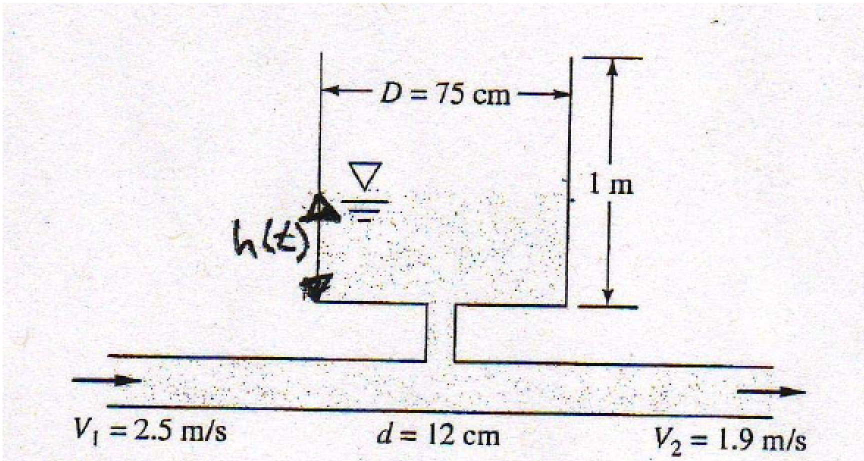
3.12 The pipe flow in Fig. P.3.12 fills a cylindrical tank as shown. At time $t = 0$, the water depth in the tank is $30cm$. Estimate the time required to fill the remainder of the tank.



P3.12

Solution:

$$\begin{aligned} 0 &= \frac{d}{dt}\int_{CV} \rho dV - \rho Q_1 + \rho Q_2 \\ &= \frac{d}{dt}\int_{CV} \rho dV - \rho V_1 \frac{\pi d^2}{4} + \rho V_2 \frac{\pi d^2}{4} \end{aligned}$$

where

$$\frac{d}{dt}\int_{CV} \rho dV = \rho \frac{\pi D^2}{4}\frac{dh}{dt}$$

$$V(t) = h(t)\frac{\pi D^2}{4}$$

Therefore,

$$\begin{aligned} 0 &= \frac{\rho \pi D^2}{4}\frac{dh}{dt} + \rho \frac{\pi d^2}{4}(V_2 - V_1) \\ \therefore \frac{dh}{dt} &= \left(\frac{d}{D}\right)^2 (V_2 - V_1) = 0.0153 \\ dt &= \frac{dh}{0.0153} = \frac{0.7}{0.0153} = 46s \end{aligned}$$

Alternate solution approach: Assume steady flow, one inlet and two exits with uniform flow.

$$0 = -Q_1 + Q_2 + Q_3$$

$$Q_3 = Q_1 - Q_2 = \frac{V_3}{dt}$$

$$dt = \frac{V_3}{Q_1 - Q_2} = \frac{dh\frac{\pi D^2}{4}}{\frac{\pi D^2}{4}(V_1 - V_2)} = 46s$$