Buoyancy and Stability of Immersed and Floating Bodies

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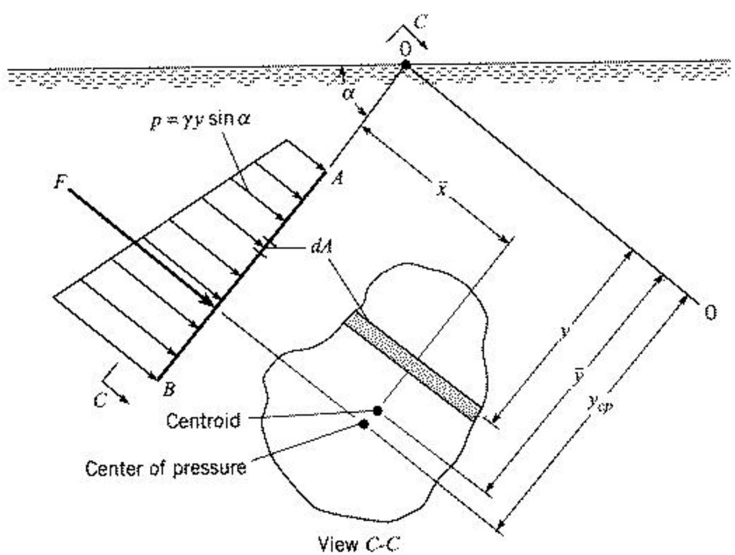
Hyunse Yoon, Ph.D.

Associate Research Scientist
IIHR-Hydroscience & Engineering

Review: Pressure Force on a Plane Surface

- The resultant force F_R (or F) acting on a (completely submerged) plane surface is equal to the product of the pressure at the centroid of the surface (equivalent to the average pressure on the surface) and the surface area, and its line of action passes through the center of pressure y_{cp}.
- Care is needed when the plate is partially submerged; take account only the wet part of the plate in to the calculations.

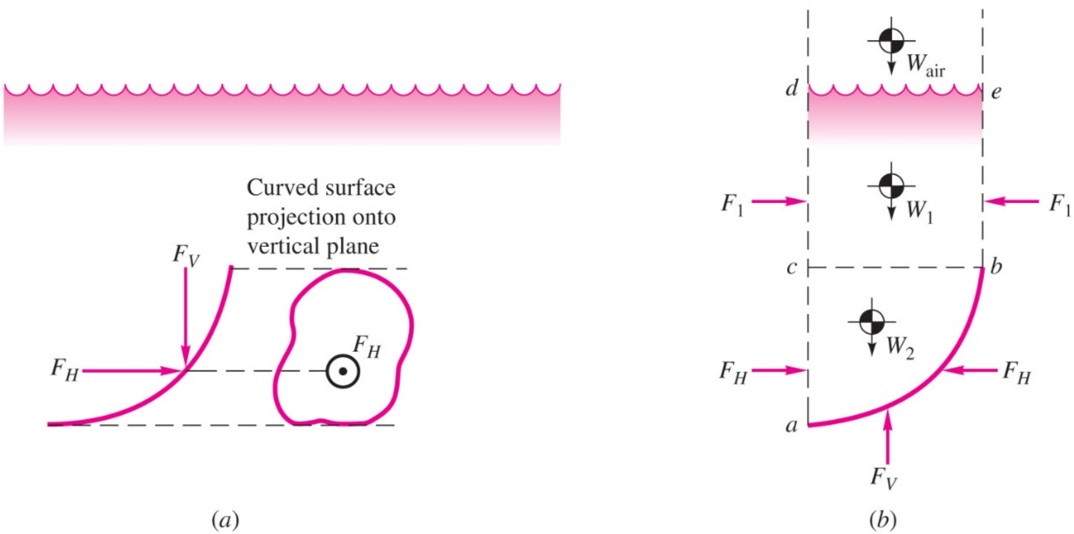


$$F = \bar{p}A = \gamma\bar{y}\sin\alpha\, A$$

$$y_{cp} = \bar{y} + \frac{I}{\bar{y}A}$$

Review: Pressure Force on a Curved Surface

- The resultant force acting on a curved surface is determined by determining the horizontal and vertical components F_H and F_V separately.
  - F_H: Equal to the force acting on a vertical projection of the curved surface including both the magnitude and the line of action.
  - F_V: Equal to the net weight of the column of fluid above the curved surface with the line of action through the centroid of that fluid volume.



