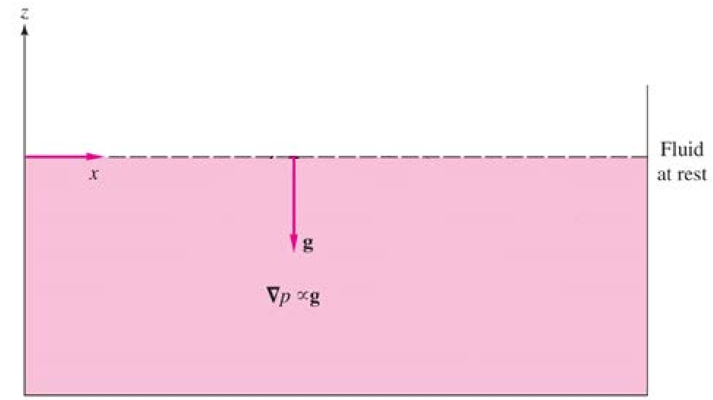
Stability: (2) Floating bodies

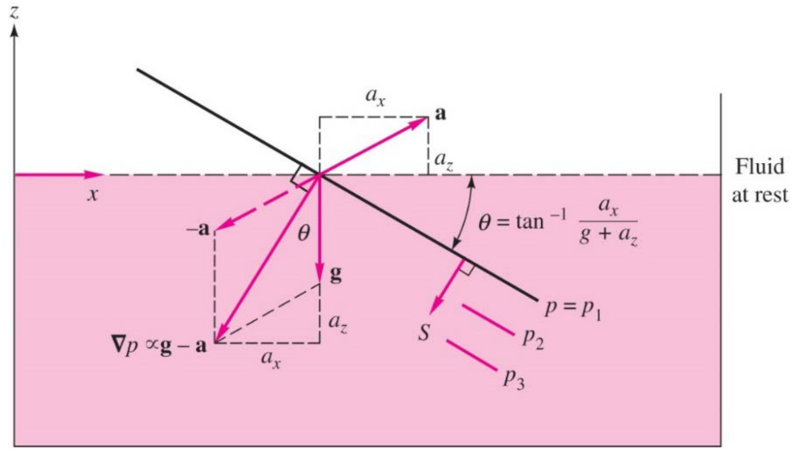


- $GM > 0$: Stable (M is above G)
- $GM < 0$: Unstable (G is above M)

$$GM = \frac{I_{00}}{\forall} - CG$$

Rigid-body motion: (1) Translation





- Fluid at rest
  - $\frac{\partial p}{\partial z} = -\rho \mathrm{g}$
  - $p = \rho \mathrm{g} z$

- Rigid-body in translation with a constant acceleration,

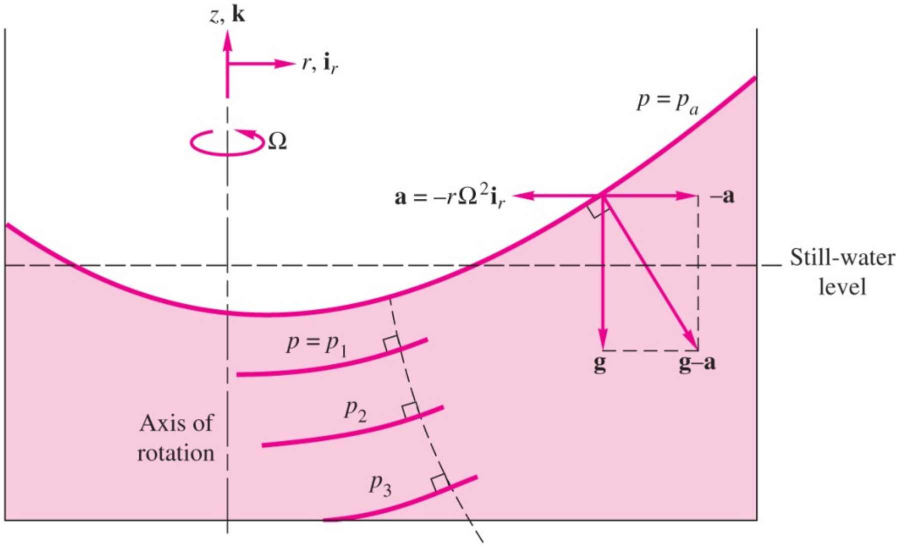
  $$\underline{a} = a_x \hat{\boldsymbol{\imath}} + a_z \hat{\boldsymbol{k}}$$

