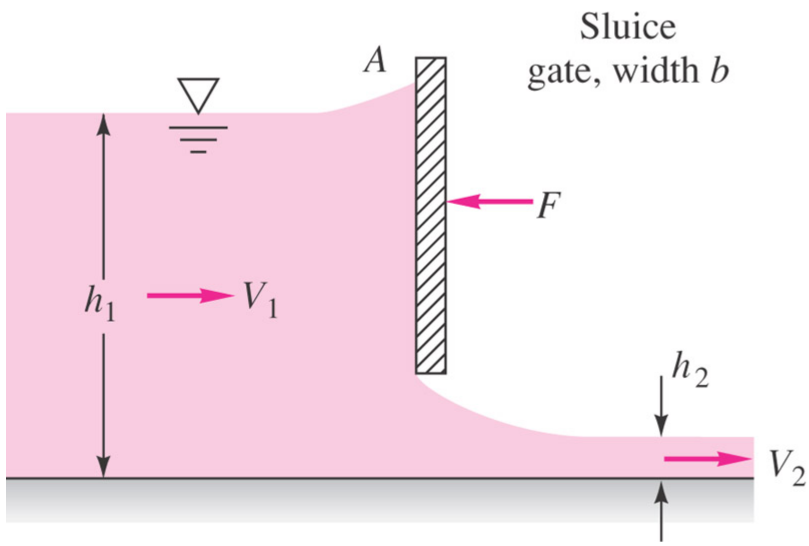
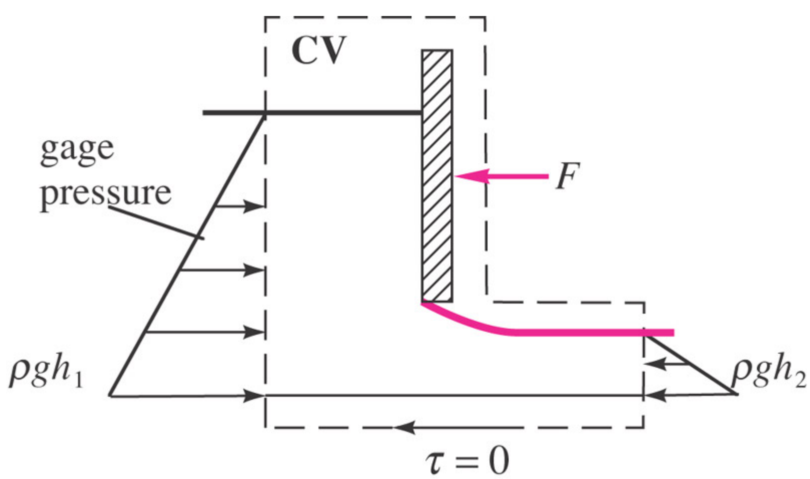
EXAMPLE (Sluice gate)

A sluice gate controls flow in open channels. At sections 1 and 2, the flow is uniform and the pressure is hydrostatic. Neglecting bottom friction and atmospheric pressure, decide the velocities V_1 and V_2, and the horizontal force F required to hold the gate if $h_1 = 6\ \mathrm{m}$, $h_2 = 1\ \mathrm{m}$, and $b = 5\ \mathrm{m}$.



Solution

Choose a control volume that cuts through known regions (section 1 and section 2, the bottom and the atmosphere) and that cuts along regions where unknown information is desired (the gate, with its force F).



Assume steady incompressible flow with no variation across the width b. The inlet and outlet mass flows balance:

$$\dot{m} = \rho V_1 h_1 b = \rho V_2 h_2 b$$

or

$$V_2 = \frac{h_1}{h_2} V_1$$

We may use gage pressures for convenience because a uniform atmospheric pressure causes no force. With x positive to the right, equate the net horizontal force to the x-directed momentum change:

$$\sum F_x = -F_{gate} + \bar{p}_1 A_1 - \bar{p}_2 A_2 = \dot{m}(V_2 - V_1)$$

or

$$-F_{gate} + \gamma \frac{h_1}{2}(h_1 b) - \gamma \frac{h_2}{2}(h_2 b) = \dot{m}\left(\frac{h_1}{h_2} V_1 - V_1\right)$$

$$\dot{m} = \rho h_1 b V_1$$

Solve for F_{gate} and eliminate V_2 using the mass flow relation. The desired result is:

$$F_{gate} = \frac{1}{2}\rho g b h_1^2 \left[1 - \left(\frac{h_2}{h_1}\right)^2\right] - \rho h_1 b V_1^2 \left(\frac{h_1}{h_2} - 1\right)$$

Now, if we apply energy (Bernoulli) equation to the two points at the free surface ($p_1 = p_2 = 0$) before and after the gate:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_2$$

Eliminate V_2 using the mass flow relation, then:

$$V_1 = \sqrt{\frac{2g(h_1 - h_2)}{(h_1/h_2)^2 - 1}} = \sqrt{\frac{2gh_2}{1 + h_1/h_2}}$$

where, $h_1/h_2 = 6$

$$V_1 = \sqrt{\frac{2\times 9.81\mathrm{m/s^2}\times 1\,m}{1+6}} = \mathbf{1.674\,m/s} \qquad \textit{Ans.}$$

and

$$V_2 = \frac{h_1}{h_2} V_1 = 6 \times 1.674\,\mathrm{m/s} = \mathbf{10.045\,m/s} \qquad \textit{Ans.}$$

Finally the gate force is:

$$F_{gate} = \frac{1}{2}\rho g b h_1^2 \left[1 - \left(\frac{h_2}{h_1}\right)^2\right] - \rho h_1 b V_1^2 \left(\frac{h_1}{h_2} - 1\right)$$

$$= \frac{1}{2} \times 998 \times 9.81 \times 5 \times 6^2 \times \left[1 - \left(\frac{1}{6}\right)^2\right]$$

$$-998 \times 6 \times 5 \times 1.674^2 \times (6 - 1) = \mathbf{437\ kN} \qquad \textit{Ans.}$$