For the curved surface ab,

$$F_H = \bar{p}A_{\text{proj}}$$
$$= \gamma\left(cd + \frac{ac}{2}\right)(ac \times \text{width})$$

$$F_V = W_1 + W_2 = \gamma V\!\!\!\!-_{abedc}$$

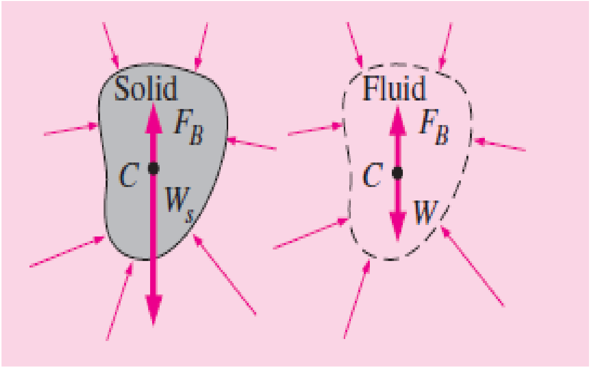
Note: W_{air} is ignorable

Buoyancy

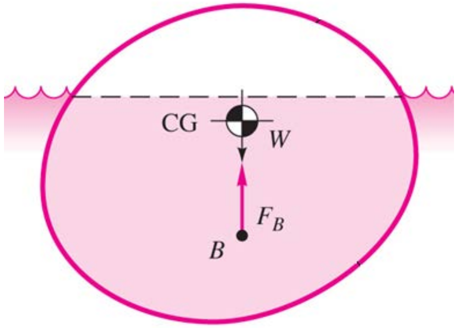
- A body immersed in a fluid experiences a vertical buoyant force F_B equal to the weight of the fluid it displaces.

$$F_B = \gamma_f \forall$$

- F_B is due to the increase of pressure in a fluid with depth and acts upward through the centroid of the displaced volume.
- A floating body displaces its own weight in the fluid in which it floats, i.e., $W = F_B$.



Immersed body (The buoyant forces acting on a solid body submerged in a fluid and on a fluid body of the same shape at the same depth are identical. The F_B is equal in magnitude to the weight W of the displaced fluid, but is opposite in direction. The solid weight W_s is not necessarily the same as W, i.e., $W_s > W$ (sink), $W_s = W$ (neutrally buoyant, i.e., suspending), or $W_s < W$ (float)).



Floating body (In this figure, W is the weight of the floating body and CG is the center of gravity. The point B is the centroid of the displace fluid volume; W must be the same as F_B).

Example: Buoyancy

2.135 The homogeneous timber *AB* of Fig. P2.135 is 0.15 m × 0.35 m in cross section. Determine the specific weight of the lumber and the tension in the rope.

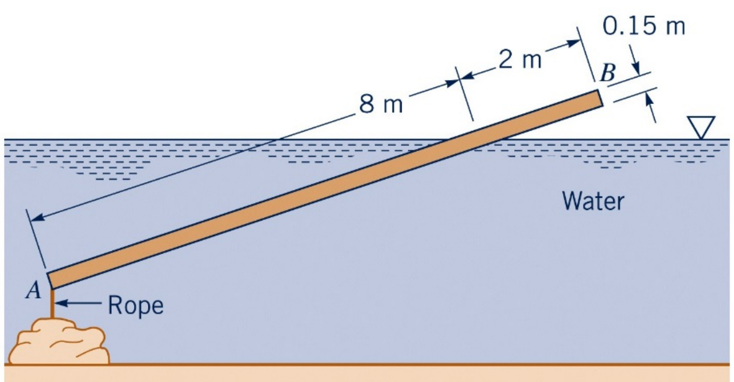
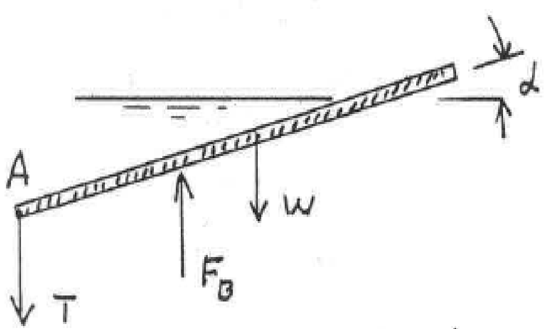