  - $\frac{\partial p}{\partial s} = -\rho \mathrm{G}$
  - $p = \rho \mathrm{G} s$

$$\mathrm{G} = (a_x^2 + (\mathrm{g} + a_z)^2)^{\frac{1}{2}}$$

$$\theta = \tan^{-1} \frac{a_x}{\mathrm{g} + a_z}$$

Rigid-body motion: (2) Rotation



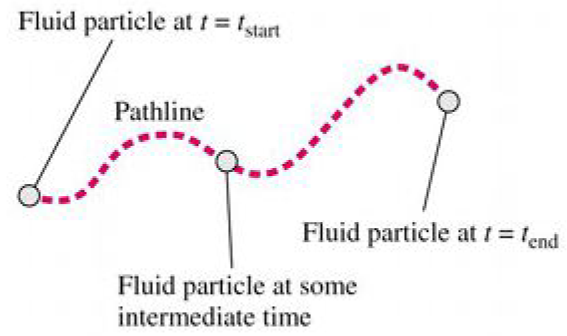
- Rigid-body in translation with a constant rotational speed Ω,

$$\underline{a} = -r\Omega^2 \hat{\boldsymbol{e}}_{\boldsymbol{r}}$$

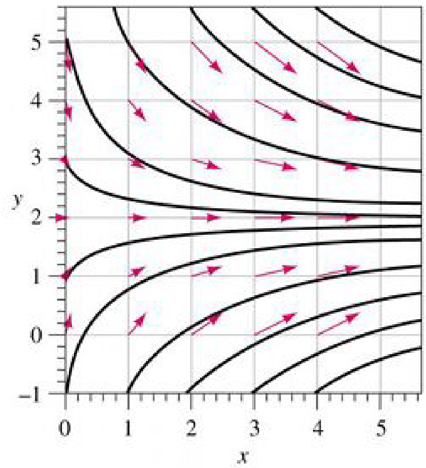
  - $\frac{\partial p}{\partial r} = \rho r \Omega^2$ and $\frac{\partial p}{\partial z} = -\rho g$
  - $p = \frac{\rho}{2} r^2 \Omega^2 - \rho g z + C$
  - $z = \frac{p_0 - p}{\rho g} + \frac{\Omega^2}{2g} r^2$

Flow Patterns

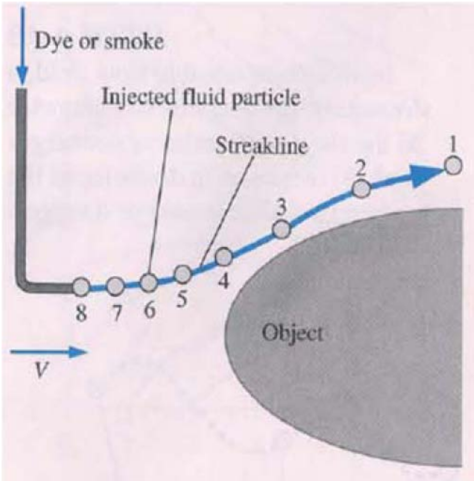
- **Pathline**: The actual path traveled by a given fluid particle.
- **Streamline**: A line that is everywhere tangent to the velocity vector at a given instant.
- **Streakline**: The locus of particles which have earlier passed through a particular point.
- For steady flow, all three lines coincide.



