5.104

5.104 A siphon is used to draw water at 70°F from a large container as indicated in Fig. P5.104. The inside diameter of the siphon line is 1 in. and the pipe centerline rises 3 ft above the essentially constant water level in the tank. Show that by varying the length of the siphon below the water level, h, the rate of flow through the siphon can be changed. Assuming frictionless flow, determine the maximum flowrate possible through the siphon. The limiting condition is the occurrence of cavitation in the siphon. Will the actual maximum flow be more or less than the frictionless value? Explain.

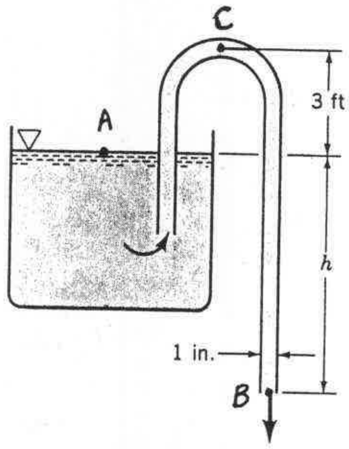

FIGURE P5.104

The flowrate, Q, can be determined with

$$Q = A_B V_B = \frac{\pi D_B^2}{4} V_B \qquad (1)$$

To obtain V_B we apply the energy equation (Eq. 5.82) between points A and B in the sketch above to obtain

$$\cancelto{0}{\frac{p_B}{\rho}} + \frac{V_B^2}{2} + g z_B = \cancelto{0}{\frac{p_A}{\rho}} + \cancelto{0}{\frac{V_A^2}{2}} + g z_A + \cancelto{0}{w_{\substack{shaft\\ net\ in}}} - loss \qquad (2)$$

or

$$\frac{V_B^2}{2} = g(z_A - z_B) - loss$$

and

$$V_B = \sqrt{2\left[g(h) - loss\right]} \qquad (3)$$

With Eq. 3 we conclude that as h varies, so does V_B and thus Q. For no loss, the maximum flow will occur when the pressure at point C is just equal to the vapor pressure of water at 0°C. We apply the energy equation (Eq. 5.82) between points A and C to get

$$\frac{p_C}{\rho} + \frac{V_C^2}{2} + g z_C = \frac{p_A}{\rho} + \cancelto{0}{\frac{V_A^2}{2}} + g z_A + \cancelto{0}{w_{\substack{shaft\\ net\ in}}} - \cancelto{0}{loss} \qquad (4)$$

Using absolute instead of gage pressures we obtain with Eq. 4

$$V_C = \sqrt{2g(z_A - z_C) + \frac{p_A - p_C}{\rho}}$$

or

$$V_C = \sqrt{2\left(9.81\,\frac{m}{s^2}\right)(-3\,ft)\left(0.3048\,\frac{m}{ft}\right) + \frac{\left(101{,}000\,\frac{N}{m^2} - 1228\,\frac{N}{m^2}\right)}{\left(999.7\,\frac{kg}{m^3}\right)\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = 9.048\,\frac{m}{s}$$

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Since

$$Q = A_c V_c = \frac{\pi D_c^2}{4} V_c$$

we have for the maximum flowrate through the siphon,

$$Q = \frac{\pi (1 \text{ in.})^2}{4 \left(144 \frac{\text{in.}^2}{\text{ft}^2}\right)} \left(0.3048 \frac{\text{m}}{\text{ft}}\right)^2 \left(9.048 \frac{\text{m}}{\text{s}}\right) = \underline{\underline{4.58 \times 10^{-3} \frac{\text{m}^3}{\text{s}}}}$$

With Eqs. 3 and 4 we conclude that any loss would act to lower the value of V in the siphon and thus make the actual maximum flowrate with friction less than the maximum flowrate without friction.