Figure P2.135



For equilibrium, $\sum M_A = 0$,

$$W_t \cdot \left(\frac{L}{2}\cos\alpha\right) = F_B \cdot \left(\frac{\ell}{2}\cos\alpha\right)$$

where, t denotes the timber, $W_t = \gamma_t V_t$, $F_B = \gamma \mathcal{V}$, L = 10 m, ℓ = 8 m. Also, $V_t = L \cdot A$ and $\mathcal{V} = \ell \cdot A$, where are A = (0.15)(0.35) = 0.0525 m^2. Thus,

$$\gamma_t L A \cdot \left(\frac{L}{2}\cos\alpha\right) = \gamma V \ell A \cdot \left(\frac{\ell}{2}\cos\alpha\right)$$

$$\therefore \gamma_t = \gamma \cdot \left(\frac{\ell}{L}\right)^2 = (9{,}800)\left(\frac{8}{10}\right)^2 = 6{,}272\ \mathrm{N/m^3}$$

Also, $\sum F_{\text{vertical}} = 0$,

$$F_B - T - W = 0$$

$$\begin{aligned}\therefore T &= F_B - W = \gamma \ell A - \gamma_t L A \\ &= (9800)(8)(0.0525) - (6272)(10)(0.0525) \\ &= 823\ \mathrm{N}\end{aligned}$$

Stability: The “Ball on the floor” Analogy

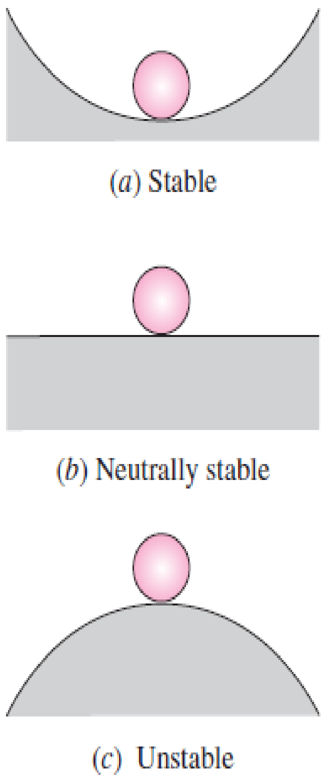


FIGURE 3–43
Stability is easily understood by analyzing a ball on the floor.

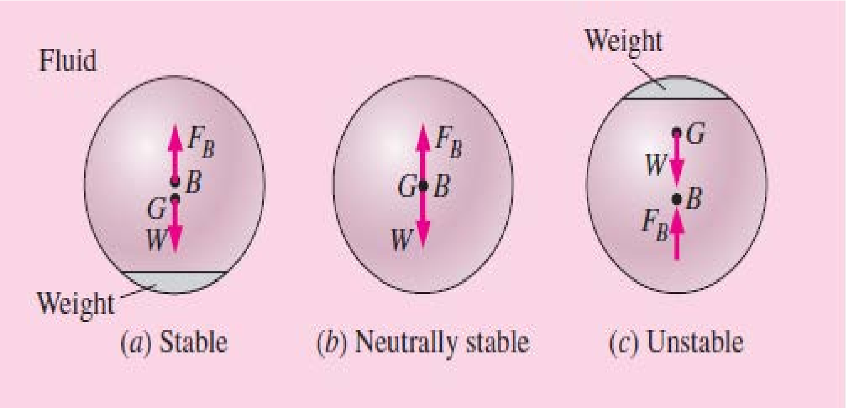
Three balls at rest on three different types of floor: The ball is

a) **STABLE** since any small disturbance (someone moves the ball to the right or left) generates a restoring force (due to gravity) that returns it to its initial position.