Pathline

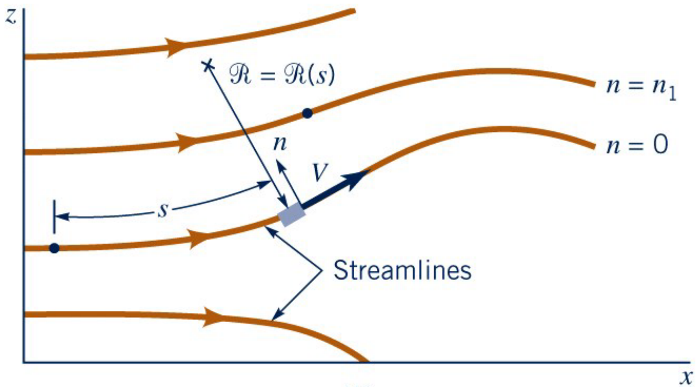


Streamline



Streakline

Streamline coordinates



- Velocity

$$\underline{V} = v_s\hat{\mathbf{s}} + v_n\hat{\mathbf{n}}$$

where

$$v_s = V$$
$$v_n = 0$$

- Acceleration in streamline coordinates:

$$\underline{a} = a_s\hat{\mathbf{s}} + a_n\hat{\mathbf{n}}$$

where,

- $a_s = \frac{\partial v_s}{\partial t} + v_s\frac{\partial v_s}{\partial s}$
- $a_n = \frac{\partial v_n}{\partial t} + \frac{v_s^2}{\Re}$

Note:

$$\rho g\hat{\boldsymbol{k}} = \frac{\partial(\gamma z)}{\partial z}\hat{\boldsymbol{k}} = \nabla(\gamma z)$$

- Euler equation in the streamline coordinates

$$\rho\underline{a} = -\nabla(p + \gamma z)$$

or

$$\rho\left(\frac{\partial v_s}{\partial t} + v_s\frac{\partial v_s}{\partial s}\right) = -\frac{\partial}{\partial s}(p + \gamma z)$$
$$\rho\left(\frac{\partial v_n}{\partial t} + \frac{v_s^2}{\Re}\right) = -\frac{\partial}{\partial n}(p + \gamma z)$$

Bernoulli Equation

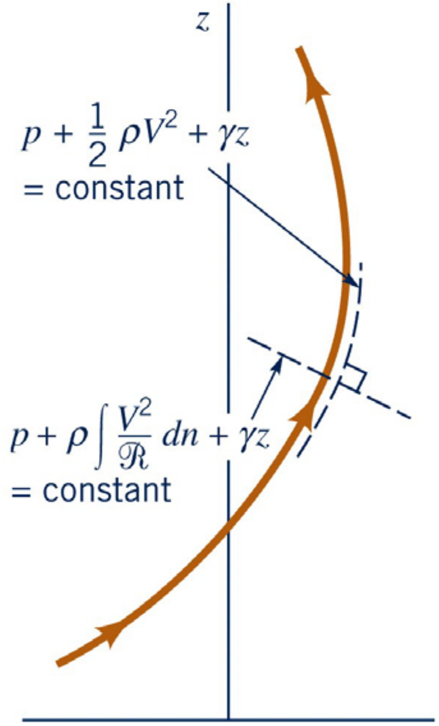
Integration of the Euler equation for a **steady incompressible** flow:

- Along a streamline:

$$p + \frac{1}{2}\rho V^2 + \gamma z = \text{Constant}$$

- Across the streamline:

$$p + \rho \int \frac{V^2}{\mathfrak{R}} dn + \gamma z = \text{Constant}$$



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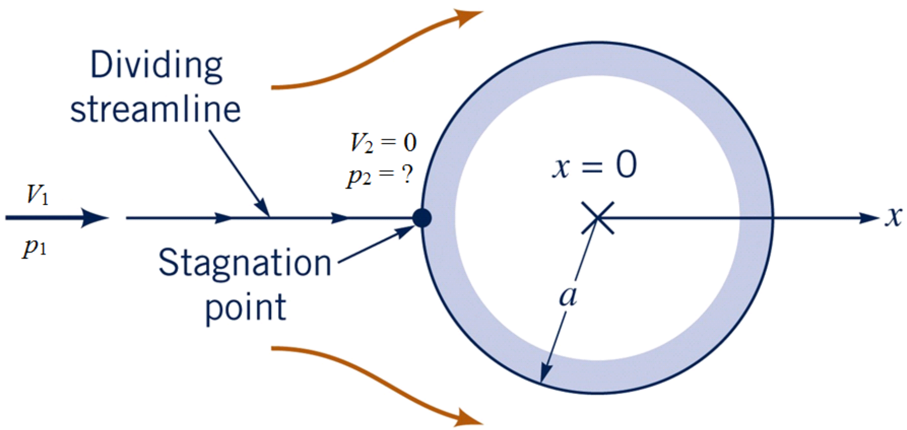
Alternative Forms and Restrictions of Bernoulli equation

