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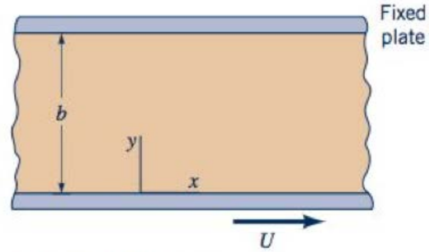
NAME



Quiz 10. The viscous, incompressible flow between the parallel plates shown in Figure is caused by both the motion of the bottom plate and a constant pressure gradient $\partial p/\partial x$. Starting from the following equations,

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Navier Stokes: $$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

(a) drive an expression for u and (b) calculate the shear stress τ_{xy} at bottom wall ($y=0$) if $\mu = 1.12\times 10^{-3}\ N\,s/m^2$, $\partial p/\partial x = 1\ N/m^3$, $U = 2\ m/s$ and $b = 1\ m$. Assume the flow is steady state, laminar, purely two-dimensional (w = 0 and $\partial/\partial z = 0$) and parallel to the walls (v = 0).

Note: Attendance (+2 points), format (+1 point)

Solution

(a) For steady flow, $\partial/\partial t$ = 0. As the flow is laminar and parallel, v = w = 0 and $\partial/\partial z = 0$. In this case $\partial u/\partial x$ = 0 from the continuity equation. With these conditions the Navier-Stokes equation reduces to

$$0=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial y^2}\right)$$ (+3 points)

By integrating the equation twice with respect to y,

$$u=\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)y^2+C_1 y+C_2$$ (+1 points)

At $y=0, u=U$

$$\therefore C_2=U$$

At $y=b, u=0$

$$\therefore C_1=-\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)b-U/b$$

Thus,

$$u=\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2-by)+U\left(1-\frac{y}{b}\right)$$ (+1 points)

(b) Deriving expression for the shear stress

$$\tau_{xy} = \mu \frac{du}{dy}$$

$$\tau_{xy} = \frac{1}{2}\left(\frac{\partial p}{\partial x}\right)(2y - b) + \mu U\left(-\frac{1}{b}\right)$$

(+1 points)

Evaluating shear stress at the bottom wall

$$\tau_{xy}(y = 0) = \frac{1}{2}\left(\frac{\partial p}{\partial x}\right)(-b) - \frac{\mu U}{b}$$

(+0.5 points)

$$\tau_{xy}(y = 0) = -\frac{1}{2}\left(1\,\frac{N}{m^3}\right)(1\ m) - \frac{1.12 \times 10^{-3}\ N\frac{s}{m^2} \times 2\,\frac{m}{s}}{1\ m} = \mathbf{0.502\ N/m^2}$$

(+0.5 points)