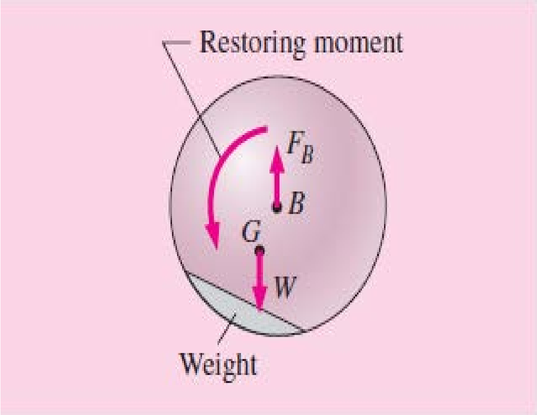
b) **NEUTRALLY STABLE** because if someone moves the ball to the right or left, it would stay put at its new location. It has no tendency to move back to its original location, nor does it continue to move away.

c) **UNSTABLE** since any disturbance, even an infinitesimal one, causes the ball to roll off the hill – it does not return to its original position; rather it *diverges* from it.

Stability of Immersed Bodies



An immersed neutrally buoyant body is (a) stable if G is directly below B, (b) neutrally buoyant if G and B are coincident, and (c) unstable if G is directly above B.



When G is not vertically aligned with B, the body would rotate to its stable state, even without any disturbance.

The stability of an immersed body depends on the relative locations of the center of gravity G of the body and the center of buoyancy B that is the centroid of the displaced volume*.

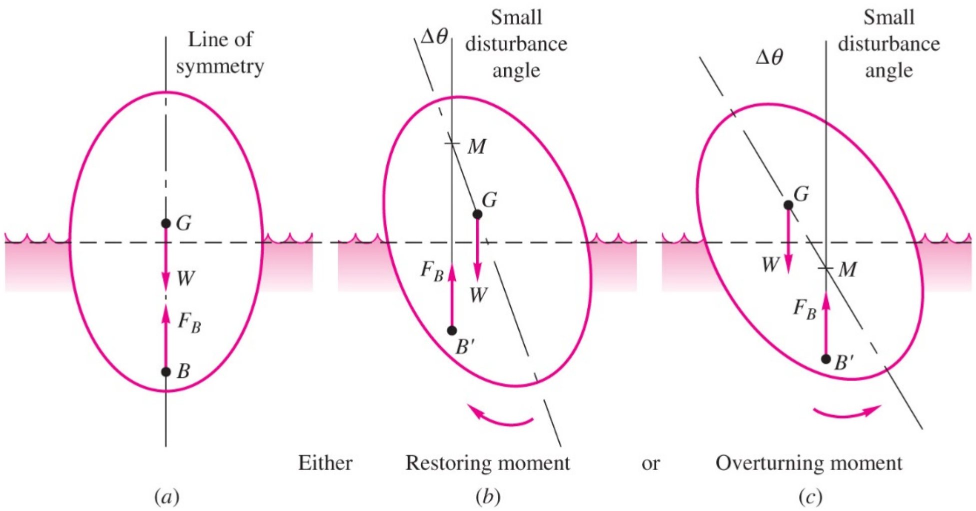
a) The body is **stable** if the body is bottom-heady (G below B); A disturbance produces restoring moment to return the body to its original stable position (E.g., submarines or hot-air balloons).

b) Bodies of homogeneous density are **neutrally stable**, for which G and B coincide.

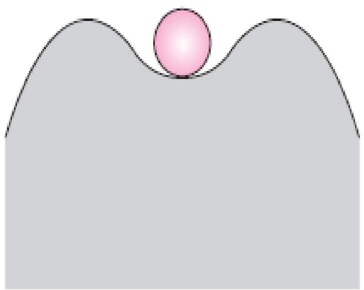
c) The body is **unstable** if G is directly above B; any disturbance will cause this body to turn upside down.