- Static, stagnation dynamic, and Total pressure

$$\underbrace{\underbrace{p}_{\text{static pressure}} + \underbrace{\frac{1}{2}\rho V^2}_{\text{dynamic pressure}}}_{\text{stagnation pressure}} + \underbrace{\gamma z}_{\text{hydrostatic pressure}} = p_T = \text{constant}$$



Since $V_2 = 0$ and $z_1 = z_2$,

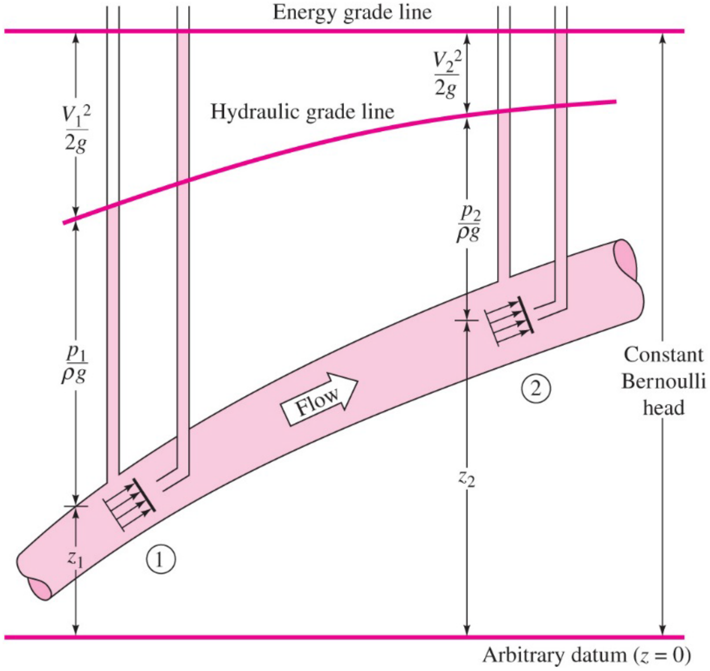
$$p_1 + \frac{1}{2}\rho V_1^2 + 0 = p_2 + 0 + 0$$

$$\therefore p_2 = p_1 + \frac{1}{2}\rho V_1^2$$

Alternative Forms and Restrictions of Bernoulli equation – Contd.

- Head form

$$\therefore \underbrace{\frac{p}{\gamma}}_{\text{pressure head}} + \underbrace{\frac{V^2}{2g}}_{\text{velocity head}} + \underbrace{z}_{\text{elevation head}} = \text{constant}$$



- Restrictions
  1) Inviscid flow (i.e., no friction)
  2) Incompressible flow (i.e., ρ = constant)
  3) Steady flow

Pressure Variation in a Flowing Stream

Bernoulli equation across the streamline:

$$p + \int \rho \frac{V^2}{\Re} dn + \gamma z = \text{Constant}$$

- From A to B, $\Re = \infty$

$$p_1 = p_2 + \int \rho \frac{V^2}{\Re} dn + \gamma (z_2 - z_1)$$

$$\therefore p_1 = \gamma h_{2-1}$$

- Let $dn = -dz$ for the portion from C to D

$$p_4 + \rho \int_{z_3}^{z_4} \frac{V^2}{\Re} (-dz) + z_4 = p_3 + \gamma z_3$$

$$\therefore p_3 = \gamma h_{4-3} - \underbrace{\rho \int_{z_3}^{z_4} \frac{V^2}{\Re} dz}_{>0}$$

