3.3

3.3 Water flows steadily through the variable area horizontal pipe shown in Fig. P3.3. The velocity is given by $\mathbf{V} = 10(1 + x)\hat{\mathbf{i}}$ ft/s, where x is in feet. Viscous effects are neglected. **(a)** Determine the pressure gradient, $\partial p/\partial x$, (as a function of x) needed to produce this flow. **(b)** If the pressure at section (1) is 50 psi, determine the pressure at (2) by: (i) integration of the pressure gradient obtained in **(a)**; (ii) application of the Bernoulli equation.

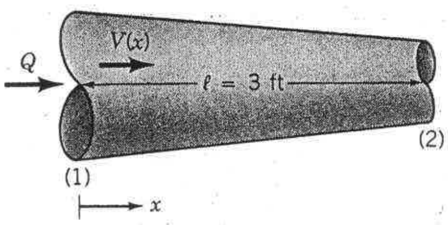

FIGURE P3.3

(a) $-\gamma \sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial V}{\partial s}$ but $\theta = 0$ and $V = 10(1+x)$ ft/s

$$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s} \quad \text{or} \quad \frac{\partial p}{\partial x} = -\rho V \frac{\partial V}{\partial x} = -\rho\,(10(1+x))(10)$$

Thus, $\frac{\partial p}{\partial x} = -1.94 \frac{\text{slugs}}{\text{ft}^3}\left(10\,\frac{\text{ft}}{\text{s}}\right)^2(1+x)$, with x in feet

$$= -194(1+x)\,\frac{\text{lb}}{\text{ft}^3}$$

(b)(i) $\frac{dp}{dx} = -194(1+x)$ so that $\int_{p_1 = 50\,\text{psi}}^{p_2} dp = -194\int_{x_1=0}^{x_2=3}(1+x)\,dx$

or $p_2 = 50\,\text{psi} - 194\left(3 + \frac{3^2}{2}\right)\frac{\text{lb}}{\text{ft}^2}\left(\frac{1\,\text{ft}^2}{144\,\text{in.}^2}\right) = 50 - 10.1 = \underline{39.9\ \text{psi}}$

(ii) $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$ or with $z_1 = z_2$

$p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2)$ where $V_1 = 10(1+0) = 10\,\frac{\text{ft}}{\text{s}}$

$V_2 = 10(1+3) = 40\,\frac{\text{ft}}{\text{s}}$

Thus,

$$p_2 = 50\,\text{psi} + \frac{1}{2}\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)(10^2 - 40^2)\frac{\text{ft}^2}{\text{s}^2}\left(\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right) = \underline{39.9\ \text{psi}}$$