* Note: Class lecture note uses a character C, instead of B, to denote the center of buoyancy.

Stability of Floating Bodies



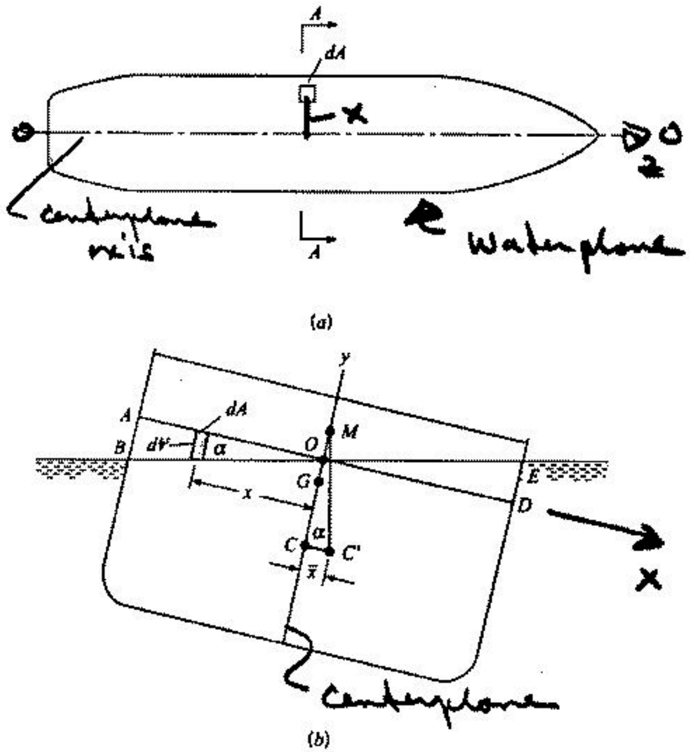
Calculations of the metacenter M of the floating body shown in (a). Tilt the body a small angle $\Delta\theta$. Either (b) B′ moves far out (point M above G denotes stability; or (c) B′ moves slightly (point M below G denotes instability).



A ball in a trough between two hills is stable for small disturbances, but unstable for large disturbances.

- Unlike immersed bodies, a floating body may still be stable when G is directly above B; This is because the centroid of the displaced volume shifts to the side to a point B′ while G remains unchanged.
- If B′ is sufficiently far, F_B and W create a restoring (or righting) moment.
- A measure of stability is the metacentric height GM, the distance between G and the metacenter M – the intersection point of the lines of action of F_B before and after rotation.
- A floating body is STABLE if M is above G (GM > 0) and UNSTABLE if M is below G (GM < 0; overturning moment and capsize).
- The body can tilt to some maximum angle without capsizing, but beyond that angle it overturns (for a ship, it may sink).

Stability Related to Waterline Area



α = small heel angle
C = center of buoyancy
$\bar{x}$ = CC' = lateral displacement of C
L = Variable depth *L*(*x*) into paper
I_{OO} = moment of inertia of waterplane about z-axis O-O

- Tilting the body a small angle α then submerges the small wedge OED and uncovers an equal wedge AOB.
- The new position C' of the center of buoyancy is calculated as the centroid of the submerged portion BOEDB:

$$\bar{x}V_{BOEDB} = \int_{AODBA} x dV + \int_{OED} x dV - \int_{AOB} x dV$$

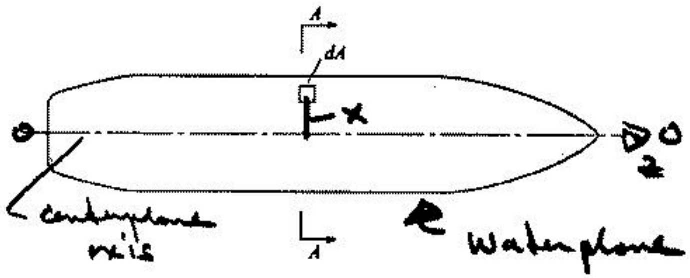
$$= 0 + \int_{OED} xL(x \tan\alpha \, dx) - \int_{AOB} xL(-x \tan\alpha dx)$$

$$= \tan\alpha \underbrace{\int_{\text{waterline}} x^2 dA_{\text{waterline}}}_{=I_{OO}}$$