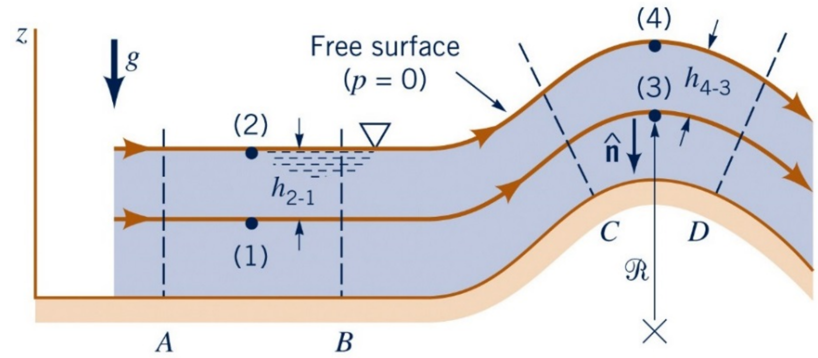
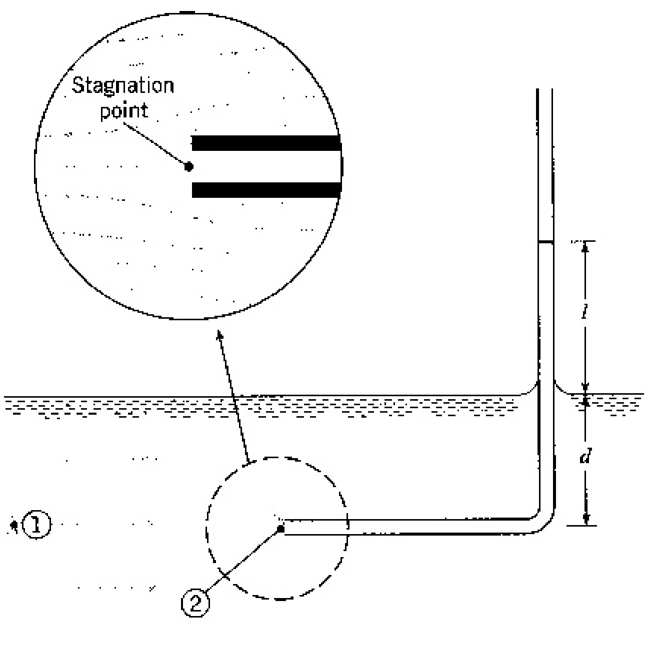


Figure E3.5b

- For the portion from A to B, where the flow is parallel, the pressure variation in the vertical direction is the same as if the fluid were stationary.
- For the portion from C to D, the pressure at (3) is less than the hydrostatic value, γh_{4-3}, due to the curved streamline.

Application of Bernoulli equation
(1) Stagnation tube



$$p_1 + \rho \frac{V_1^2}{2} + \gamma z_1 = p_2 + \rho \frac{V_2^2}{2} + \gamma z_2$$

Since V_2 = 0 (stagnation point) and $z_1 = z_2$,

$$p_1 + \rho \frac{V_1^2}{2} = p_2$$

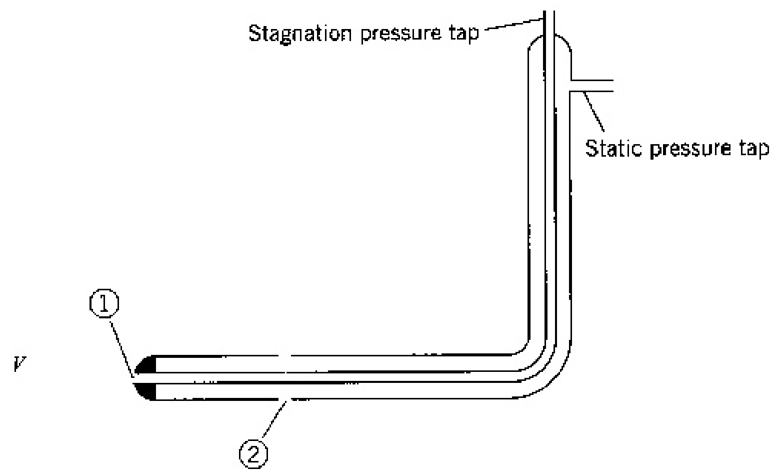
Solve for V_1:

$$V_1 = \sqrt{\frac{2(p_2 - p_1)}{\rho}}$$

Also, $p_1 = \gamma d$ and $p_2 = \gamma(d + \ell)$

$$\therefore V_1 = \sqrt{2 g \ell}$$

Application of Bernoulli equation
(2) Pitot tube



$$p_1 + \rho \frac{V_1^2}{2} + \gamma z_1 = p_2 + \rho \frac{V_2^2}{2} + \gamma z_2$$

where $V_1 = 0$ (stagnation point),

$$p_1 + \gamma z_1 = p_2 + \rho \frac{V_2^2}{2} + \gamma z_2$$

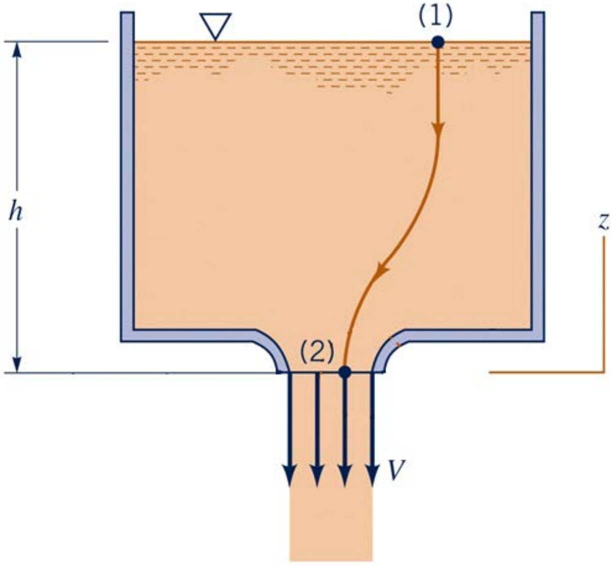
Solve for V_2:

$$V_2 = \sqrt{2g\left[\underbrace{\left(\frac{p_1}{\gamma} + z_1\right)}_{=h_1} - \underbrace{\left(\frac{p_2}{\gamma} + z_2\right)}_{=h_2}\right]}$$

Thus,

$$\therefore V = V_2 = \sqrt{2\mathrm{g} \cdot \underbrace{(h_1 - h_2)}_{\text{from manometer}}}$$

Application of Bernoulli equation
(3) Free jets



Applying the B.E. between (1) and (2),

$$p_1 + \rho \frac{V_1^2}{2} + \gamma z_1 = p_2 + \rho \frac{V_2^2}{2} + \gamma z_2$$

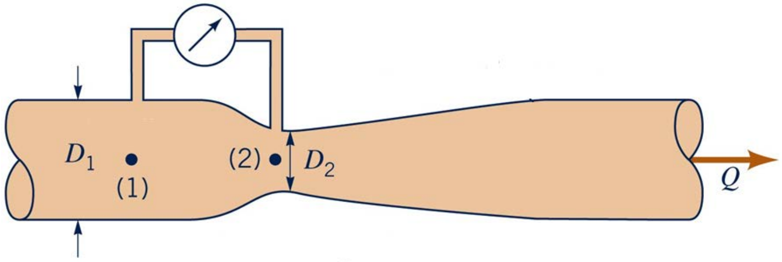
Since $p_1 = p_2 = 0$ and $V_1 \approx 0$, and $z_1 - z_2 = h$,

$$\gamma h = \rho \frac{V_2^2}{2}$$

Solve for V_2:

$$V_2 = \sqrt{2 \frac{\gamma h}{\rho}} = \sqrt{2gh}$$

Application of Bernoulli equation
(4) Venturimeter



- Volume flow rate
$$Q = VA$$
- Mass flow rate
$$\dot{m} = \rho Q = \rho VA$$
- Conservation of mass, $\dot{m}_1 = \dot{m}_2$,
$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$
- Incompressible flow (ρ = const.),
$$V_1 A_1 = V_2 A_2$$

