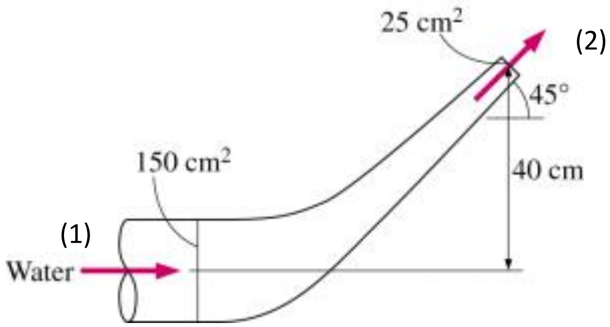
October 31, 2016

NAME

Quiz 9. A reducing elbow shown in Figure is used to deflect water (ρ = 998 kg/m^3) flow at a rate of 0.03 m^3/s in a horizontal pipe upward by an angle θ = 45° from the flow direction while accelerating it. The elbow discharges water into the atmosphere ($p_2 = 0$). The cross-sectional area of the elbow is 150 cm^2 at the inlet and 25 cm^2 at the exit. The elevation difference between the centers of the exit and the inlet is 40 cm. Determine (a) the mass flow rate $\dot{m}$ and water velocity at sections 1 and 2, (b) the pressure at section 1, and (c) the horizontal component of the anchoring force, F_{Ax}, needed to hold the elbow in place. Assume frictionless, incompressible and steady flow.



Momentum equation:

$$\Sigma \underline{\boldsymbol{F}} = \frac{\partial}{\partial t}\int_{CV} \underline{\boldsymbol{V}}\rho d\cancel{V} + \int_{CS} \underline{\boldsymbol{V}}\rho\underline{\boldsymbol{V}}\cdot d\underline{\boldsymbol{A}}$$

Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

Note: Attendance (+2 points), format (+1 point)

Solution:

a) Continuity:

$$\dot{m} = \rho Q = \left(998\frac{kg}{m^3}\right)\left(0.03\frac{m^3}{s}\right) = \boldsymbol{30\ kg/s}$$

(+0.5 point)

$$V_1 = \frac{Q}{A_1} = \frac{0.03\ m^3/s}{0.015\ m^2} = \boldsymbol{2\ m/s}; \qquad V_2 = \frac{Q}{A_2} = \frac{0.03\ m^3/s}{0.0025\ m^2} = \boldsymbol{12\ m/s}$$

(+0.5 point)

b) Bernoulli equation:

$$p_1 = p_2 + \frac{1}{2}\rho(V_2^2 - V_1^2) + \gamma(z_2 - z_1)$$

(+2 point)

$$p_1 = (0) + \frac{1}{2}\left(998\frac{kg}{m^3}\right)\left(\left(12\frac{m}{s}\right)^2 - \left(2\frac{m}{s}\right)^2\right) + \left(9790\frac{N}{m^3}\right)(0.4\ m) = \boldsymbol{74\ kPa}$$

(+0.5 point)

c) x-momentum:

$$F_{Ax} + p_1A_1 - p_2A_2 = \left(-\underbrace{\rho V_1 A_1}_{\dot{m}}\right)(V_1) + \left(\underbrace{\rho V_2 A_2}_{\dot{m}}\right)(V_2\cos 45°)$$

$$F_{Ax} = \dot{m}(V_2\cos 45° - V_1) - p_1A_1$$

(+3 points)

$$F_{Ax} = \left(30\frac{kg}{s}\right)\left(\left(12\frac{m}{s}\right)\cos 45° - \left(2\frac{m}{s}\right)\right) - \left(74{,}000\frac{N}{m^2}\right)(0.015\ m^2) = \boldsymbol{915\ N}$$

(+0.5 point)