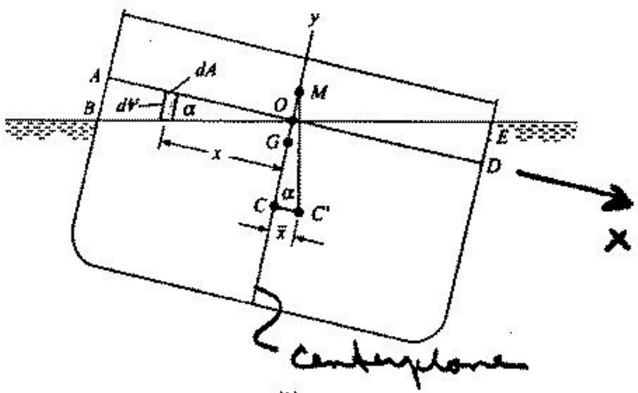
or,

$$\bar{x}V_{BOEDB} = I_{OO} \tan\alpha \qquad (1)$$

Stability of Related to Waterline Area – Contd.



(a)



(b)

α = small heel angle
C = center of buoyancy
$\bar{x}$ = CC' = lateral displacement of C
L = Variable depth $L(x)$ into paper
I_{OO} = moment of inertia of waterplane about z-axis O-O

Since V_{AOB} = V_{OED}, the tilted volume V_{BOEDB} is the same as the volume at rest V_{AODBA} or the displaced volume $\not{V}$. Thus, equation (1) can be rewritten as

$$\bar{x}\not{V} = \tan\alpha \, I_{OO} \qquad (2)$$

By rearranging equation (2),

$$\frac{\bar{x}}{\tan\alpha} = CM = \frac{I_{OO}}{\not{V}} = GM + CG$$

Solve for GM,

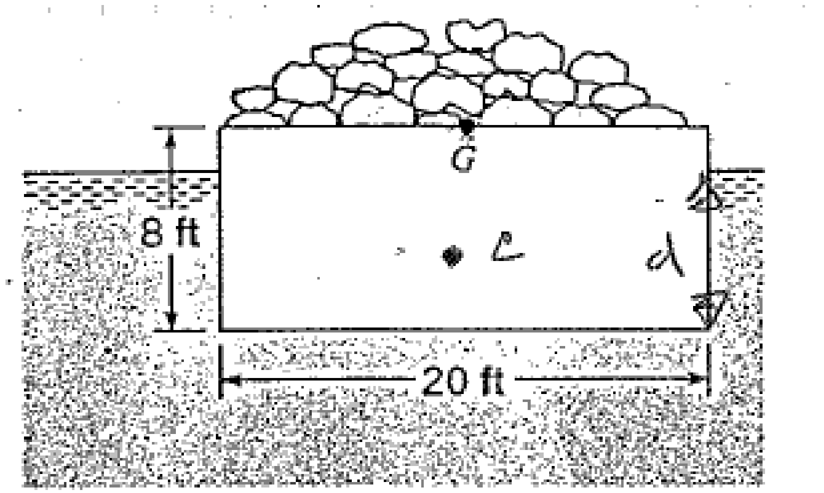
$$\therefore GM = \frac{I_{OO}}{\not{V}} - CG$$

Recall,

$$GM > 0 \qquad \text{Stable}$$
$$GM < 0 \qquad \text{Unstable}$$

Example: Stability

3.135 A barge 20ft wide and 50ft long is loaded with rock as shown. Assume that the center of gravity of the rock and barge is located along the centerline at the top surface of the barge. If the rock and the barge weigh 400,000lbf, will the barge float upright or tip over?



PROBLEM 3.135