Bernoulli eq. with $z_1 = z_2$,

$$p_1 + \rho \frac{V_1^2}{2} = p_2 + \rho \frac{V_2^2}{2}$$

Continuity eq.,

$$V_1 = \frac{A_2}{A_1} V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2$$

Thus,

$$p_1 + \frac{1}{2}\rho \left(\left(\frac{D_2}{D_1}\right)^2 V_2\right)^2 = p_2 + \rho \frac{V_2^2}{2}$$

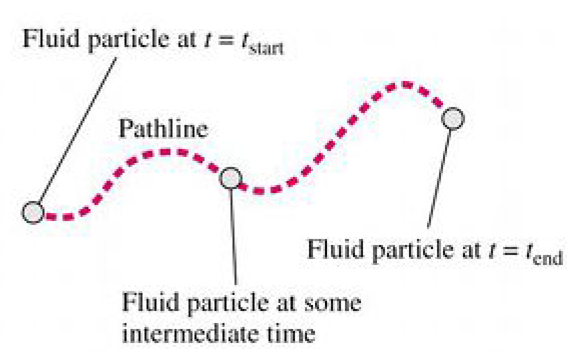
Solve for V_2,

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (D_2/D_1)^4]}}$$

Then,

$$Q = V_2 A_2$$

Flow Kinematics: (1) Lagrangian Description



- Keep track of individual fluid particles

$$\underline{V_p}(t) = \frac{d\underline{x}}{dt} = u_p(t)\hat{\boldsymbol{\imath}} + v_p(t)\hat{\boldsymbol{\jmath}} + w_p(t)\widehat{\boldsymbol{k}}$$

$$u_p = \frac{dx}{dt}, v_p = \frac{dy}{dt}, w_p = \frac{dz}{dt}$$

$$\underline{a_p} = \frac{d\underline{V_p}}{dt} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\widehat{k}$$

$$a_x = \frac{du_p}{dt}, a_y = \frac{dv_p}{dt}, a_z = \frac{dw_p}{dt}$$

Flow Kinematics: (2) Eulerian Description

- Focus attention on a fixed point in space

$$\underline{V}(\underline{x},t) = u(\underline{x},t)\hat{\boldsymbol{\imath}} + v(\underline{x},t)\hat{\boldsymbol{\jmath}} + w(\underline{x},t)\hat{\boldsymbol{k}}$$

$$\underline{a} = \frac{D\underline{V}}{Dt} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

Or,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Acceleration and material derivatives –Contd.

- Acceleration

$$\underline{a} = \frac{D\underline{V}}{Dt} = \underbrace{\frac{\partial \underline{V}}{\partial t}}_{\substack{\text{Local} \\ \text{acc.}}} + \underbrace{\left(\underline{V} \cdot \nabla\right)\underline{V}}_{\substack{\text{Convective} \\ \text{acc.}}}$$

  - $\frac{\partial \underline{V}}{\partial t}$ = Local or temporal acceleration. Velocity changes with respect to time at a given point
  - $\left(\underline{V} \cdot \nabla\right)\underline{V}$ = Convective acceleration. Spatial gradients of velocity

- Material derivative:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \left(\underline{V} \cdot \nabla\right)$$

where

$$\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$$

4. Flow classification

- One-, Two-, and Three-dimensional flow
- Steady vs. Unsteady flow
- Incompressible vs. Compressible flow
- Viscous vs. Inviscid flow
- Rotational vs. Irrotational flow
- Laminar vs. Turbulent viscous flow
- Internal vs. External flow
- Separated vs. Unseparated flow

Reynolds Transport Theorem (RTT)

General RTT (for moving and deforming CV):

$$\frac{dB_{\text{sys}}}{dt} = \frac{d}{dt}\left(\int_{\text{CV}} \beta\rho dV\right) + \int_{\text{CS}} \beta\rho \underline{V_r} \cdot \hat{\boldsymbol{n}} dA$$

Special Cases:

1) Non-deforming (but moving) CV

$$\frac{dB_{\text{sys}}}{dt} = \int_{\text{CV}} \frac{\partial}{\partial t}(\beta\rho) d\forall + \int_{\text{CS}} \beta\rho \underline{V_r} \cdot \hat{\boldsymbol{n}} dA$$

2) Fixed CV

$$\frac{dB_{\text{sys}}}{dt} = \int_{\text{CV}} \frac{\partial}{\partial t}(\beta\rho) d\forall + \int_{\text{CS}} \beta\rho \underline{V} \cdot \hat{\boldsymbol{n}} dA$$

3) Steady flow:

$$\frac{\partial}{\partial t} = 0$$

4) Flux terms for uniform flow across discrete CS's (steady or unsteady)

$$\int_{\text{CS}} \beta\rho \underline{V} \cdot \hat{\boldsymbol{n}} dA = \sum (\beta\dot{m})_{\text{out}} - \sum (\beta\dot{m})_{\text{in}}$$

RTT Summary

For fixed CV's:

Parameter (B)	$\beta = B/m$	RTT	Remark
Mass (m)	1	$0 = \frac{d}{dt}\int_{\mathrm{CV}} \rho d\forall + \int_{\mathrm{CS}} \rho \underline{V} \cdot \hat{\boldsymbol{n}} dA$	Continuity eq. (Ch. 5.1)
Momentum ($m\underline{V}$)	$\underline{V}$	$\sum \underline{F} = \frac{d}{dt}\int_{\mathrm{CV}} \underline{V} \rho d\forall + \int_{\mathrm{CS}} \underline{V} \rho \underline{V} \cdot \hat{\boldsymbol{n}} dA$	Linear momentum eq. (Ch. 5.2)
Energy (E)	e	$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{\mathrm{CV}} e \rho d\forall + \int_{\mathrm{CS}} e \rho \underline{V} \cdot \hat{\boldsymbol{n}} dA$	Energy eq. (Ch. 5.3)

Continuity Equation

RTT with B = mass and β = 1,

$$\underbrace{0 = \frac{Dm_{\text{sys}}}{Dt}}_{\text{mass conservatoin}} = \frac{d}{dt}\int_{\text{CV}} \rho d\forall + \int_{\text{CS}} \rho \underline{V} \cdot \hat{\boldsymbol{n}} dA$$

or

$$\underbrace{\int_{\text{CS}} \rho \underline{V} \cdot \hat{\boldsymbol{n}} dA}_{\substack{\text{Net rate of outflow} \\ \text{of mass across CS}}} = \underbrace{-\frac{d}{dt}\int_{\text{CV}} \rho d\forall}_{\substack{\text{Rate of decrease of} \\ \text{mass within CV}}}$$

Note: Incompressible fluid (ρ = constant)

$$\int_{\text{CS}} \underline{V} \cdot \hat{\boldsymbol{n}} dA = -\frac{d}{dt}\int_{\text{CV}} d\forall$$ (Conservation of volume)

Simplifications

1. Steady flow

$$\int_{CS} \rho \underline{V} \cdot \hat{\boldsymbol{n}} dA = 0$$

2. If $\underline{V}$ = constant over discrete CS's (i.e., one-dimensional flow)

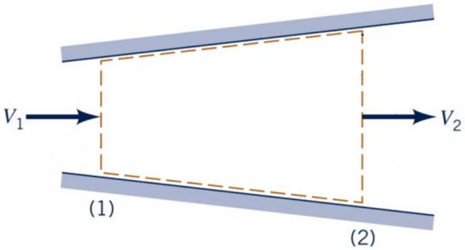
$$\int_{CS} \rho \underline{V} \cdot \hat{\boldsymbol{n}} dA = \sum_{\text{out}} \rho VA - \sum_{\text{in}} \rho VA$$

3. Steady one-dimensional flow in a conduit

$$(\rho VA)_{\text{out}} - (\rho VA)_{\text{in}} = 0$$

or

$$\rho_2 V_2 A_2 - \rho_1 V_1 A_1 = 0$$



For ρ = constant

$$V_1 A_1 = V_2 A_2 \quad (\text{or } Q_1 = Q_2)$$